Damping material distribution analysis for structural-acoustic optimization

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Abstract. The damping material topology optimization problem of structural-acoustic radiation is discussed. According to the artificial material hypothesis, a novel optimization approach of the damping materials distribution method is introduced. The mathematical formulation about damping material topology optimization is established. Finally, a hexahedral box structures for example, the distribution topology optimization method is illustrated. With topology optimization, material conversion between damping and artificial materials is achieved. Numerical results indicate that the proposed artificial material hypothesis for topology optimization is effective and feasible.

1. Introduction

The structural-acoustic radiation problems are a hot research issue in engineering design and manufacturing. The aim of acoustic radiation optimization is to minimize the acoustic radiation from a vibration structure in the acoustic medium. Several techniques, such as active, passive, and semi-active methods, are utilized to reduce vibration and acoustic radiation. As a kinds of passive approach, attachment of damping material on the structure surface, which is relatively simple and economical, is widely utilized in engineering design.

Topology optimization of a structure, i.e., the arrangement of materials or positioning of structural elements, is crucial because of its optimality. According to topology optimization, optimal modifications of holes and connectivity in the design domain can be achieved; these modifications can be implemented by redistributing the structure materials. Topology optimization of continuum structures first appeared in literature about 20 years ago in the study of Bendsøe and Kikuchi, which employed homogenization material theory [1].

According to the SIMP method, the topology optimization of laminated composite structures for minimization of acoustic radiation power is studied, in which the element-wise constant densities are defined as design variables [2]. The acoustic radiation level can be reduced in a certain part of the room by the optimized distribution of reflecting materials in the design domain along the ceiling or by the distribution of absorbing and reflecting materials along walls [3]. To establish an effective optimal damping treatment, topology optimization for damping layout is analyzed, in which the objective function is maximization of the modal loss factor[4]. The acoustic pressure on a prescribed reference plane/domain in the acoustic field is minimized by applying two-phase damping material distribution optimization, and the penalization models with respect to the acoustic transformation matrix and/or the damping matrix are proposed [5]. To reduce acoustic pressure in acoustic domain, the thickness distribution optimization problem of a multilayered structure is analyzed, in which a continuous approximation method is applied [6].

2. Structural-acoustic radiation formulations

In the structural-acoustic analysis, the structural response (harmonic normal velocity) defining as the acoustic boundary condition, the acoustic radiation behaviors (acoustic pressure and acoustic radiation power) are calculated. Harmonic external loading is expressed as F(x,t). By assuming that fluid-structural coupling is weak, and it can be neglected. In structure domain Ω^S, the differential equation governs the behavior of the structural dynamic system can be written as:

$$[\mathrm{M}]\{\ddot{\mathrm{U}}\}+[\mathrm{C}]\{\ddot{\mathrm{U}}\}+[\mathrm{K}]\{\mathrm{U}\}=\mathrm{F}(x,t), \qquad x\in\Omega^{\mathrm{S}}, t>0, \tag{1}$$

where {U} is the nodal displacement vector matrix, [M] is the structural mass matrix, [K] is the structural stiffness matrix, and [C] is the viscous damping matrix.

Only the low–middle frequency domain exterior acoustic radiation problem is discussed in this paper. Acoustic radiation field quantities are relevant to acoustic radiation pressure and acoustic radiation power. In the acoustic field domain, acoustic power describes the energy flow of an assumed integral surface and it can be defined as:

$$\Pi=\frac{1}{2}\int_{\mathrm{S}}\mathrm{Re}\{\mathrm{p}(r)\cdot \mathrm{v}_{\mathrm{n}}^{*}(r)\}\mathrm{d}S, \tag{2}$$

where Re represents the use of the real part of a complex variable, $p(r)$ is acoustic pressure at the acoustic field point, $\mathrm{v}_{\mathrm{n}}^{*}(r)$ is the field-point normal complex conjugate operator velocity of a fluid particle, and S is the integral surface of the acoustic computational domain. By omitting acoustic transmission loss and acoustic absorption of the acoustic boundary and acoustic source, the acoustic radiation power of the exterior acoustic field becomes equal to acoustic source radiation power.

