

Hybrid Frequency-dependent Hysteresis Model of Magnetostrictive Actuator

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Abstract. The inherent frequency-dependent hysteresis nonlinearities of magnetostrictive actuator prominently limit its performance. In this paper, a hybrid hysteresis model comprising a frequency-independent hysteresis model cascaded an adaptive finite impulse response filter is developed to improve the performance of actuators. For the frequency-independent part, a generalized Prandtl-Ishlinskii model that have analytical inversion is adopted. Besides, a modified coral reefs optimization algorithm is applied to identify the parameters to improve the model accuracy. The adaptive filter is utilized to adapt to the rate-dependent characteristic. The model validation results show that the hybrid model can satisfactorily describe the frequency-dependent hysteresis.

1. Introduction

Magnetostrictive actuators (MAs) are widely used in micropositioning, manufacturing applications and active vibration control systems due to their fast response and relatively large stroke and force capacity. Efficient precision positioning with magnetostrictive actuators is, however, limited by the intrinsic frequency-dependent hysteretic behaviour [1]. Inverse compensation method is the most popular approach to handle hysteresis by constructing a right inverse of the hysteresis model. So far, several models have been proposed to characterize the hysteresis phenomenon of MAs. In general, these models can be categorized into hysteresis models based on physic, such as Jiles-Atherton model and Duhem model, and hysteresis models based on phenomenology, such as the Preisach model and Prandtl-Ishlinskii (PI) model.

Phenomenological models can describe the behaviours of actuators well without physical mechanism, hence they can be convenient used in hysteresis compensation system. Among them, PI model with an analytical inverse model is the most commonly used. The classical PI model is a weighted superposition of several symmetric backlash operators, so it cannot describe the asymmetric hysteresis phenomenon, which is widely found in the MAs. To account for the asymmetry and saturation properties of hysteresis which are always observed for MA, Kuhnien [2] proposed a modified PI model with deadzone operators. Gu [3] introduced a generalized input function to describe the asymmetric hysteresis nonlinearity. Al Janaideh [4] proposed a generalized PI model with different envelope functions for increase curve and decrease curve, which can describe the symmetric and asymmetric hysteresis properties of MA with output saturation. However, the hysteresis nonlinearities of MA are dependent on the frequency of input, the above-mentioned models are frequency-independent.

To characterize the frequency-dependent hysteresis, several modified models, which formulate dynamic density functions in the classical models, have been developed. For instance, Al janaideh [5] proposed a PI model with dynamic density functions. As another choice, Al janaideh [6] modeled rate-dependent hysteresis of MA by PI model with dynamic thresholds. For all the rate-independent PI models, the analytical inverse can be obtained by the exact inversion law. However, great difficulties arise in the inversion calculation for the rate-dependent PI models that make those models difficult to



compensate hysteresis in real-time applications. In some literature, the influence of the input frequency is due to the system dynamics of MA [7], thus the rate-dependent hysteresis nonlinearity of MA can be decomposed to a static hysteresis part and a linear dynamic part [8]. The adaptive filter can be utilized to model the rate-dependent characteristic of hysteresis by treat the frequency as an variable parameter. Besides, the adaptive filter can be identified online, that making the model can adapt the time-varying parameter. In this way, a rate-independent hysteresis model cascade an adaptive finite impulse response (FIR) filter is introduced to describe the rate-dependent hysteresis.

In this paper, the rate-dependent hysteresis is described by a cascaded model with an adaptive FIR filter and rate-independent hysteresis model with an generalized PI model that have analytical inversion. The hybrid model is discussed in section 2. Besides, a modified coral reefs optimization algorithm is utilized to identify the parameters to improve the model accuracy. The identification method and model validation are presented in section 3. Finally, conclusions are given in section 4.

