

The study of non-smooth dynamics of a class of multi-degree-of-freedom vibro-impact system with clearance

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Abstract: The period doubling bifurcation and the Neimark-Sacker bifurcation of the multi-degree-of-freedom vibro-impact system with clearance are studied in this paper. The periodic solution of vibro-impact system is solved and a six-dimensional Poincaré map is established, and the doubling bifurcation and the Neimark-Sacker bifurcation are analyzed. The study of bifurcation and chaos of the system provides reliable basis for the design and fault diagnosis and provides theoretical guidance and technical support for the actual design in the safe and stable operation of some rotating machinery.

1. Introduction

The non-smooth bifurcation phenomena widely exist in mechanical engineering field. The research on complicated dynamic behaviors of vibro-impact systems has become the most active branch of nonlinear dynamics. Shaw and Holmes [1] studied the oscillator system with single piece rigid constraint under the action of a harmonic force by using modern dynamics theoretical methods and analyzed the bifurcation behaviors of periodic motions by using center manifold and normal form theory. Luo and Xie [2] studied the complexity co-dimension-two bifurcation behaviors of a two-degree-of-freedom vibro-impact system. Ding and Xie [3] studied the path from torus bifurcations to chaos in a three-degree-of-freedom vibro-impact system. Karin Mora [4] analysed the novel dynamics arising in a nonlinear rotor dynamic system by investigating the discontinuity-induced bifurcations corresponding to collisions with the rotor housing. Keogh & Cole [5] show that a rotor-stator system with damping and friction can exhibit various forms of stable and unstable synchronous single impact limit cycles. Similar discontinuity-induced Hopf bifurcations, exhibiting a bifurcation of a non-impacting equilibrium to a limit cycle with impact, have been observed in planar piecewise smooth continuous systems [6] with sliding [7] and with biological applications [8].

The paper is organized as follows: the mechanical mode of three-degree-of-freedom vibro-impact system with clearance and equations of solution, the period doubling bifurcation and the Neimark-Sacker bifurcation are respectively studied and the conclusions.



2. Mechanical model and equation of motion

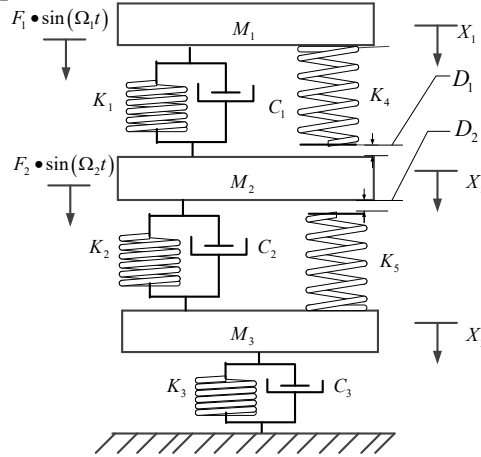


Figure 1. The multi-degree-of-freedom vibro-impact system with clearances

According to the model in Figure1, masses M_1 , M_2 and M_3 are connected by linear springs with stiffness K_1 , K_2 and K_3 , and linear viscous dampers C_1 , C_2 and C_3 , respectively, stiffness K_4 , K_5 are connected with masses M_1 , M_2 with the clearances D_1 and D_2 . When fault occurs, $F_1 \sin(\Omega_1 T)$ is the equivalent force. Ω_1 is the frequency.

In order to describe the motion process of the system, a switching boundary need be introduced. The boundary function is defined $E_1 = X_1 - X_2 - D_1$, and $E_2 = X_2 - X_3 - D_2$ then the switching boundary can be expressed as follows:

$$\Sigma_1 = \{(X_1, X_2) \in R^2 | E_1 = 0\}, \quad \Sigma_2 = \{(X_2, X_3) \in R^2 | E_2 = 0\} \quad (1)$$

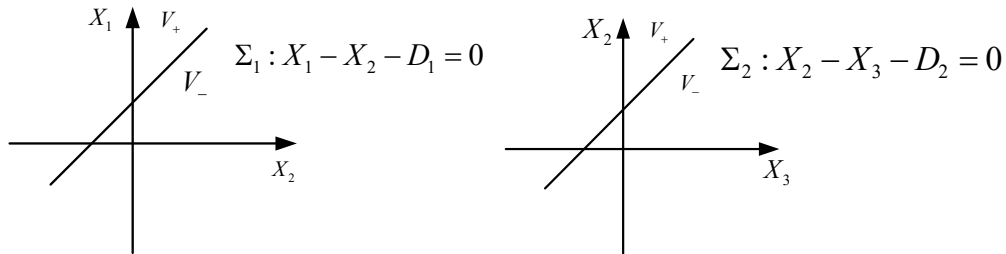


Figure 2. The subspaces of the system

Thus the state space is divided into two subspaces by the switching boundary Σ_1 and Σ_2 , just as shown in figure 2.