Acoustic radiation power is a function of radiation frequency, which varies over frequency band. The objective of structural-acoustic radiation optimization is to minimize radiation power over this band. The frequency averaged acoustic radiation power over this band can be obtained by integrating Π over the frequency band as follows:

$$\Pi_a=\frac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2}\Pi(\omega_{\mathrm{i}})\mathrm{d}\omega\cong\frac{1}{\omega_2-\omega_1}\sum_{\mathrm{i}=1}^{\mathrm{m}}\Pi(\omega_{\mathrm{i}})\Delta\omega_{\mathrm{i}}, \tag{3}$$

where ω_2 and ω_1 are the upper and lower bounds of the frequency band, respectively. $\Pi(\omega_{\mathrm{i}})$ is the acoustic power in frequency ω_{i}.

3. Dynamic properties of damping materials

The continuum structure is meshed into many finite elements at elemental level. Thickness and material topological distribution in each element are defined as design variables. For a vibration structure covered with damping materials, the system consists of a base material layer structure and a damping material layer. Matrices [M], [K], and [C] can be further expressed as:

$$[\mathbf{M}]^{\mathrm{e}}=[\mathbf{M}]_1^{\mathrm{e}}+[\mathbf{M}]_2^{\mathrm{e}}, [\mathbf{K}]^{\mathrm{e}}=[\mathbf{K}]_1^{\mathrm{e}}+[\mathbf{K}]_2^{\mathrm{e}}, [\mathbf{C}]^{\mathrm{e}}=[\mathbf{C}]_1^{\mathrm{e}}+[\mathbf{C}]_2^{\mathrm{e}}, \tag{4}$$

where $[\mathbf{M}]_1^{\mathrm{e}}, [\mathbf{K}]_1^{\mathrm{e}}$, and $[\mathbf{C}]_1^{\mathrm{e}}$ are the element stiffness matrix, element mass matrix, and element damping matrix of the base structure material layer, respectively. Their parameters remain unchanged during optimization. $[\mathbf{M}]_2^{\mathrm{e}}$, $[\mathbf{K}]_2^{\mathrm{e}}$, and $[\mathbf{C}]_2^{\mathrm{e}}$ are the element stiffness matrix, element mass matrix, and element damping matrix of the damping material layer, respectively. The damping effect of the conventional base material layer is very small compared with that of the damping material layer and can thus be neglected. In Eq. (4), if $[\mathbf{M}]_2^{\mathrm{e}}=0$, $[\mathbf{K}]_2^{\mathrm{e}}=0$, and $[\mathbf{C}]_2^{\mathrm{e}}=0$, no damping material layer exists on the base structure element.

4. Artificial material hypothesis

Topology optimization of damping material distribution on a base structure is a 0-1 integer discrete design variable optimization problem. In the design domain, if there is an existing damping material layer, the design variable is expressed as 1. Without an existing damping material layer, the design variable is expressed as 0. If we define the domain without an existing damping material layer as a material too, the damping material distribution optimization problem is the optimal distribution of these two problems of materials in the design domain.

Elastic modulus is the main mechanical property of a structure material. Poisson's ratio, loss factor, and material density are important for material mechanical properties. Consider a material with very small elastic modulus, loss factor, and material density. When the base structure is attached to this damping material, the change in structure mechanical parameters is so small that it can be neglected; the material is then defined as artificial. With this definition, the optimization problem of damping material distribution is transformed into a problem that requires distinguishing between artificial and damping materials.

To apply the material conversion approach in topology optimization in which material elastic modulus is defined as the design variable, several presumptions about mechanical property hypothesis for artificial and damping materials are established as follows.

(1) Cellular material theory is introduced. A cellular material is attached to the base structure layer, and the thickness of the cellular material is *t*. In the present study, each design variable is defined as a cellular material, and the material is arbitrary between artificial and damping materials. For the cellular material, artificial and damping materials can be defined as candidate materials. This idea is illustrated in Fig. 2.

(2) The mechanical properties of artificial materials are defined as follows: elastic modulus (shear modulus) and density are very small, and the loss factor is zero.

(3) The material elastic modulus of the structure cellular material is regarded as a discrete design variable. The elastic modulus of the damping material is defined as E_D. The elastic modulus of artificial materials is defined as E_A, and the elastic modulus of the structure cellular material is defined as discrete design variable E_i. Damping and artificial materials are defined as candidate materials that satisfy the discrete design variable set $E_i \in \{E_D, E_A\}$. If $E_i = E_D$, the cellular material is a damping material, and if $E_i = E_A$, the cellular material is an artificial material.

