

# Dynamic response analysis of a single degree of freedom model under seismic

**Jing Zhao, Ruo Chen Liu**

Beijing Geely University, Beijing 102202, China

Email: zhaojing1980821@163.com

**Abstract:** Numerical simulation is the basic research method that lays equal effect on the theoretical analysis and experimental testing in the field of scientific research and engineering technology, and it is the most powerful analysis tool for the complex physical problems and engineering problems. A single degree of freedom model will be used to study the dynamic response of the mechanical equipment by combining ABAQUS under seismic in this paper. The results show that the model of numerical simulation is correct due to deviation of the self-vibration frequency between the results of numerical simulation and mathematical calculation is 0.2%. The method of numerical simulation is correct because deviation of the displacement between the results of numerical simulation and mathematical calculation is 0.3% and deviation of the absolute acceleration between the results of them is 0.4%. The results of the numerical simulation are larger than the results of the mathematical calculation.

## 1. Introduction

Numerical simulation is the basic research method that lays equal effect on the theoretical analysis and experimental testing in the field of scientific research and engineering technology, and it is the most powerful analysis tool for the complex physical problems and engineering problems. The real digital equipment will be done by numerical simulation of advanced design according to mechanical, mechanical, material, computer, physics, and other disciplines, it is an important means to realize the equipment innovation design and independent research and development, and it also is the comprehensive embodiment of advanced digital design, knowledgeable, and intelligent. Therefore, the design technology based on numerical simulation is a necessary means of developing advanced equipment at present and in the future, and it is great significance for improving the independent innovation ability of our equipment manufacturing industry and ensuring national strategic safety.

All design and manufacture can not be separated from the finite element analysis and mathematical calculation from bicycle to space shuttle, and the finite element analysis and mathematical calculation will be paid more and more attention in the engineering design and analysis. Because of the great difference between the displacement and the acceleration parameters of the mechanical equipment, the scholars at home and abroad have carried out a lot of research work. Nason G P et al[1] studied Local Stationary Wavelet of random process for multi degrees of freedom. Zheng jia-shu et al[2] separated a multi-freedom structure system based on the FEM method basic principles model. Yuan wen-jun et al[3] studied the characteristics of dynamic response in single-degree-of-freedom(SDOF) system acted by earthquake based on the technique of wavelet pack decomposition and the dynamic calculation of harmonic wave. Zheng jia-shu et al[4] studied the multiple time scale method in order to improve the calculation accuracy of analytical methods for equivalent damping and frequency of the SDOF structure with Maxwell damper.



ABAQUS is a set of advanced general finite element program system, and the purpose of ABAQUS is to carry on the numerical calculation and analysis to the mechanical problem. A SDOF model will be used to study the dynamic response of the mechanical equipment by ABAQUS under seismic in the paper.

## 2. Numerical Investigation

### 2.1. Finite Element Model

A model of SDOF is shown in Figure 1, the height of the model is 10m, and the model includes elastic rod and lumped mass block. The section of elastic rod is square ( $0.5\text{m}\times 0.5\text{m}$ ), density of it is  $0\text{ kg/m}^3$ , on the top of lumped mass block is 1000 kg, and the parameters of elastic rod and lumped mass block are shown in Table 1. The material of model is steel, elastic modulus is  $2.1\times 10^5\text{ MPa}$ . The passive control technology had been widely used in practical engineering such as viscous and viscoelastic dampers [5][6], but damping is not considered in order to simplify the complexity of the model.

Table 1 The Parameter of Model

Part	E (MPa )	Quali ty (kg)
Elastic rod	2100 00	0
Lumped mass block	2100 00	1000

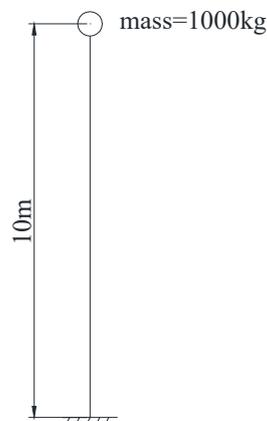


Figure 1 A Model of SDOF

The model of SDOF is established by ABAQUS and shown in Figure 2. The elastic rod of the model is simulated by the element of beam, and lumped mass block is simulated by setting up the quality point with the reference point. Simulation of interaction between elastic rod and lumped mass block are simulated through tie connection.

