A method for three-dimensional precise modelling of spiral bevel gear

Zhao Hua[1]

[1] Nan Yang Institute of Technology, Nan yang, 473004, China

nylm1996@126.com

Abstract. A method of three-dimensional precise modelling is given based on the basic meshing principle of spiral bevel gear. With the help of the spherical involute equation and the base conic helix equation, a series of data points of tooth profile and tooth trace are calculated by programming with MATLAB, according to the known basic parameters of spiral bevel gear. Then the exact curve is generated, and it is introduced into the UG environment for modelling. Taking the automobile rear axle driven gear as an example, the concrete method of the 3D solid modelling is illustrated. It is proved that the 3D accurate modelling method is feasible and effective by its application in the measurement and evaluation of the tooth surface error of the spiral bevel gear.

1. Introduction

Spiral bevel gears are important transmission parts in automobiles, machine tools, metallurgy and mining machinery. Error detection and evaluation can not be made by means of general gear measuring device and evaluation method because of the complex shape of tooth surface. According to reference [1], the 3D solid model of spiral bevel gear can be regarded as an ideal factor for the global error detection and evaluation of tooth surface, and the difference surface can be used to quantitatively reflect the overall error of gear tooth surface. In this paper, a method of three-dimensional precise modeling is given based on the basic meshing principle of spiral bevel gear. By means of the spherical involute and base conical spiral，a series of data points of tooth profile and tooth trace are calculated and collected with MATLAB software, according to the basic parameters of spiral bevel gear and cutter head parameters. Then an accurate three-dimensional solid model is constructed to determine the ideal element of the spiral bevel gear in UG. Taking the rear axle driven gear as an example, the steps of 3D solid modeling and the application of 3D model in error detection and evaluation of spiral bevel gear are explained in detail.

The key of gear modeling lies in the drawing of the spherical involute tooth profile [2]. The coordinate value of any number of tooth profile points can be obtained accurately by MATLAB software, which has significant advantages in matrix operation and numerical calculation. Then the tooth profile point coordinate data file is imported into UG to make up for the shortcomings of UG itself, that is, the number of fitting points of the involute (spline curve) generated by the equation is less and not accurate. After that the three-dimensional modeling of the tooth profile surface of the spiral bevel gear is completed by the powerful complex surface modeling function of UG software [3].

2. Spherical involute equation

The theoretical tooth profile of spiral bevel gear is spherical involute, the formation of a spherical involute is a pure rolling of a circular plane on a cone, and the radius of the circular plane is equal to the conical cone distance. The trajectory of any point on a circular plane is a spherical involute [4].

In the coordinate system shown in Figure 1, the coordinate system (O; X, Y, Z) is the right-handed Cartesian coordinate system with the small circular plane as the OXY surface and the small circle center as the center of the coordinate system. It is assumed that the coordinate system (O$_1$; X$_1$, Y$_1$, Z$_1$) that is consolidated with the PMB plane rotates the φ angle around the Z axis, moves up $R_b \sin \delta_b$, and then rotates the δ_b angle around the Y2 axis to arrive at the coordinate system (Op; Xp, Yp, Zp), the plane PMB coincides with the YOZ plane, P reaches P', the coordinates of the P' points are (0, $-R_b \sin \psi$, $-R_b \cos \psi$). The spherical involute equation is the reverse transformation of the P' point to the coordinate system (O; X, Y, Z).

The parameter equation of the spherical involute can be obtained by the coordinate transformation and the matrix operation, as shown below:

$$x = R_b (\sin \delta_b \cos \phi \cos \psi + \sin \phi \sin \psi)$$
$$y = R_b (\sin \delta_b \sin \phi \cos \psi - \cos \phi \sin \psi)$$
$$z = R_b \cos \delta_b \cdot (1 - \cos \psi) \tag{1}$$

Here δ_b is the base cone semiangle; r_b is the radius of a small circle, and R_b is the radius of the large circle; ϕ and ψ are the spreading angles of the small circle and the large circle, respectively; and α is pressure angle.

3. Base conical spiral equation

The base conical spiral is the guide line of the spiral bevel gear, Figure 2 shows its formation process. At the beginning, point P and point E coincide. Point P rotates around the Z axis while moving along the cone generatrix. When the length of OP increases to R_{b1}, point O moves to point P, at the same time point P rotates the θ_i angle around the Z axis to arrive at point p_i, and the trajectory of point p_i is a conical spiral [5].