(4) Material conversion between damping and artificial materials is implemented. If the cellular material is defined as an artificial material, damping material attachment on the surface of the base structure is absent. If the cellular material is defined as a damping material, then the damping material is attached on the surface of the base structure with defined thickness. With such processing, optimization of damping material distribution is conducted.

In this study, material elastic modulus and thickness (total number of plies) of damping materials are defined as discrete design variables assigned in elements. The acoustic power of a structure is defined as an objective function. In the optimization model, elastic modulus is defined as a design variable, with a change in elastic modulus E_i. All other mechanical properties of materials are modified, and optimization of damping material distribution is obtained.

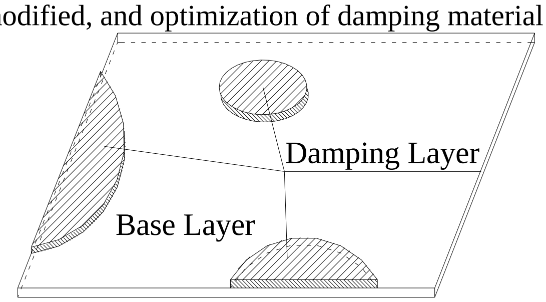

Fig.1 Sketch of damping material distribution

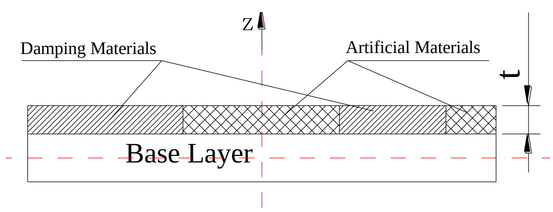

Fig.2 Method of cellular material

The artificial material hypothesis followed four steps. First, the designed domain is replaced with a cellular material so that artificial and damping materials could be defined as candidate materials. Second, a finite element model is constructed for the cellular material. Third, an optimization model for damping material distribution optimization is established by defining the design variables and objective function. Finally, thickness optimization of the damping material is carried out.

5. Damping material topology distribution optimization formulations

Considering that the structural acoustic radiation power equation has been defined, the structural-acoustic optimization problem can be converted into searching a group of design variables in design scope that minimize acoustic radiation power.

The optimal design of damping material distribution is more complicated than conventional structure designs. The objective is to minimize acoustic radiated power over a frequency band, which can be obtained by integrating Π over a frequency band. Then, the structural-acoustic optimization mathematical formulation based on artificial material hypothesis approach can be stated as

$$\begin{aligned}
\text{Find}\quad & E=[E_1,E_2,\cdots,E_\mathrm{i}]^\mathrm{T},\ i=1,2,\cdots,k\,; \\
& t=[t_1,t_2,\cdots,t_\mathrm{i}]^\mathrm{T},\ i=1,2,\cdots,k\,; \\
\text{Min}\quad & \prod\nolimits_a=\frac{1}{\omega_2-\omega_1}\sum_{\mathrm{i}=1}^{\mathrm{m}}\Pi(\omega_\mathrm{i})\Delta\omega_\mathrm{i}\,; \\
\text{s.t.}\quad & \sum_{i=1}^{k}(t_\mathrm{iD}\cdot\rho_\mathrm{D}\cdot s_\mathrm{iD}+t_\mathrm{iA}\rho_\mathrm{A}\cdot s_\mathrm{iA})\le G_0\,; \\
& t_\mathrm{D}^\mathrm{l}\le t_\mathrm{iD}\le t_\mathrm{D}^\mathrm{u},\ t_\mathrm{A}^\mathrm{l}\le t_\mathrm{iA}\le t_\mathrm{A}^\mathrm{u}\,; \\
& g(E_\mathrm{i},t_\mathrm{i})\le 0; \\
& E_\mathrm{i}\in\{E_\mathrm{D},E_\mathrm{A}\}.
\end{aligned}\tag{5}$$