2. Frequency-dependent hysteresis describing

As mentioned before, the hysteresis loops of the magnetostrictive actuator (MA) are frequency-dependent. In this work, a generalized Prandtl-Ishlinskii (GPI) model that have analytical inversion is applied as the rate-independent hysteresis model. Besides, an adaptive FIR filter model is introduced to series the GPI because it can adapt the variation of the frequency of input and conveniently implement the tracking algorithm. The structure of that hybrid model is shown in Fig. 1.

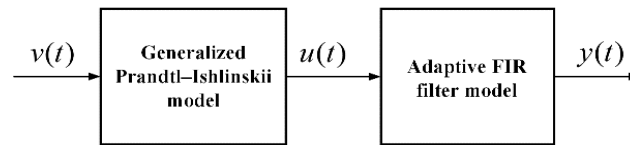


Figure 1. Frequency-dependent hysteresis model structure.

2.1. Frequency-independent GPI model

The PI model describe the hysteresis input-output relationships based on play operator and density function. But the operator used in the classical PI model is symmetric, thus it cannot be applied for predicting asymmetric hysteresis of MAs. The generalized play operator is a nonlinear play operator resulting in asymmetric hysteresis loops, for the output increase and decrease, respectively, along different curves. The more details can be referred in [4].

A bias magnetic field is usually used to eliminate the frequency doubling effect of MA. Under a right bias field, the output displacement and input current of MA maintain positive. In this work, a one-side generalized play operator, as shown in Fig. 2, is adopted. The model based on this operator is a one-side generalized PI model (OGPI). The operator for input $v(t)$ and threshold r is defined by [9]

$$G_{or}[v](t) = \begin{cases} g_{or}^{\zeta}(v(t), r, 0) & t = 0 \\ g_{or}^{\zeta}(v(t), r, G_{or}[v](t_i)) & 0 < t_i < t < T \end{cases} \quad (1)$$

$$g_{or}^{\zeta}(v(t), r, s) = \max(\zeta_1(v) - r, \min(\zeta_r(v), s))$$

where ζ_1 and ζ_r are envelope functions to characterize the observed nonlinearities of MAs. The hyperbolic tangent envelope functions are chosen, since they can describe the output saturation properties of the actuators and have unique inverse function, which means the analytical inverse model can be obtained. The envelope functions are expressed as

$$\begin{aligned} \zeta_1 &= a_0 \tanh(a_1 v + a_2) + a_3 \\ \zeta_r &= a_4 \tanh(a_5 v + a_6) + a_7 \end{aligned} \quad (2)$$

where all the coefficients which can be identified are constant and $a_0, a_1, a_4, a_5 > 0$.

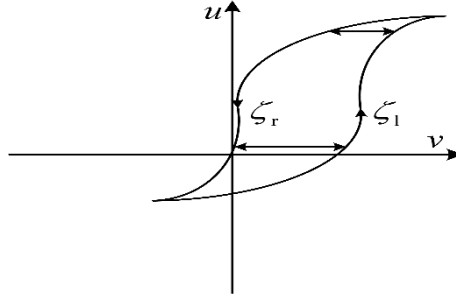


Figure 2. One-side Generalized play operator.

ζ_r always pass through the coordinate origin, therefore the coefficient should meet $a_4 \tanh(a_6) + a_7 = 0$. The finally output of the OGPI is

$$u(t) = \mathbf{w}^T \mathbf{G}_{or}[v](t) \quad (3)$$

And the thresholds and weights: $r_i = \alpha i$ and $w_i = \beta e^{-\tau r_i}$, for $i = 0, 1, \dots, n$. There α , β and τ are positive constants identified from experimental data of actuator.

