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On code verification of 2D transient heat conduction in composite wall

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Abstract. The objective of the present research paper is to verify and validate the C++ code to solve a 2D transient heat conduction problem. A 2-D wall comprised of four different materials was chosen as a domain to analyse numerically in a commercial software FLUENT and C++. The results were compared and found that the non-uniformity in conductivity arises when there exists non-homogeneity in the material. It is also found that C++ results varies on 'f' value in finite volume method's (FCM's) discretization scheme which determines the solution scheme as explicit or implicit. The code was executed for explicit and implicit scheme and made sure that it follows Courant–Friedrichs–Lewy (CFL) conditions.

1. Introduction

Multilayer materials are composite media composed of several layers. Because of the additional benefit of combining various mechanical, physical, and thermal properties of different substances, a construction using multilayer elements is of interest [1]. The increased use of composite materials in the automotive and aerospace industries has stimulated research into experimental techniques and solution methods to determine the thermal properties of such materials. [2]. In the present paper, all available data on the problem have been reviewed and discussed in details. In addition, the result developed from C++ code has been compared with commercial ANSYS (FLUENT) software results.

The purpose of the existing review was to make some of the most important work done on the unsteady state composite material and validate the C++ code using CFD Finite control volume method to the composite wall problem. A literature survey revealed that majority of the earlier studies address the thermal conduction in one-dimensional, multi-layer bodies and very few researchers worked on analytical solutions of heat conduction problems in composite media [3,4,5]. Salt [6] examined the transient temperature solution in a two-dimensional, isotropic-composite slab. Mikhailov and Ozisik [7] analysed the three-dimensional form of the problem published by Salt [6]. Yan et al. [8] worked on exact series solutions for three-dimensional temperature distributions in two-layer bodies subject to various types of boundary conditions. The numerical steps leading to one-dimensional temperature solution in a two-layer body is reportedly Haji-sheikh and Beck [9].

Several decades ago, analytical solutions for one-dimensional time dependent composite heat conduction problems were developed and mathematical theory for such problems in more than one dimension was also developed during that time. Several methods were based on separation of variable and finite integral transform. However, the application of these methods was hindered by the fact that the eigenvalue problems, which are essential for this methodology, are difficult to solve. Suneet Singh et. al. [10] discussed the methodology as well as possible application in nuclear reactors of analytical solutions of two-dimensional multilayer heat conduction in spherical and cylindrical coordinates and solutions obtained with separation of variables. Moreover, in two and three dimensional Cartesian coordinates these eigenvalues were imaginary rendering their solutions even more difficult [10].

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Time dependent temperature distribution in such components, with the presence of sources (with all three types of boundary conditions) can be obtained analytically or numerically. Though not always available, exact analytical solutions are desirable since: (1) better insight can be gained through the mathematical form of an analytical solution compared to a discrete numerical solution; (2) these analytical solutions can be used as benchmark or reference results to verify numerical algorithms and codes [11].Lu et al. [12] combined the Laplace transform and the Separation of variables methods to determine the temperature distribution in 2D rectangular and cylindrical media. Haji-Sheikh and Beck [13] used the Green's function approach to determine the temperature distribution in a 3D two-layer orthotropic slab.

2. Problem statement

A two dimensional transient heat conduction problem is solved numerically using C++. A very long rod shown in figure 1 is composed of four different materials and each rod interacts with the surrounding in a different manner; the boundary condition is different at different faces of the rod. At the bottom of the rod, the boundary condition is isotherm (constant temperature at 18°C). At the top of the rod, there is a uniform heat flow at the rate of 89 W/m of length. At the left side of the rod, it contacts with the fluid which has 35°C temperature and 8 W/m^{2}K heat transfer coefficient, while on the right side it interacts with the temperature (11+0.006*t°C where t=time) which changes along with the time (transient temperature). Initial temperature of the rod is 11°C. The exercise is to simulate a temperature distribution throughout the rod with the time step up to 10,000 and to make sure that the solution convergence as well.Table 1 illustrates the dimensions of the figure 1. M1, M2, M3 & M4 are the four different materials with different properties. The physical properties of these materials are represented in table 2.