The objective function is derived from the criteria for damping material topology distribution optimization formulations. $E=[E_1, E_2, \ldots, E_\mathrm{i}]^\mathrm{T}$ is an i-dimensional elastic modulus design variable vector. $E_\mathrm{i}\in\{E_\mathrm{D}, E_\mathrm{A}\}$ is the allowable elastic modulus for the materials set. In this case, these equations indicate damping and artificial materials, respectively. This parameter defines the shear modulus, loss factor, and density of two candidate materials. $t=[t_1, t_2\ldots, t_\mathrm{i}]^\mathrm{T}$ is the k-dimensional design variable of cellular material thickness. Πa is acoustic radiation power at the frequency band of interest, which is defined by Eq. (5). t_iD, ρ_D, and s_iD are damping material plate thickness, material density, and plate area, respectively. t_iA, ρ_A, and s_iA are artificial material plate thickness, material density, and plate area, respectively. G_0 is damping material mass in the initial design. t_D^u and t_D^l correspond to the lower thickness and upper thickness bounds of ith design variable t_iD of the damping material, respectively. t_A^u and t_A^l correspond to the lower thickness and upper thickness bounds of ith design variable t_iA of the artificial material, respectively. $g(E_\mathrm{i},t_\mathrm{i})\le 0$ includes the other constraints of formulation, such as structural dynamic stress, structural displacement, and structural fundamental frequency.

6. Numerical experiments and discussions

To demonstrate damping topology optimization for the structural-acoustic problem with artificial material hypothesis, a hexahedral box structure under external harmonic excitation in air is considered. Both finite element method (FEM) and boundary element method (BEM) are applied.

6.1. Model description

Figure 3 shows the hexahedral box. The rectangular Cartesian coordinate system (x, y, and z) is applied to describe the loadings and structure. The coordinate origin is located at the bottom center of the hexahedron. Structural vibration is excited by time harmonic loading with the prescribed excitation frequency, amplitude, and direction. The design parameters of the hexahedral box structure are as follows: height of 0.6 m, length in x direction is 0.6 m, and length in y direction is 0.6 m.

6.2. Calculation parameter

In the initial design, the material of the hexahedral box structure is steel, and the plate thickness is 5 mm. The covering with a damping material is labeled as SA on the box structure surface. The uniform thickness of the damping material layer is 2 mm. The attachment of the damping material and steel material structure is an ideal connection without any defects. According to the artificial material hypothesis, the steel, damping, and artificial materials are all homogeneous and isotropic. Table 1 shows the mechanical properties of the steel, damping, and artificial materials.

Table. 1 The mechanical properties of materials

Material	Elasticity modulus/MPa	Poisson ration	Loss factor	Density/kg·m^{-3}
Steel	E=210000	0.3	η=0	7800
Damping materials	E=280	0.49	η=0.3	1800
Artificial materials	E=0.01	0.3	η=0	1

In dynamic response analysis, the four bottom corners of the hexahedral box are supported, as shown in Fig. 3. Time-harmonic concentrated loading $p(t)=P\sin(2\pi ft)$ with prescribed amplitude P < 600, 0, −800 > N is applied to the center of the box top surface, and the frequency of loading is f = 81 Hz. The velocity of the node is defined as boundary condition for acoustic radiation analysis.

The admissible design domain of the hexahedral box structure is discretized by the finite element mesh, as shown in Fig. 3. The damping material layer is modeled with two types of elements. In the first step, damping material distribution optimization is carried out, and the cellular material is modeled with four node shell elements. In the second step, thickness optimization of damping is carried out, and the cellular material is modeled with eight node solid elements.

In damping material distribution topology optimization, the entire surface of the base structure is defined as the design domain for topology optimization. A total of 500 quadrilateral elements and 521 nodes exist in the finite element model. Each element in the damping layer is assigned a design variable whose value indicates the composition in the discretized domain. Consequently, the total number of design variables (N) equals the number of cellular materials in the FEM model.

In structural-acoustic analysis, the acoustic properties of fluid (i.e., air) are isotropic and homogeneous. The density is ρ=1.225 kg/m^3, the speed of sound is c=343m/s, the acoustic power reference value is $W_0=1\times10^{-12}$ Watt, and the acoustic pressure reference value is $p_0=1\times10^{-6}$ Pa.