2.2. Adaptive FIR filter

In order to characterize the rate-dependent hysteresis, an adaptive FIR filter model is cascaded the OGPI model. An adaptive filter is a linear filter with transfer function controlled by variable parameters and those parameters is adjusted by an optimization algorithm. A delayed adaptive filter has been widely used for its simple structure and ease of implementation. Since the output generated by any given input is determined by these parameters, determining their values is the central task. The output of an adaptive tapped delay line FIR filter model is

$$y_k = \sum_{l=0}^L \omega_{lk} u_{k-l} = \mathbf{\omega}_k \mathbf{u}_k \quad (4)$$

where L is the order of the filter, ω_{lk} is the l 'th weight at k 'th time. To get minimum error (e_k), $\mathbf{\omega}_k = \{\omega_{0k}, \omega_{1k}, \dots, \omega_{Lk}\}$ needs to be updated by an optimization algorithm. In this way, adaptive FIR filter model can track the variation of the input frequency.

In concluded that adaptive FIR filter model is suitable for characterize the rate-dependent phenomenon of MA. Furthermore, FIR filter model can realize online modeling which endows the model with better adaptability. In conclusion, the hybrid model with an adaptive FIR filter cascade with a rate-independent OGPI model is introduced to describe the rate-dependent hysteresis. In this research, the order of FIR filter model is chose as $L = 20$.

3. Model validation

3.1. Frequency-independent OGPI Parameter identification

To identify the whole hybrid frequency-dependent hysteresis model, we identify the frequency-independent model at first. In OGPI model, there are 11 parameters to be identified

$$\mathbf{X} = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, \beta, \tau, \alpha\} \quad (5)$$

To identify the parameters of the frequency-independent model, a current at 0.1 Hz is applied. NI CompactRIO 9082 system is used to generate the control signal and acquired displacement signal. The control signal amplified by NF BP4610 is the input current of MA. A laser sensor (KEYENCE LK-G80) is used to measure the output displacement.

The degree of matching between target loop and simulated hysteresis loop is the judge standard of model accuracy. In this paper, the objective function is chosen as an error squared function between model output $u(i)$ and experimental displacement $s(i)$.

The coral reef optimization (CRO) algorithm is a novel global optimization algorithm, which mimics the behaviors of corals' reproduction and coral reef formation [10]. However, the local search capabilities of CRO is not well. In order to solve this problem, Rosenbrock's method (RM) [11] is introduced to fine-tune the solution around optimal.

First, a jump conditions is defined to determine whether to enter the RM process. In this work, the CRO process terminates and the RM process takes effect based on the best coral and the worst coral when ΔAF is bigger than the set threshold $S_{break} = 1 \times 10^{-6}$. The population size and maximum iterations are set as 50 and 300. The values of the step adjustment factors recommended by Rosenbrock are $\eta = 3.0$ and $\theta = -0.5$. The details about the MCRO can refer to our recent work [12].

3.2. OGPI Identification Results

The identified X are list in Table 1. The identified hysteresis loops are shown in Fig. 3. For comparison, the identified loop using genetic algorithm (GA) is also plotted. Relative error (RE) is used to represent the accuracy:

$$RE = \sqrt{\sum_{i=1}^N (s(i) - u(i))^2 / \sum_{i=1}^N (s(i))^2} * 100\% \quad (6)$$

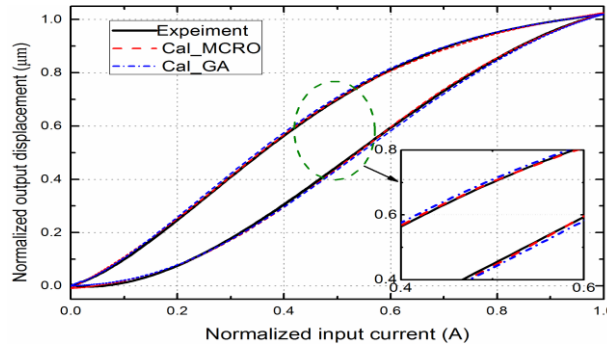


Figure 3. Identification results. Cal_MCRO: hysteresis loop identified by MCRO. Cal_GA: hysteresis loop identified by GA.

Table 1. Identified parameters of OGPI model.