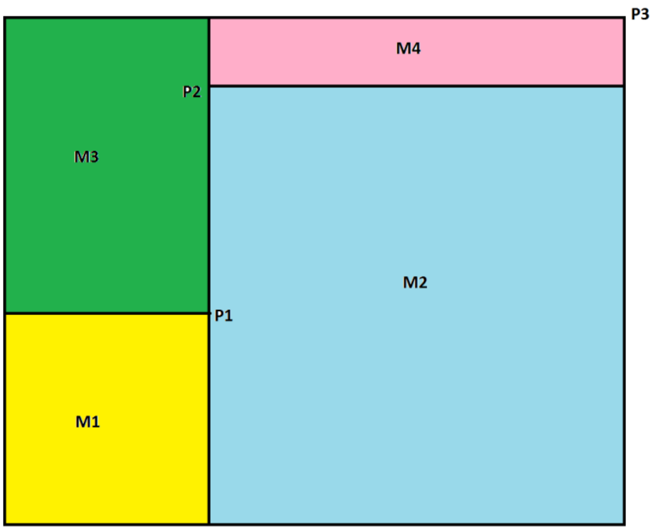

Figure 1.2D Transient heat conduction material

Table 1. Dimensions of geometry

Points	X (m)	Y(m)
P1	0.3	0.4
P2	0.3	0.8
P3	0.9	0.9

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Table 2. Properties of materials

S.No	Material	Density	Specific heat	Conductivity
1	M1	2500	970	180
2	M2	2700	930	140
3	M3	2200	710	150
4	M4	1700	920	140

3. Numerical method

The code is developed in C++, by implementing Finite control volume method to solve this exercise. A uniform grid of 10x10was designed. Due to the composite nature of material i.e the non-homogeneity of the material, there exists a possibility of araising non-uniform conductivity between the materials [13]. So, by assuming a linear variation of conductivity between two neighbouring grid points, we can overcome the non-uniform conductivity. Figure 2 shows an equation used in code to overcome the non-uniform conductivity of the material.

```
KE[i][j]=((0.5*k[i][j])+(0.5*k[i+1][j]));
KW[i][j]=((0.5*k[i][j])+(0.5*k[i-1][j]));
KN[i][j]=((0.5*k[i][j])+(0.5*k[i][j+1]));
KS[i][j]=((0.5*k[i][j])+(0.5*k[i][j-1]));
```

Figure 2. Equations used in code to calculate thermal conductivity

Transient heat conduction equation (Fourier heat law) has been discretizedand applied for each control volume and the boundary conditions are assumed to all control volumes appearing in the boundary. Since this is an unsteady (transient) problem, solution will depend on time as we march the equation. We need a solver to iterate the solution and the error can be fixed to some (10^{-6}), so that solver will subtract the solution at $(n)^{th}$ time and $(n-1)^{th}$ time and check with the error and run accordingly.

The general unsteady two dimensional heat conduction equation:

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) \quad (1)$$

The discretization equation is now derived by integrating Eq. (1) over the control volume and over the time interval from t to t+Δt. Thus

$$\rho c\int_{w}^{e}\int_{t}^{t+\Delta t}\frac{\partial T}{\partial t}dt\,dx = \int_{t}^{t+\Delta t}\int_{w}^{e}\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)dx\,dt + \int_{t}^{t+\Delta t}\int_{s}^{n}\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)dy\,dt \quad (2)$$

Fully explicit scheme is used to solve this exercise. By having a weighting factor (f) zero in the FCM’s discretizes equation, problem can be solved fully explicit and by varying the f factor from 0 to 1, we can reach fully explicit to fully implicit scheme respectively [15,16]. Once the fully explicit scheme is chosen, then the solution for the succeeding time step will only depend on the preceding time step's solution. Since, this is an explicit method, it should satisfy CFL condition.

The governing equation is discretized into linear algebraic equation and this equation is solved for each node at each time step by using Tri-diagonal -Matrix Algorithm (TDMA) method.

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4. Result and discussion

Simulation in ANSYS Fluent as well as C++ code runs for the time of 10000s. Time step chosen to run the simulation satisfies the Courant–Friedrichs–Lewy (CFL) condition. By bi-linear interpolation, the values of temperature are obtained at two different points and the same is shown in figure 3 as an equation used in code.

```
TP1=((((y1-0.62)/(y1-y2))*(TV1))+(((0.62-y2)/(y1-y2))*(TV2)));
TP2=((((y22-0.16)/(y22-y11))*(T111))+(((0.16-y11)/(y22-y11))*(T121)));
```

Figure 3. Equations used in code to calculate temperature at two different points ((0.73,0.62) and (0.6,0.16))

Mesh convergence study was also included in the present work. By increasing the mesh size until 10x10 from 1x1, it showed a change in the values of temperature. Increasing the mesh six further, it was found that the values of temperature remain unchanged. Therefore, it is concluded that the solution obtained at 10x10 mesh size is a mesh convergence values of temperature. Both simulations using C++ code and Fluent as shown in Fig. 4 & 5 achieve the variations of temperature with respect to time at specific points (0.73, 0.62) & (0.60, 0.16) for uniform grid size 10x10. In Fluent, the values of temperature are taken at exact points. This may be one of the reasons for the deviation in temperature value at some time steps. In addition, the right side boundary condition of the problem is transient, which means the wall temperature changes with respect to the time. In C++, it is possible to give the transient condition and the value of the temperature of the wall can get updated with respect to time. While simulating in Fluent we have to update the value of temperature of right boundary at every time step according to the given condition (11+0.006*t). This may also cause the difference in temperature value measured at same time step in C++ & Fluent.