6.3. *Acoustic topology optimization of the hexahedral box*

According Eq. (2), the structural-acoustic radiation power of the structure and the acoustic pressure of any field in the frequency range can be calculated. Given that each of the cellular material is defined as a design variable, the numerical example comprised 500 design variables. According to Eq. (5), structural-acoustic topology optimization of the hexahedral box can be changed as follows:

$$\begin{aligned}
&\text{Find} \quad E=[E_1,E_2,\cdots,E_{\mathrm{i}}]^{\mathrm{T}},\ \mathrm{i}=1,2,\cdots,500; \\
&\qquad\quad t=[t_1,t_2,\cdots,t_{\mathrm{i}}]^{\mathrm{T}},\ \mathrm{i}=1,2,\cdots,500; \\
&\text{Min} \quad \prod\nolimits_a=\frac{1}{\omega_2-\omega_1}\sum_{\mathrm{i}=1}^{\mathrm{m}}\Pi(\omega_{\mathrm{i}})\Delta\omega_{\mathrm{i}}\le 135.4\mathrm{dB}; \\
&\text{s.t.} \quad \sum_{i=1}^{k}(t_{\mathrm{iD}}\cdot\rho_{\mathrm{D}}\cdot s_{\mathrm{iD}}+t_{\mathrm{iA}}\rho_{\mathrm{A}}\cdot s_{\mathrm{iA}})\le 6.48\mathrm{kg}; \\
&\qquad\quad 0.001\le t_{\mathrm{iD}}\le 0.01,\ 0.001\le t_{\mathrm{iA}}\le 0.01; \\
&\qquad\quad E_{\mathrm{i}}\in\{280\mathrm{MPa},0.01\mathrm{MPa}\}.
\end{aligned} \tag{6}$$

The parameters of Eq. (6) have the same meaning as those in Eq. (5). According to Eq. (5), if material elastic modulus E_{i} is changed, then the other mechanical properties (shear modulus, Poisson's ratio, loss factor, density, and so on) will be transformed too.

6.4. *Damping material topology optimization*

In damping distribution topology optimization, the variation in acoustic radiation power in frequency ranges from 0 Hz to 300 Hz; the frequency step size is 1 Hz.

The covering area of damping after topology is 40% of the initial design. Generally, the thickness of each lamina is the same and not varied during topology optimization. After topology optimization, the thickness of the damping materials is 5 mm. Fig. 4 shows the distribution of the damping materials on the hexahedral box surface. The black domain denotes damping material distribution.

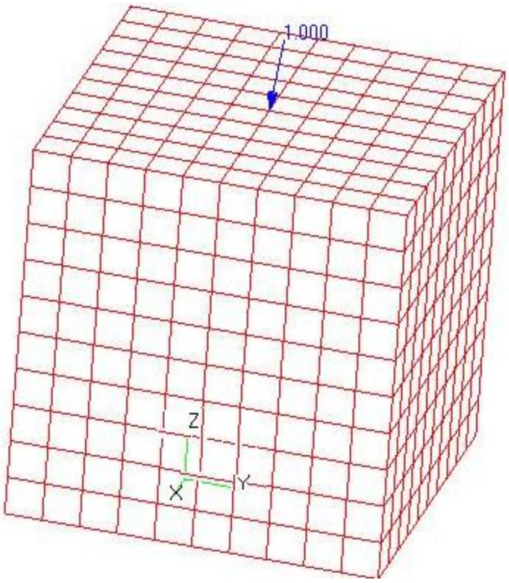

Fig.3 FEM model of the hexahedral box

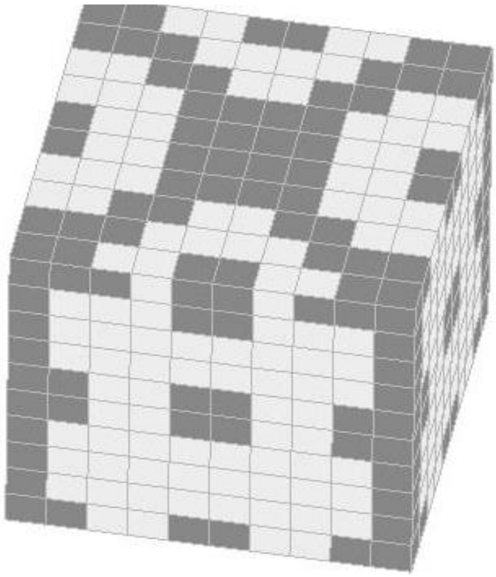

Fig.4 Topological distribution of damping materials

With topology optimization, the acoustic radiation of the hexahedral box structure is reduced. Fig. 5 shows that the peak amplitude value of acoustic power is reduced by 7.5 dB. Compared with the initial design, the optimization effect is obvious.