The equation of a conical spiral line can be derived from Figure 2, as shown below:

$$x = \frac{T_b}{2\pi} \theta_i \sin \delta_b \cos \theta_i$$
$$y = \frac{T_b}{2\pi} \theta_i \sin \delta_b \sin \theta_i$$
$$z = -\frac{T_b}{2\pi} \theta_i \cos \delta_b \tag{2}$$

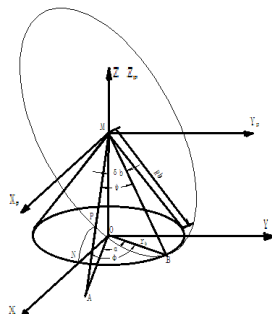Here T_b is the helical pitch of a base conical spiral.

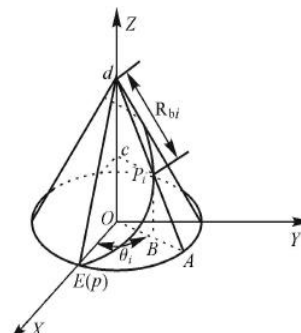Figure 1. The forming diagram of the spherical involute.

Figure 2. Schematic diagram of the formation of conical helix.

4. Three-dimensional precise modelling of spiral bevel gear

In this paper, a method of three-dimensional precise modeling is given based on the basic meshing principle of spiral bevel gear. According to the spherical involute equation and the base conic helix equation, a series of data points of tooth profile and tooth trace are calculated by programming with MATLAB, then the exact curve is generated, and it is introduced into the UG environment for modeling.

Now, taking the automobile rear axle driven gear produced by a factory as an example, three dimensional precise modeling steps of spiral bevel gear are illustrated in detail.

The basic parameters of the gear are shown in Table 1.

Table 1. The basic parameters of the gear.

Parameter	Value
Number of teeth	40
Modulus	12 mm
Offset distance of driving gear	40 mm
pitch diameter	480 mm
Average pressure angle	22.30°
Mean spiral angle	37.5°
Hand of spiral	Right
Pitch angle	78°48′
Face angle	79°18′
Root angle	74°46′
Tooth addendum	1.65 mm
Whole depth	17.01 mm

4.1. Building the gear blank model

In the UG environment, the root cone is first set up using the sketch function.

(1) The plane line of the root cone of the spiral bevel gear is first made according to the bottom diameter of the root cone, the cone height and the root cone angle, as shown in Figure 3.

(2) The plane line of the root cone is rotated around the Z axis for 360° to get the root cone of the spiral bevel gear, as shown in Figure 4.
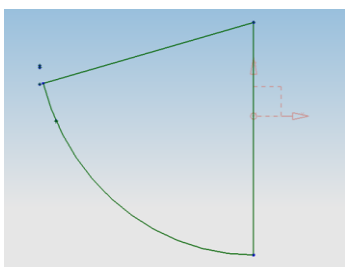

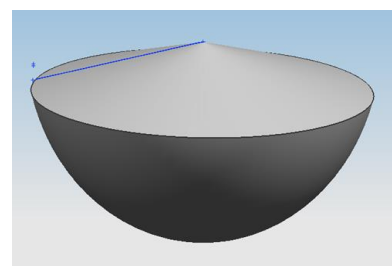Figure 3. The plane line of the root cone.

Figure 4. The root cone.

(3) Make a small circle with a radius of 141.5mm and a large circle with a radius of more than 190mm, and the two circles intersect with the root cone entity, and then stretch them, so the gear blank of the gear is obtained, as shown in Figure 5.

4.2. Generating the tooth profile of big end

The steps of generating the tooth profile are as follows：

(1) On the basis of the spherical involute equation, the spherical involute is made. The part program of concave and convex spherical involute of spiral bevel gear used by MATLAB software is as follows:

%Procedure 1:

The concave and convex spherical involute program of large gear

d_e=480;

r_e=480/2;

alpha=22.30;   %The pressure angle of the concave surface is 17.5

r_f=r_e*cos(alpha*pi/180);

f_j=80.6*pi/180;

deta=asin(cos(alpha*pi/180)*sin(f_j));

h=r/tan(deta);

r_b=r/sin(deta);

fa=linspace(0,3,100);

kesa=r*fa/r_b;

x=r_b*(sin(deta)*cos(fa)*cos(kesa)+sin(fa)*sin(kesa));

y=r_b*(sin(deta)*sin(fa)*cos(kesa)-cos(fa)*sin(kesa));

z=(r_b*cos(deta)*cos(kesa));

plot3(x,y,z)

[y',-x',-z']