In Figure2, $V_+ = \{X \in R^2 | E_1(X_1, X_2) > 0\}$, $V_+ = \{X \in R^2 | E_2(X_2, X_3) > 0\}$, stands for contact state of mass M_2 and broken spring K_4 and K_5 respectively, $V_- = \{X \in R^2 | E_1(X_1, X_2) < 0\}$ and $V_- = \{X \in R^2 | E_2(X_2, X_3) < 0\}$ stand for separation state respectively.

According to the above analysis, the dynamical equation is established as follows:

$$\begin{bmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \end{bmatrix} + \begin{bmatrix} C_1 & -C_1 & 0 \\ -C_1 & C_1 + C_2 & -C_2 \\ 0 & -C_2 & C_2 + C_3 \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} + \begin{bmatrix} K_1 & -K_1 & 0 \\ -K_1 & K_1 + K_2 & -K_2 \\ 0 & -K_2 & K_2 + K_3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + H(X) \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + G(X) \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} F_1 \sin(\Omega_1 T) \\ F_2 \sin(\Omega_2 T) \\ 0 \end{bmatrix}$$

$$H(X) = \begin{cases} 0, & [X_1, X_2]^T \in V_- \\ K_4(X_1 - X_2 - D_1), & [X_1, X_2]^T \in V_+ \end{cases}$$

$$G(X) = \begin{cases} 0, & [X_2, X_3]^T \in V_- \\ K_5(X_2 - X_3 - D_2), & [X_2, X_3]^T \in V_+ \end{cases} \quad (2)$$

where the dots “ $\dot{}$ ” and “ $\ddot{}$ ” in the expressions (1) and (2) denote the first and second order differentiations with respect to the time T , respectively.

Introduce the following non-dimensional quantities

$$m_i = M_i/M_1, (i=2,3), \zeta_i = C_i/(2\sqrt{K_1 M_1}), (i=1,2,3), \quad k_i = K_i/K_1, (i=2,3,4), f_1 = F_1/(F_1 + F_2),$$

$$f_2 = F_2/(F_1 + F_2), \quad \omega_1 = \Omega_1 \sqrt{M_1/K_1}, \quad \omega_2 = \Omega_2 \sqrt{M_1/K_1}, \quad t = T \sqrt{K_1/M_1}, \quad (3)$$

$$x_i = X_i K_1/(F_1 + F_2) (i=1,2,3), \quad d_1 = D_1 K_1/(F_1 + F_2), d_2 = D_2 K_2/(F_1 + F_2)$$

With the non-dimensional quantities (3), the original equation (1) and (2) can be transformed into the following non-dimensional forms:

$$\begin{bmatrix} 1 \\ m_2 \\ m_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2\zeta_1 & -2\zeta_1 & 0 \\ -2\zeta_1 & 2\zeta_1 + 2\zeta_2 & -2\zeta_2 \\ 0 & -2\zeta_2 & 2\zeta_2 + 2\zeta_3 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + h(x) \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + g(x) \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} f_1 \sin(\omega_1 t) \\ f_2 \sin(\omega_2 t) \\ 0 \end{bmatrix} \quad (4)$$

in which,

$$h(x) = \begin{cases} 0, & [x_1, x_2]^T \in v_- \\ k_4(x_1 - x_2 - d_1), & [x_1, x_2]^T \in v_+ \end{cases} \quad (5)$$

$$g(x) = \begin{cases} 0, & [x_2, x_3]^T \in v_- \\ k_5(x_2 - x_3 - d_2), & [x_2, x_3]^T \in v_+ \end{cases} \quad (6)$$

in equation (5),

$$v_+ = \{[x_1, x_2]^T \in R^2 | e_1(x_1 - x_2) > 0\}, \quad v_- = \{[x_1, x_2]^T \in R^2 | e_1(x_1 - x_2) < 0\},$$

$$e_1 = x_1 - x_2 - d_1$$

in equation (6),

$$v_+ = \{[x_2, x_3]^T \in R^2 | e_2(x_2 - x_3) > 0\}, \quad v_- = \{[x_2, x_3]^T \in R^2 | e_2(x_2 - x_3) < 0\}, \quad e_2 = x_2 - x_3 - d_2$$