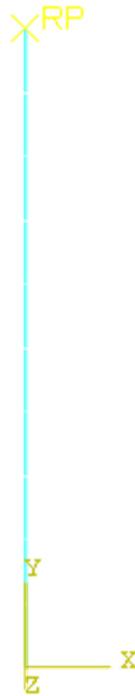


Figure 2 The Model of ABAQUS

### 2.2 Analysis of the self-vibration frequency

The self-vibration frequency of the structure is the dynamic characteristic of itself, and it is related to the height and the width of the structure. When the period of the vibration frequency of the structure is close to the period of the earthquake action, the resonance will occur, which has a great influence on the structure, and the damage of it increases with the enhance of earthquake degree. Stiffness coefficient of structure in Figure 1 can be calculated by formula 1, the self-vibration period of it can be calculated by formula 2, and the self-vibration frequency of it can be calculated by formula 3. The results of the artificial calculation are shown in Table 2.

$$k = 3 \frac{EI}{l^3} \quad \text{formula 1}$$

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{formula 2}$$

$$f = \frac{1}{T} \quad \text{formula 3}$$

where  $k$  is the stiffness coefficient of the model,  $I$  is the inertial moment of section for the model,  $l$  is the length of elastic rod,  $T$  is the self-vibration period of the model, and  $f$  is the self-vibration frequency of the model.

Table 2 The Mathematical Calculation Results

Stiffness coefficient (N)	The self-vibration period (s)	the self-vibration frequency (Hz)
$3.28 \times 10^6$	0.11	9.09

The first two frequencies of the model are 0 Hz and 9.11 Hz, and the first two modes of model are shown in Figure 3, respectively. Deviation of the self-vibration frequency between the results of numerical simulation and mathematical calculation is 0.2%, so the results of numerical simulation are correct.

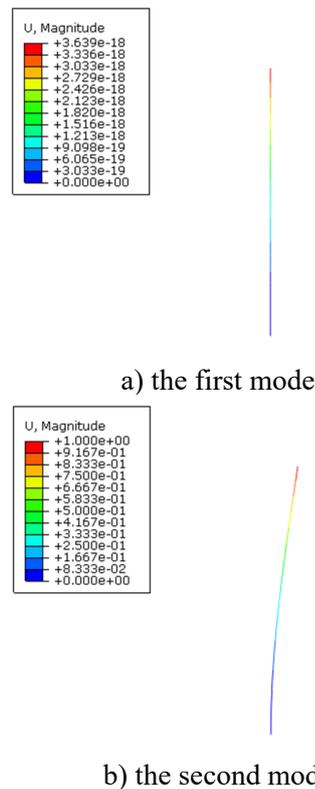


Figure 3 The First Two Modes of Model

### 3. Analysis of the absolute displacement and acceleration for model under seismic

#### 3.1. Mathematical calculation

The relationship between absolute displacement and relative displacement of a SDOF model is shown in Figure 4. The equation of motion for a SDOF model is shown in formula 4, and the formula 5 is obtained after transformation. The initial displacement is 0m, and the initial velocity is 0m/s, so the solution of the equation is formula 6, formula 7 and formula 8.

$$my(t) + mu(t) + ky(t) = 0 \quad \text{formula 4}$$

$$y(t) + \omega^2 y(t) = -u(t) \quad \text{formula 5}$$

$$y(t) = \frac{1}{\omega(\omega^2 - 1)} \sin(\omega t) - \frac{1}{\omega^2 - 1} \sin t \quad \text{formula 6}$$

$$y(t) = \frac{1}{\omega^2 - 1} \cos(\omega t) - \frac{1}{\omega^2 - 1} \cos t \quad \text{formula 7}$$

$$y(t) = \frac{\omega}{\omega^2 - 1} \sin(\omega t) - \frac{1}{\omega^2 - 1} \sin t \quad \text{formula 8}$$

where  $\omega$  is the frequency of circle,  $t$  is the time,  $y(t)$  is the relative displacement of model,  $u(t)$  is the difference between absolute and relative displacement;  $y(t)$  is the absolute acceleration, and

$y(t)$  is the absolute speed.

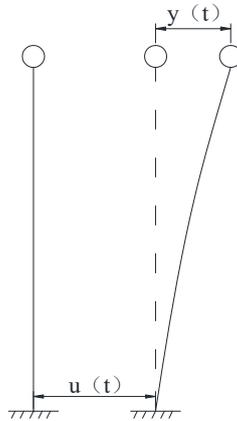


Figure 4 The Displacement of on the top of Model for the Mathematical Calculation

### 3.2 Type of seismic wave

In order to analyze the dynamic response for the SDOF model under seismic, a sinusoidal function seismic wave as the input load was selected. The seismic acceleration time history was shown in Figure 5.

