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A Topology Optimization Algorithm Based on the Overhang Sensitivity Analysis for Additive Manufacturing

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Abstract. The integration of topology optimization(TO) and additive manufacturing(AM) has the huge potential to expand the design freedom of parts. However, traditional topology optimization approaches are not tailored to the particular characteristics of AM processes, resulting in a large number of overhangs in these designs, which lead to use of sacrificial support structures and increase the manufacturing cost. This paper presents an AM-restrained topology optimization approach based on the overhang sensitivity analysis. The overhangs can be explicitly expressed by the design variable and considered to be a minimization goal of the optimization process, in which the overhang sensitivity is combined with other performance sensitivities to update design variables. The effectivity and robustness of the proposed method is demonstrated by the typical cantilever beam and the MBB beam, then validated through the fused deposition modelling(FDM) 3D printing.

1. Introduction

Topology optimization can find the optimal material layouts within a given design space, so as to maximize the structure performance according to the applied loads, boundary conditions, and constraints [1,2]. Compared with shape and size optimization method, topology optimization does not require any priori assumption about the material distribution.

Additive manufacturing refers to a process by which a digital 3D model is used to build up a component in layers by depositing material[3]. In contrast with the traditional manufacturing methods, AM has the superiorly near-net-shape ability and excellent flexibility of geometric design, thus it can shorten the design and manufacturing cycle and improve the material utilization, which also provides an opportunity for the physical implementation of topology optimization structures[4].

The integration of TO design method and the AM technology can play the advantages of creativity and advanced manufacturing, expand the design freedom, shorten the development cycle, and achieve the lightweight design requirements. Although AM can produce complex part, there are still some limitations, for example, the results of topology optimization usually include a large number of overhangs, which are prone to collapse or warping during some AM processing such as FDM and SLM. The overhangs require to design some support structures so as to be successfully manufactured, which will increase the manufacturing cost. As shown in Figure 1, the angle between the overhang and the construction direction exceeds the critical value(usually 40°~50°)[5]. In this regard, Brackett et al. proposed a combination of overhanging angles and overhanging distances for two-dimensional optimization models[6]; Leary et al. added the additional structure to the original optimization result to make the final geometry fully self-supporting[7]; Gaynor et al. proposed a wedge-shaped spatial filter

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to control the boundary direction of the topology optimization design[8]; Matthijs et al. proposed the use of hierarchical filtering scheme to design self-supporting structures[9].

In this paper, a novel AM-restrained topology optimization method is proposed to reduce overhanging regions, which makes the design more suitable for additive manufacturing. We use the design variable to express the overhanging structure explicitly, and combine the minimization of overhanging regions as a new objective function with the original optimization problem, which transforms the original problem into a multi-objective optimization problem. The remainder of this paper will introduce the explicit expression of the overhanging structure, the calculation of sensitivities and the mathematical model of the optimization problem. In the third section, the effectivity and robustness of the proposed method will be demonstrated.

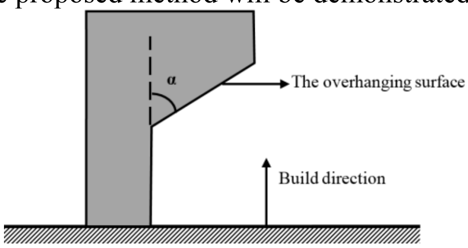


Figure 1. The overhanging surface needs to be supported.

2. Multi-objective Optimization with the Overhang Minimization

2.1. Overhang Expressed with Design Variables

In order to control the generation of overhangs, it is necessary to find a way that can be easily applied in the topology optimization process. From the point of view of discrete element, if there is no material within some certain angle of the underlying layer of a solid element, then the element cannot be supported and should be considered as the overhanging region. For the sake of simplicity, we discuss the situation of a 2D rectangular model that is evenly separated into quadrilateral elements (Figure 2(a)). The density of the element is denoted by $x_{i,j}$, where *i* and *j* represent the vertical and horizontal location of the element. The three elements under the element (*i, j*) are defined as the support elements. By adjusting the aspect ratio of the finite element, it can be ensured that the elements are always supported by the three elements at any overhanging angles (Figure 2(b)). We define the sum of the densities of the three elements below the element $x_{i,j}$ as the "*support value*" for $x_{i,j}$. The difference between the density of $x_{i,j}$ and its support value determine an overhanging element: when the difference is positive, $x_{i,j}$ is an overhanging element, otherwise, $x_{i,j}$ is not an overhanging element. Therefore, whether the element (*i, j*) is an overhanging element can be expressed by the following equations.