3. Bifurcations of periodic motion in the system

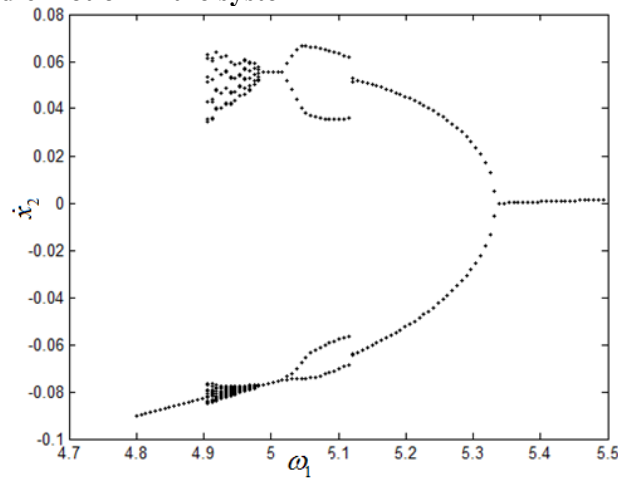
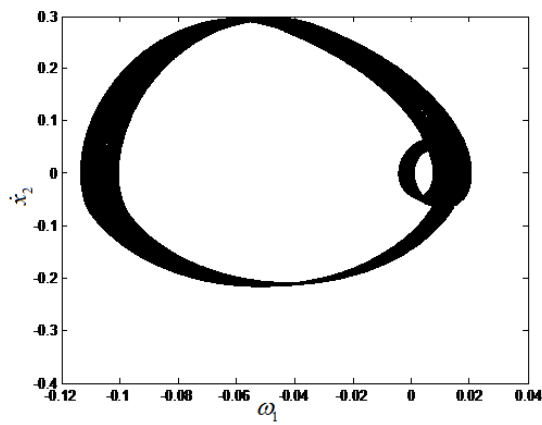
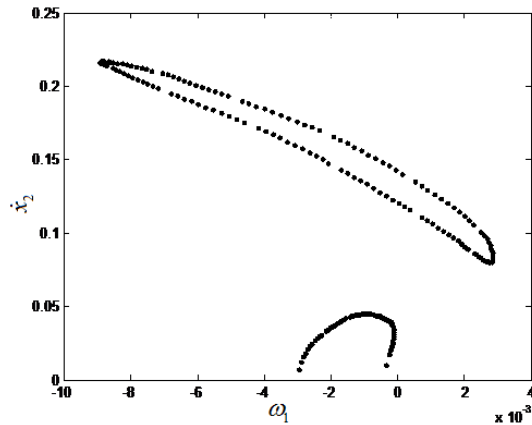


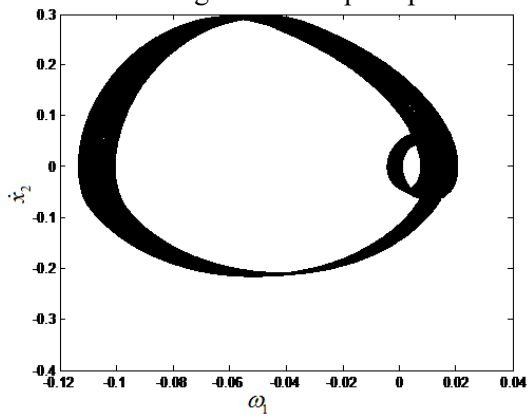
Figure 3. The global bifurcation diagram of the system