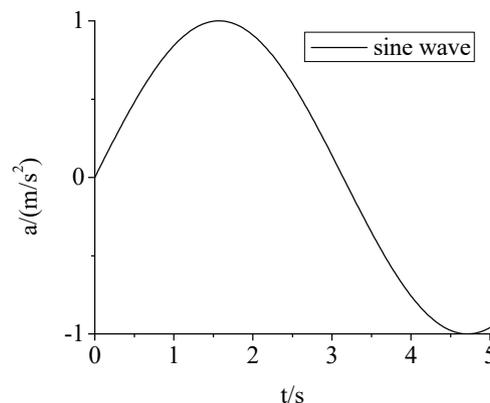


Figure 5 Time-history Curve of Acceleration

### 3.3 Comparison and analysis of the results for the absolute displacement and acceleration

Time-history curve of acceleration is imposed on the constraint of the base and relieved the direction of acceleration according to ABAQU. The absolute displacement and acceleration on the top of the SDOF model can be obtained by numerical simulation. The comparison between the results of numerical simulation and mathematical calculation for the absolute displacement is shown in Figure 6, and deviation of the displacement between the results of numerical simulation and mathematical calculation is 0.3%. Meanwhile, the comparison between the results of numerical simulation and mathematical calculation for the absolute acceleration is shown in Figure 7, and deviation of the absolute acceleration between the results of numerical simulation and mathematical calculation is 0.4%, so the method of numerical simulation is correct. The results of the numerical simulation are larger than the results of the mathematical calculation according to Figure 6 and Figure 7.

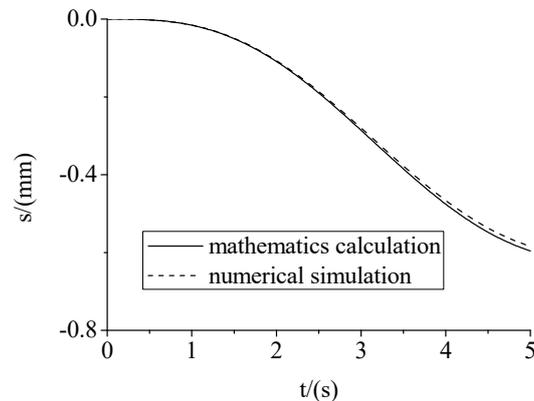


Figure 6 The Absolute Displacement on the top of Model

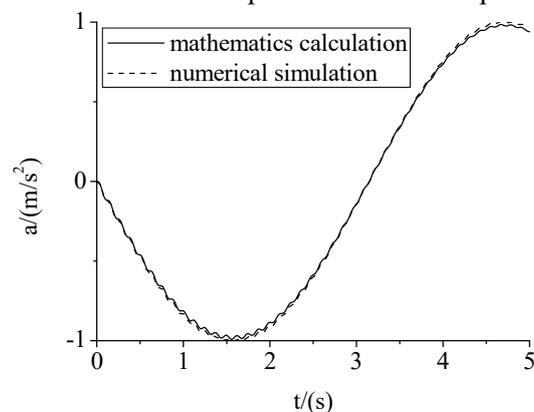


Figure 7 The Absolute Acceleration on the top of Model

#### 4. Conclusion

(1) The model of numerical simulation is correct due to deviation of the self-vibration frequency between the results of numerical simulation and mathematical calculation is 0.2%.

(2) The method of numerical simulation is correct because deviation of the displacement between the results of numerical simulation and mathematical calculation is 0.3% and deviation of the absolute acceleration between the results of them is 0.4%.

(3) The results of the numerical simulation are larger than the results of the mathematical calculation.

#### References

- [1] Nason G P, von Sachs R, Kroisandt G. Wavelet processes and adaptive estimation of the evolutionary wavelet spectrum[J]. *Journal of the Royal Statistical Society Series B-Statistical Methodology*, 2000, 62(2): 271-295.
- [2] Zheng J S, Zuo D. Dynamic response of multi-freedom structure under shock loads[J]. *Journal of Southwest University for Nationalities Natural Science Edition*, 2007, 33(2): 392-396.
- [3] Yuan W J, Huang Z L Y, Zhou R Z. Characteristics of dynamic response in single-degree-of-freedom system acted by earthquake based on wavelet packets analysis[J]. *Journal of Fuzhou University (Natural Science)*, 2008, 36(5): 752-757.
- [4] Li C D, Ban D W, Zhang Y. Equivalent damping of single-degree-of-freedom structure with Maxwell damper[J]. *Journal of Guangxi University (Nat Sci Ed)*, 2017, 42(4): 1236-1248.
- [5] Soong T T, Dargush G F. *Passive energy dissipation systems in structural engineering*[M]. England: John Wiley and Ltd, 1997.
- [6] Christopoulos C, Filiatrault A. *Principle of passive supplemental damping and seismic isolation*[M]. Pavia, Italy: IUSS Press, 2006.