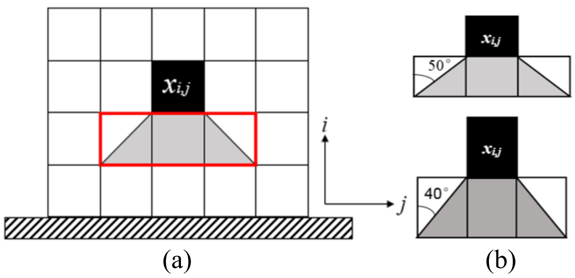


Figure 2. Definition of the overhanging on element $x_{i,j}$

$$o_{i,j} = \begin{cases} 1 & x_{\Delta i,j} > 0 \\ 0 & x_{\Delta i,j} \le 0 \end{cases} \tag{1}$$

$$x_{\Delta i,j} = x_{i,j} - \left(x_{i-1,j-1} + x_{i-1,j} + x_{i-1,j+1} \right) \tag{2}$$

$$x = \begin{cases} 1 & solid \\ 0.0001 & void \end{cases} \tag{3}$$

As in equation (1), $o_{i,j}$ represents whether the element is an overhanging element, x represents the material density of the element. In equation (2), $x_{\Delta i,j}$ represents the difference between the density of $x_{i,j}$ and its support value.

In this form, the overhanging state is not derivable, but the sensitivity information is critical in topology optimization. So we perform a smooth approximation with a logical function, then the equation (1) can be rewritten as:

$$o_{i,j} = \frac{1}{1+e^{-k \cdot x_{\Delta i,j}}} \tag{4}$$

In equation (4), k is the steepness of the fitting function curve, and the larger the k is, the more accurate the fit is. From the function curve (Figure 3), we can see that the closer the support value is to zero, the closer the overhanging state is to the intermediate value, which does not meet the expectation. Thus, we introduce a parameter α into the fitting functions to make the function more consistent with the requirements:

$$o_{i,j} = \frac{1}{1+e^{-k \cdot (x_{\Delta i,j} - \alpha)}} \tag{5}$$

The value of α can be determined by the function curve(Figure 3). For example, when $k = 40$, the intermediate value can be eliminated only if the difference between $x_{\Delta i,j}$ and α is always outside the range of -0.2 to 0.2. Here, "0.2" corresponds to a threshold β, so the range of α can be calculated by equation (6):

$$\begin{cases} x_{\Delta i,j} - \alpha > \beta, & x_{\Delta i,j} > 0 \\ x_{\Delta i,j} - \alpha < -\beta, & x_{\Delta i,j} \le 0 \end{cases} \tag{6}$$

For the whole modle, the size of the overhanging regions can be defined as:

$$O = \sum_{i=1}^{ni} \sum_{j=1}^{nj} o_{i,j} \tag{7}$$

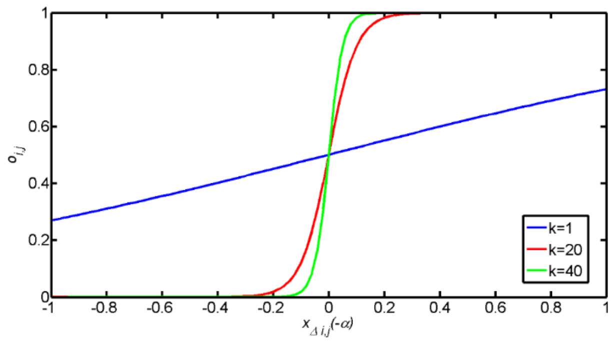


Figure 3. Approximation function

2.2. The Overhang Sensitivity

The change of the objective function caused by deleting an element is defined as "sensitivity". The value of the element sensitivity represents the contribution of the element to the objective function. The element with low sensitivity means it has little effect on structural performance, so it can be removed.