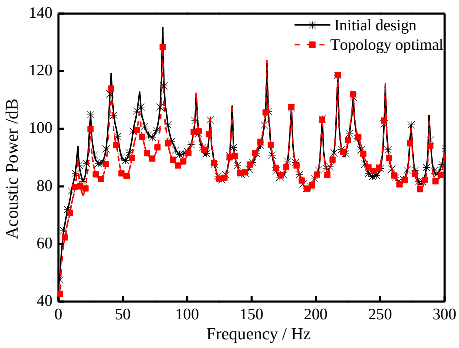

Fig. 5. Comparison of acoustic power

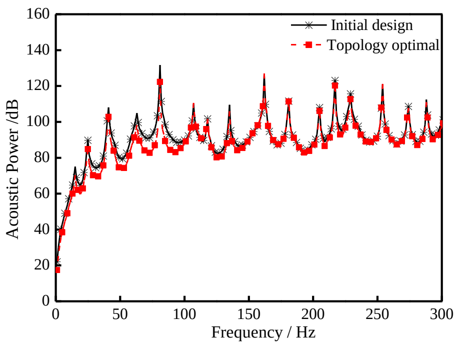

Fig. 6. Comparison of acoustic pressure

Fig. 6 shows the acoustic pressure magnitude of the acoustic field point, which is located at (0, 0, 5). Compared with the initial design, the peak amplitude value of acoustic pressure is reduced by approximately 10 dB, and the optimization effect is obvious.

6.5. Thickness optimization of damping

Damping material thickness optimization is performed to obtain a better optimization effect. Solid elements, which represent damping material patches, are obtained by extruding the corresponding four-node plate elements. The thickness of each solid element of the damping material is selected as the design variable. Considering damping attachment technology, the thickness of the damping material is a discrete variable, and thickness is limited to a small set variable, i.e., $t\in\{1mm,2mm,3mm,4mm,5mm,6mm,7mm,8mm,9mm,10,mm\}$.

With damping material thickness optimization, the thickness near the external loading application point increases, whereas that in the other domains decreases. Fig. 7 shows the acoustic radiation power magnitude of the initial design, topology optimization, and thickness optimization. Compared with topology optimization, the peak amplitude value of acoustic power is reduced by 3.0 dB by thickness optimization; thickness optimization has a certain effect.

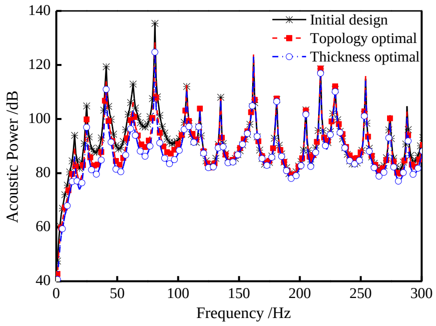

Fig. 7. Comparison of acoustic power

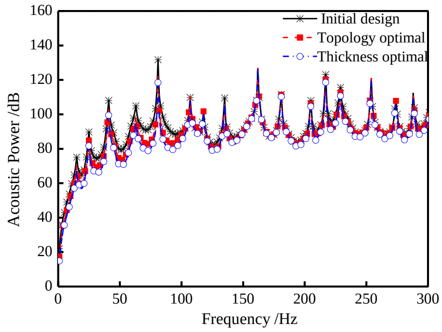

Fig. 8. Comparison of acoustic pressure

Fig. 8 shows the acoustic pressure magnitude of the acoustic field point, which is located at (0, 0, 5). Compared with topology optimization, the peak amplitude value of acoustic pressure is reduced by 3.0 dB by thickness optimization; the thickness optimization effect is obvious.

Comparison of Figs. 5 to 8 shows that acoustic radiation power is reduced significantly after damping material distribution topology and damping material thickness optimizations. Optimization achieved the purpose of reducing vibration and noise.

7. Conclusions

With acoustic radiation power defined as a performance index, topology optimization of damping materials is conducted. In the numerical examples, the peak values of the dynamic response parameter are reduced significantly in the frequency ranges of interest. The best mechanical behavior of the damping material plays a full role in topology optimization. The application of the thickness optimization approach leads to a better result. This research provides an effective and feasible method to solve multi-material distribution optimization problems.

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