The pitch circle, the addendum circle and the dedendum circle are successively made, and then the spherical involute of the concave and convex surfaces is obtained by using the MATLAB point, as shown in Figure 6.
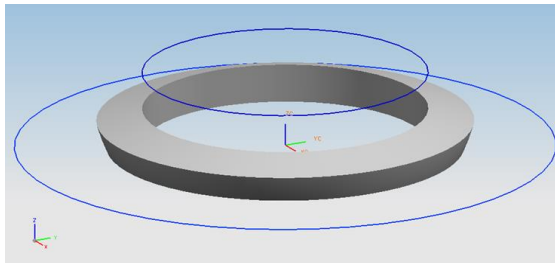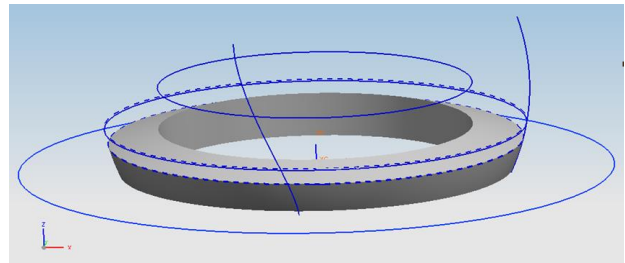

Figure 5. The gear blank.　　　　　　　　Figure 6. The generated spherical involute.

(2) An involute on the other side of the tooth profile is obtained by using the two spherical involute with the YC-ZC plane image, as shown in Figure 7.

(3) The tooth thickness S_2 on the pitch circle can be calculated using MATLAB, the calculation program is as follows:

%Procedure 2:

u=6/40;

s_1= (pi/2+2*0.39*(1-u*u)*tan(22.3*pi/180)/cos(50*pi/180)+0.265)*12

%s=(pi/2+2*x*tan(alpha)/cos(beta1)+xt)*m=(pi/2+2*0.39*(1-u*u)+0.265)*12;

%x=0.39* (1- u*u); u=6/40;

m=12;

s_2=pi*m-s_1;

(4) Take a distance from the pitch circle to make it equal to the tooth thickness, and then the generated spherical involute is rotated around the Z axis to the two ends of the tooth thickness, so the enclosed range which is surrounded by the rotated spherical involute, the addendum circle and the dedendum circle is tooth profile. The tooth profile of big end is gotten by trimming the superfluous parts, as shown in Figure 8.

4.3. Generating the guide line of tooth direction

The steps of generating the shape entity of the single tooth profile are as follows：

(1) Four conical spiral lines are generated with four endpoints as the starting point, and the four endpoints are the two endpoints of the addendum and the dedendum of the generated tooth profile, respectively. The generation of the conical spiral is also the use of MATLB, and some of the procedures for the calculation method are as follows:

```
%Procedure 3:
hudu=pi/180;
deta=74.6*hudu;
de=480;
re=480/2;
r=re;
alpha=22.3;
mj=79.3*pi/180;
rb=r/sin(deta);
h=rb*cos(mj);
dt=r*pi*tan(37.5*hudu);
w=h*2*pi/dt;
t=linspace(0,1,100);
v=w*t;
x=r*t*cos(v);
y=r*t*sin(v);
z=h*t;
plot3(x,y,z)
[x',y',z']
```
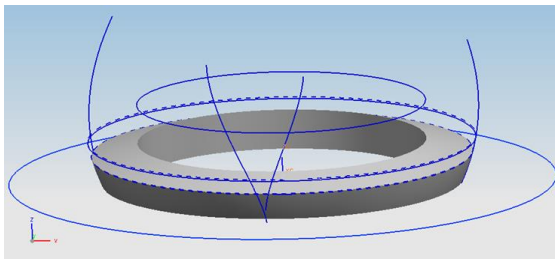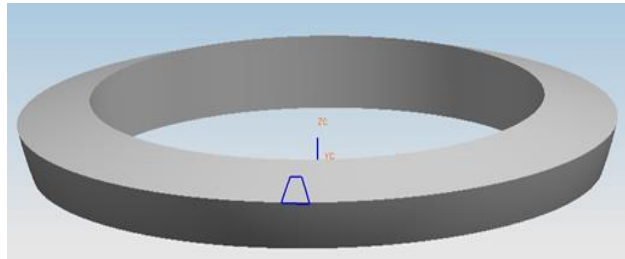

Figure 7. The mirrored spherical involute

Figure 8. The generated tooth profile of big end.