According to equation (5) and equation (7), the overhang sensitivity is as follows:

$$\frac{\partial O}{\partial x_{i,j}} = \frac{\partial o_{i,j}}{\partial x_{i,j}} + \frac{\partial o_{i-1,j-1}}{\partial x_{i,j}} + \frac{\partial o_{i-1,j}}{\partial x_{i,j}} + \frac{\partial o_{i-1,j+1}}{\partial x_{i,j}} \tag{8}$$

$$\frac{\partial o_{i,j}}{\partial x_{i,j}}=\frac{k\cdot e^{-k\cdot\left(x_{\Delta i,j}-\alpha\right)}}{\left(1+e^{-k\cdot\left(x_{\Delta i,j}-\alpha\right)}\right)^2} \tag{9}$$

For the elements located at the boundary of the design domain, the last three terms of equation (8) may not exist, and we only discuss the general situation here.

2.3. Mathematical Model

The ideal situation is to eliminate all the overhanging structures in the design, but it may not be possible. To forcibly limit the size of the overhanging region needs the designers to set the upper limit in advance, which is unreasonable. Thus, it may be a better way to take the overhanging regions as a minimization goal of the optimization process

Considering a multi-objective optimization problem integrated by the typical "compliance minimization" and the "overhanging regions minimization". The mathematical model based on the Bi-directional evolutionary structural optimization(BESO) method with the “soft kill” strategy is as follows[10].

$$\begin{aligned} &Min\text{:}\ \ g(x)=C(x)+\lambda\cdot O(x)\\ &s.t.\ \ \ \sum_{i=1}^{N}V_i x_i - V^* \le 0\\ &\qquad\ \ x_i\in\{x_{\min},1\}\end{aligned} \tag{10}$$

In equation (10), C represents the structural compliance, O represents the size of the overhanging region, V^* represents the structural target volume, V_i represents the volume of the ith element, N represents the total number of elements, x_i represents the density of the ith element, x_{min} means the void element (the value is *0.001* in this paper), and λ is a weight coefficient, which is determined by the order of the compliance sensitivity and the overhanging sensitivity.

The sensitivity of the new objective function is as follows:

$$\frac{dg(x)}{dx}=\frac{\partial C(x)}{\partial x}+\lambda\cdot\frac{\partial O(x)}{\partial x} \tag{11}$$

The sensitivity of compliance is as described in the literature [10]:

$$\frac{\partial c}{\partial x_{i,j}}=-\frac{p x_{i,j}^{p-1}}{2}u_{i,j}^{T}K_{i,j}^{0}u_{i,j} \tag{12}$$

Where p is the density penalty, $u_{i,j}$ is the displacement vector, $K^0_{i,j}$ is the element stiffness matrix which can be calculated with the solid material.

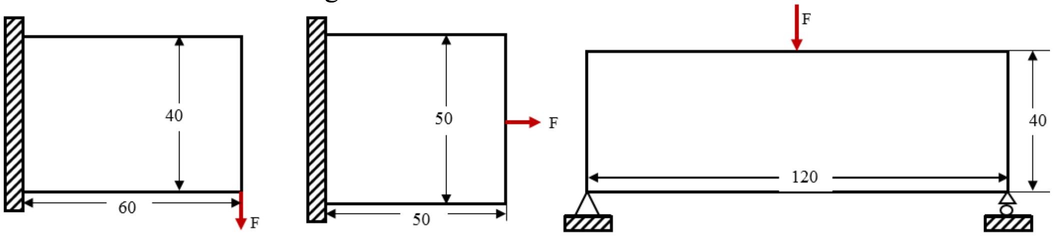
3. 3 Numerical Experiments

The novel method is tested on two well-known minimum compliance topology optimization problems [11]: the cantilever beam and the MBB beam. For all of the experiments, the default parameters are as follows: Young's modulus $E = 1GPa$, Poisson's ratio $\upsilon = 0.3$, the volume fraction $V^* = 30\%$, the density penalty value $p = 3$, the fitting function curve $k = 40$, the approximation penalty value $\alpha = 0.4$, the weight coefficient $\lambda = 0.4$, the evolution volume ratio $ER = 2\%$.