Parameters	Value
$\{a_0, a_1, a_2, a_3\}$	0.596, 1.665, -0.533, 0.317
$\{a_4, a_5, a_6, a_7\}$	0.394, 2.056, -0.674, 0.223
$\{\beta, \tau, \alpha\}$	0.196, 0.437, 0.056

The RE of loop identified by MCRO is 0.8167% is smaller than the corresponding RE of GA (1.023%). From the insert of Fig. 3, it can be observed that the loop identified by MCRO overlap the experiment loop better than the loop identified by GA.

The rate-dependent hysteresis loops of the used actuator are shown in Fig. 4. It can be observed that when the frequency increases the output displacement decreases and the loop becomes wider. Due to the bias field is introduced by the bias current, which will lead to more eddy current loss, the performance of actuator is already deteriorated at a relatively low frequency. As a consequence, the OGPI can satisfactorily describe the rate-independent hysteresis, but its performance will deteriorate when the input frequency changes.

3.3. Frequency-dependent Hybrid Model Identification

The block diagram of the identification of the hybrid model is shown as Fig. 5. In this diagram, OGPI refers to the frequency-independent generalized PI model, y_k is the output of the actuator, $\tilde{y}_k$ is the output of the identified FIR model and the optimization algorithm used to update the weights of FIR model is variable step-size normalized least mean square algorithm (VSNLMS) [13].

The weights update law can be expressed as

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu_k \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|^2} e_k \quad (7)$$

where $\mu_k = \mu_{\max} \cdot \|\hat{\mathbf{p}}_k\|^2 / (\|\hat{\mathbf{p}}_k\|^2 + c)$ is the variable step size and $\hat{\mathbf{p}}_k = \gamma \hat{\mathbf{p}}_{k-1} + (1-\gamma) \mathbf{u}_k e_k / \|\mathbf{u}_k\|^2$. At the beginning, the size is large to get fast convergence speed and when the algorithm convergence the size is small to obtain small steady state error.

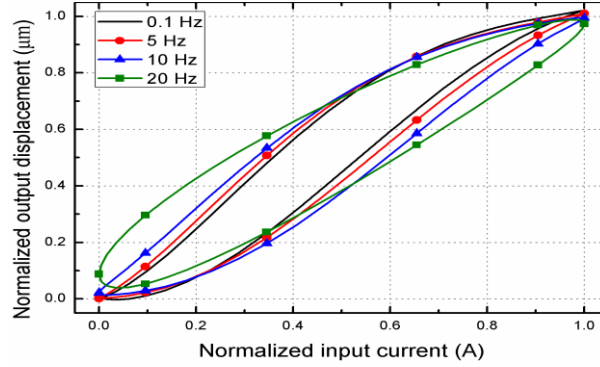


Figure 4. Frequency-dependent hysteresis loops.

An active vibration isolation used MA is used for experiment, so we mainly focused on the rate-dependent hysteresis model within a frequency range of 0.1 ~ 20 Hz. To validate the hybrid frequency-dependent hysteresis model for the MA, input current at a frequency of 0.1, 1, 10 and 20 Hz are applied to the MA, respectively. The parameters of weights update law are set as $\gamma = 0.95, c = 1, \mu_{\max} = 0.12$. The accuracy of the model is measured in terms of root-mean-square error (RMSE) and relative error (RE) defined as

$$\text{RMSE} = \sqrt{\sum_{k=1}^N (\tilde{y}_k - y_k)^2 / N}, \text{RE} = \sqrt{\sum_{k=1}^N (\tilde{y}_k - y_k)^2 / \sum_{k=1}^N (\tilde{y}_k)^2} * 100\% \quad (8)$$

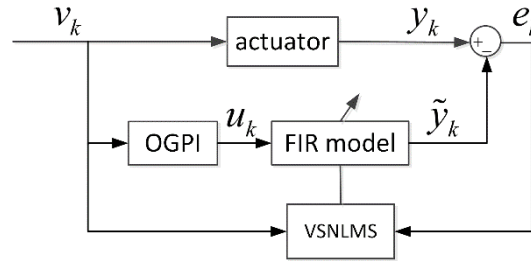


Figure 5. The block diagram of rate-dependent hybrid hysteresis model identification.