(2) Using four conical spiral lines as the guiding line of the spherical involute, the spherical involute of big end is rotated around the Z axis to the four endpoints of the tooth profile, and then using of UG's sweep and suture commands, make the spherical involute sweep along the conical spiral. The shape entity of the single tooth profile of the gear is obtained, as shown in Figure 9.

(3) The unnecessary auxiliary curves are hidden, and a circular array of the generated single teeth is carried out. So the solid model of the automobile rear axle driven gear is accurately constructed, as shown in Figure 10.

5. The application of 3D solid model in error detection and evaluation of spiral bevel gear

After building the gear entity model under the environment of UG, the path planning of the entity model can be carried out. The coordinate system which is consistent with the measuring coordinate system is established, and the coordinates of the grid nodes are the coordinates of the theoretical measurement points. The three dimensional model is used as an ideal factor, and then the error detection and evaluation of spiral bevel gear can be accomplished. Figure 11 shows the global error of

tooth surface of the automobile rear axle driven gear expressed by first order difference surface. It is based on 3D modeling to get the theoretical tooth surface. It uses the three coordinate measuring machine to measure the actual tooth surface, and according to the deviation between the actual tooth surface and the theoretical tooth surface, it is fitted out.
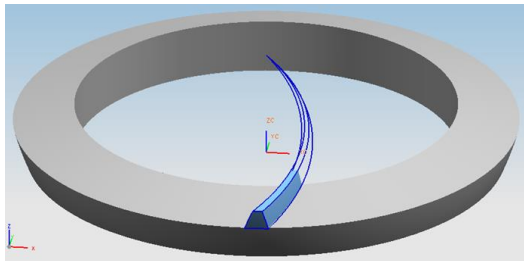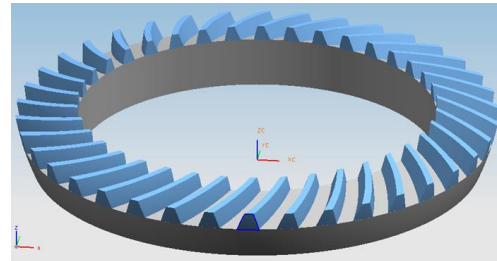

Figure 9. The generated single tooth.

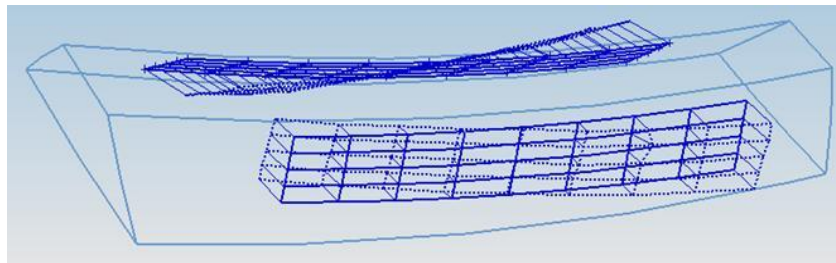Figure 10. The accurate solid model of the gear.

Figure 11. The tooth surface error expressed by the first order difference surface.

6. Conclusion

Based on the spherical involute and the base conic helix equation, the methods and steps of the three-dimensional precise modeling of the spiral bevel gear are given in this paper. The feasibility of the method is verified by the actual modeling of the automobile rear axle driven gear. The three dimensional entity model can be used as an ideal element, combined with the actual tooth surface measured by the three coordinate measuring machine, the detection and evaluation of the global error of the tooth surface of the spiral bevel gear can be realized.

References

[1]   Zhao Hua. The tooth surface error measurement and evaluation of spiral bevel gear based on three-coordinate measurement[J]. *Journal of Mechanical Transmission*, 2011, **35**(3):20-23.
[2]   Liu Pingping, Qian Danhao. Study on accurate 3D modeling of involute gear based on optimization and UG [J]. *Coal Mine Machinery*, 2011,**32**(9):227-229.
[3]   Du Xinyu, Wang Xiaolin, Yan Qianpeng. Finite element modeling and analysis of involute cylindrical gear based on Matlab and UG[J]. *Journal of Mechanical Transmission*, 2011, **36**(7):39-42.
[4]   Ji Zhenhai, Duan Jianzhong, Gu Jihua. OpenGL-based 3D parameterization modeling for involute spiral bevel gear [J]. *Journal of Engineering Design*, 2009, **16**(6):422-435.
[5]   Gu Fengmin, Zhou Liang, Duan Jianzhong. A new theoretical algorithm for spiral bevel gear [J]. *Machinery Design & Manufacture*, 2008, (12):12-13.