3.1. Examples

Example 1 is a cantilever beam with a fixed bearing at one end without axial, vertical displacement and rotation, and vertically loaded at the other end(Figure 4(a)). The entire design domain is discretized into 60 × 40 quadrilateral planar elements. Example 2 is another cantilever beam horizontally loaded at the midpoint of the right end, as shown in Figure 4(b), its design domain is discretized using 50 × 50 finite elements. The loading condition is the same as Example 1. Example 3 is shown in Figure 4(c). The lower right corner of the MBB beam is bounded by a rolling hinge and the lower left corner is supported. The load is applied vertically at the midpoint of the upper boundary.

This MBB beam is discretized using 120 × 40 elements.

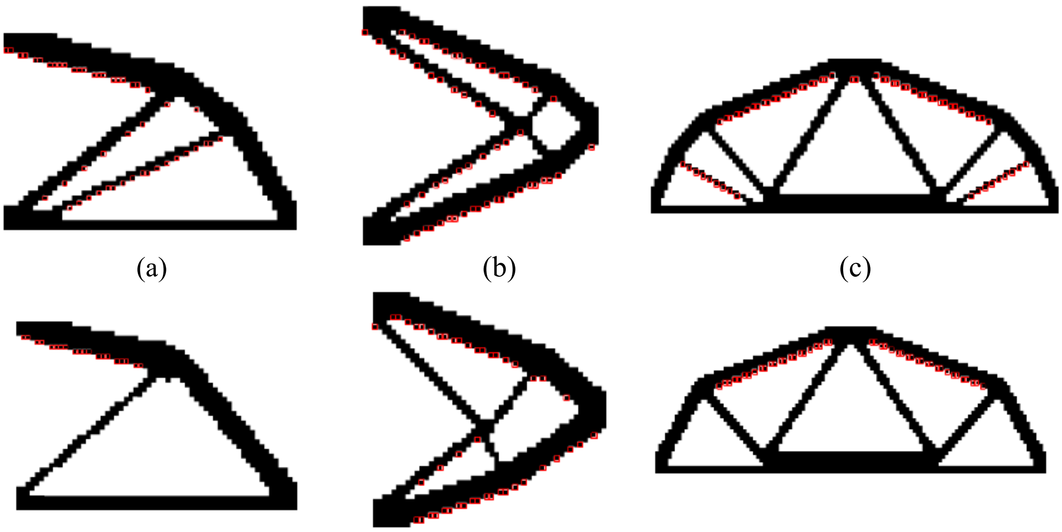


(a) (b) (c)

Figure 4. The numerical examples

Numerical results for the three examples are shown in Figure 5. The original topology optimization designs are respectively shown in (a), (b) and (c). The solutions considering the minimum overhang objective are shown in (d), (e) and (f). The red regions represent the overhanging regions. Table 1 lists the number of overhang and the compliance of the solutions.

It is clear from Figure 5 that the optimized results have less overhangs than the original design. From the data in Table 1, it can be intuitively seen that the overhanging structures of the novel designs are reduced by 52.1%, 23.3% and 28.1% respectively. This indicating the proposed method which considering the minimum overhanging constraint is effective. The structural compliance of the minimum overhang designs not only did not increase obviously, but even slightly reduced in example 2 and 3. This means that the proposed method does not rely on sacrificing performance to improve the manufacturability of the structure.