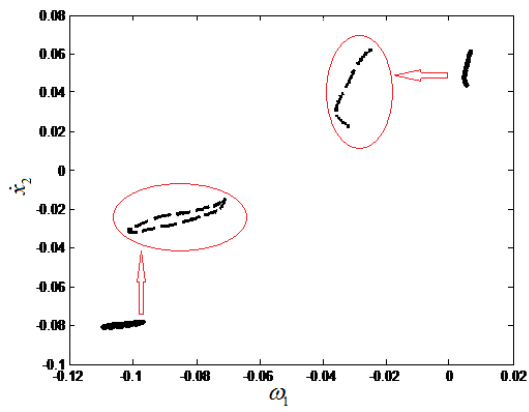
(a) The phase diagram



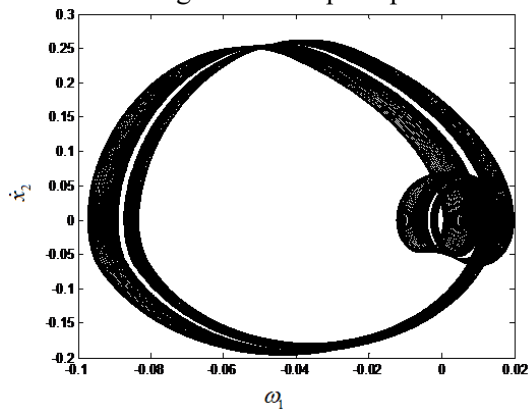
(b) The stable focus on Poincaré section

Figure 4. The quasi-periodic motion at $\omega = 4.95$ at the up collision plane

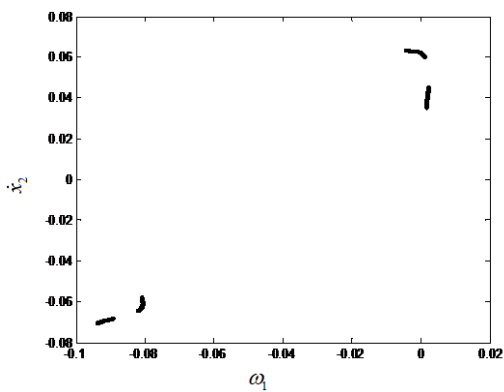
(a) The phase diagram



(b) The stable focus on Poincaré section

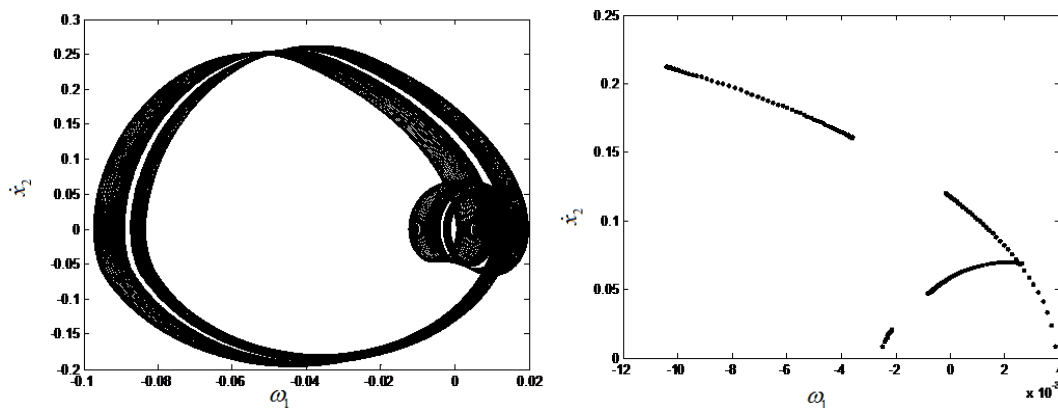
Figure 5. The quasi-periodic motion at $\omega = 4.95$ at the down collision plane

(a) The phase diagram



(b) The stable focus on Poincaré section

Figure 6. The quasi-periodic motion at $\omega = 5.1$ at the up collision plane



(a) The phase diagram

(b) The stable focus on Poincaré section

Figure 7. The quasi-periodic motion at $\omega = 5.1$ at the down collision plane

4 Conclusions

In this paper we have investigated the bifurcation and chaos of periodic motions of the system with clearances. The collisions happen among the mass M_2 and M_1 or M_3 . The complex nonlinear behaviors happen. From the results obtained, it is found that the system with clearances can exhibit the interesting and complex dynamical behaviors. Based on these nonlinear dynamical behaviors of the system, the relationship between abrupt and collisions will be revealed from bifurcation perspective by spectral analysis method to obtain the set of relevant features of vibration signal in future work. This will provide theoretical basis for fault identification.

Acknowledgements

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