The experimental hysteresis loops and loops simulated based the rate-dependent hybrid hysteresis model (RDHM) are shown in Fig. 6. As a comparison, the simulated loops of generalized PI model alone are also depicted. The accuracies of those models are listed in Table. 2. It can be observed that the accuracy of OGPI model is unsatisfactory, but the proposed model can describe the frequency-dependent hysteresis fairly accurately from 0.1 Hz (super-low frequency) to 20 Hz. The RMSE of the hybrid model are less than 1 μm . The RE of the hybrid model is 1.64% when the frequency of inputs is 20 Hz, it is less than 1% under other frequencies. Moreover, the FIR filter part of the proposed model is build online, so the hybrid model can track the variation of system. Compared to the frequency-dependent PI model [5] and [6], the proposed hybrid hysteresis model is easy to implement in the hysteresis compensation control system.

4. Conclusions

In this paper, we proposes a hybrid frequency-dependent hysteresis model for magnetostrictive actuator. This hybrid hysteresis model comprise a generalized Prandtl-Ishlinskii static hysteresis

model that has analytical inversion and an adaptive FIR filter. A one-side operator is utilized in this model because of the output displacement and input current of MA are always positive. Besides, a modified coral reefs optimization algorithm is applied to identify the parameters to improve the accuracy. The developed model can describe the hysteresis loops well, the root-mean-square error of the proposed model are less than 1 μm . In the future work, the proposed model should be applied in the hysteresis compensation control system for magnetostrictive actuators and other smart material based actuators.

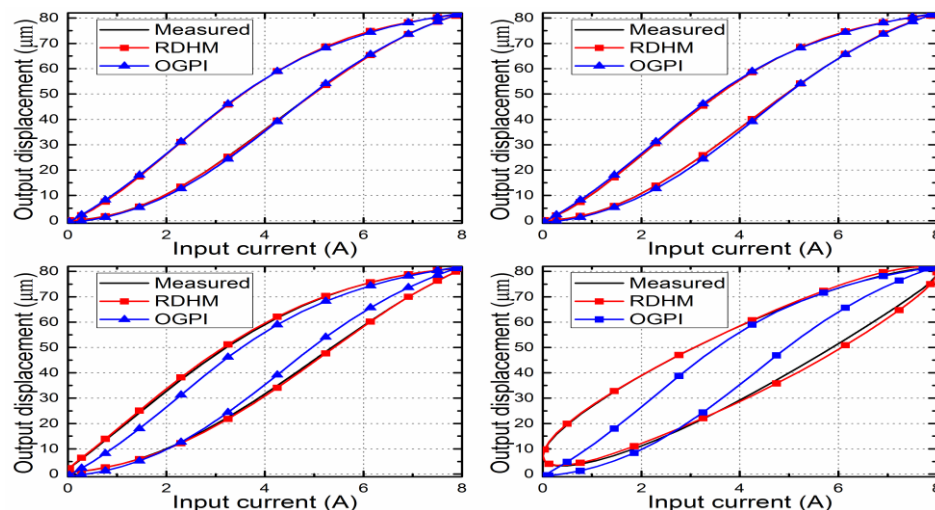


Figure 6. Model validation in view of frequency change. (a) 0.1 Hz. (b) 1 Hz. (c) 10 Hz. (d) 20 Hz. RDHM: the proposed rate-dependent hysteresis model, OGPI: one-side generalized PI model alone.

Table 2. Accuracy of model

Frequency (Hz)	RDHM		OGPI	
	RMSE(μm)	RE(%)	RMSE(μm)	RE(%)
0.1	0.074	0.142	0.478	0.876
1	0.116	0.219	0.650	1.20
10	0.442	0.825	3.24	6.05
20	0.991	1.89	7.71	14.7

5. Acknowledgments

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