(a) (b) (c)

(d) (e) (f)

Figure 5. The numerical results

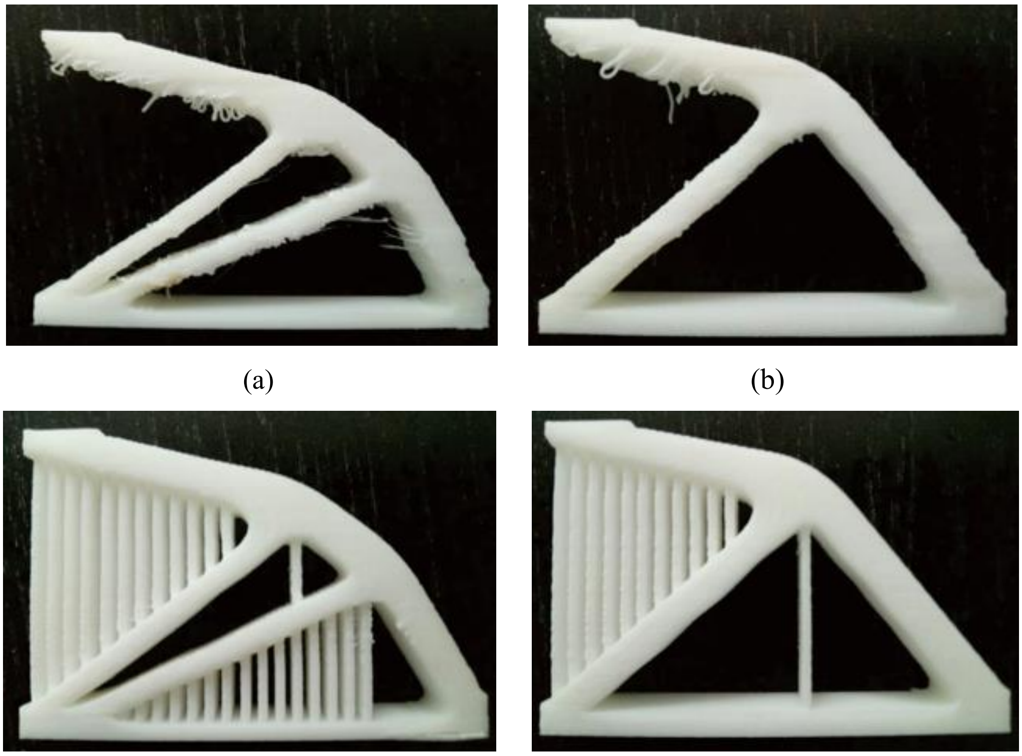
Table 1. The size of the overhanging elements and the compliance of the TO results

Examples	Design Method	The number of overhangs	Compliance
Example 1	Original design	48	30.9679
	Optimized Method	23	30.9822
Example 2	Original design	60	11.5732
	Optimized Method	46	10.5121
Example 3	Original design	64	17.2573

Optimized Method	46	17.2405

3.2. Manufacturability Experiments

The cantilever beam optimized in Example 1 of Section 3.1 were experimentally assessed and compared to the original optimized model. Both the models were manufactured by FDM 3D printing. The FDM machine type is UP Plus 2(A), the material is ABS and the slicing height is 0.2mm. Figure 6(a) and Figure 6(b) illustrating the manufacturing results without using any support material. It can be seen that the overhanging regions were failed to printed due to the lack of filament adhesion. The failure area of the original structure is larger than the overhanging minimized structure. Figure 6(c) and Figure 6(d) illustrating the results based on the support structure. The required supporting volume for the original model is 3425 mm^3 while the value can be reduced to 2258 mm^3 for the optimized model.



(a) (b)

(c) (d)

Figure 6. The manufacturing results by FDM

4. Conclusion

In this paper, a novel topology optimization method is proposed to reduce the overhangs for additive manufactured components. The overhangs are transformed into a display form expressed by design variables and considered as a minimization goal of the optimization problem, in which the overhang sensitivity is combined with other performance sensitivities to update design variables. The results of numerical experiments and fused deposition modeling AM process demonstrate the proposed method is effective for reducing the overhangs during the optimization. As a result, the volume of the support material used for the manufacturing process can be saved, the manufacturing time and the post processing time can also be reduced simultaneously.

In the course of the research, we also found the proposed optimization method will not improve the structural compliance, even beyond our expectations. This may prove that the topology optimization problem with the overhanging minimized objective will not affect the performance of the structure.

In spite of this, there is still room for improvement in this method. That is the optimization process depending on the regular finite element discrete method, in the future research, other more suitable finite element discrete method may assist us to improve the work of this article.

Acknowledgments
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