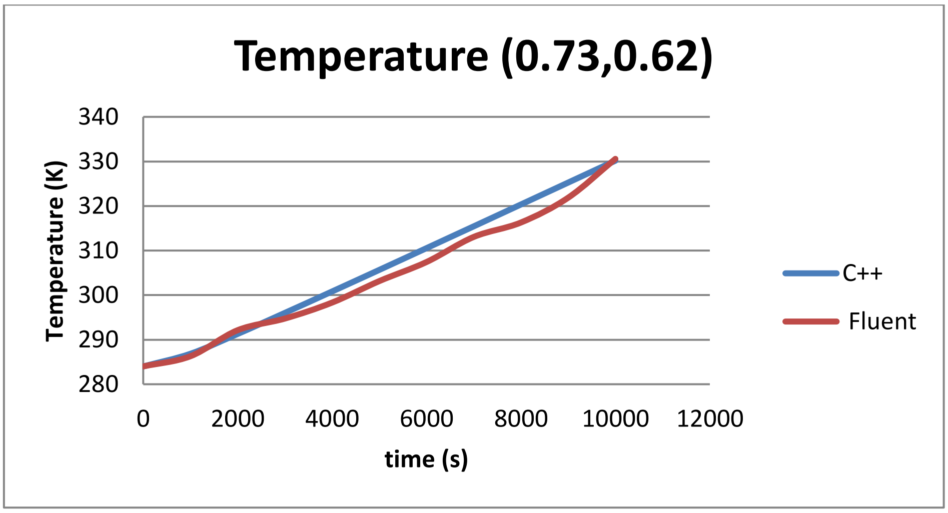

Figure 4.C++ vs fluent results for temperature at (0.73,0.62)

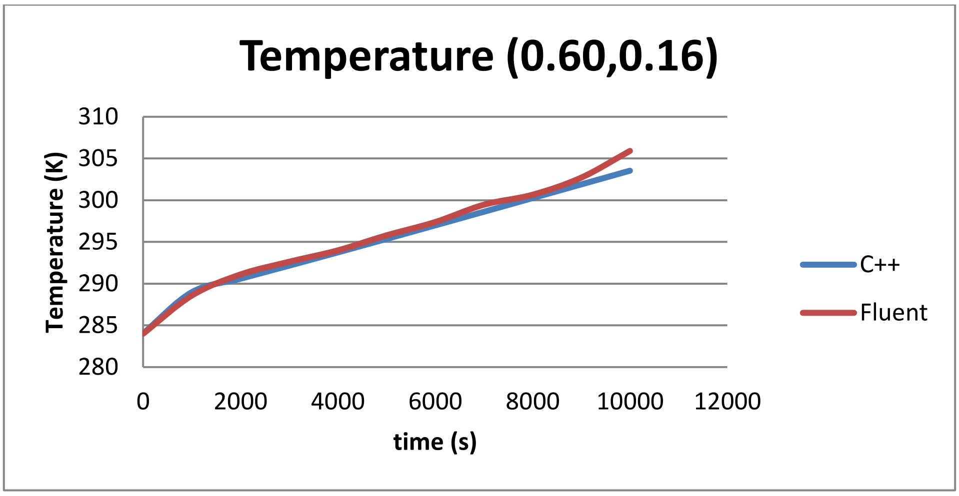

Figure 5.C++ vs fluent results for temperature at (0.60, 0.16)

5. Conclusion
Numerical solution of transient heat conduction equation which was obtained for 2D composite wallthrough the code developed in C++, using uniform grid size of 10x10 were compared with the results obtained by the simulation in Fluent. Fully explicitmethod was used to solve the algebraic equations. The C++ result shows an excellent agreementwithFluentresults. It is concluded that the values of temperature obtained at two different locations from Fluent and C++ are almost same at every time step. The slight deviation aroused from the values of temperature at two different locations is because of the usage of bi-linear interpolation equation that was used to extract the values of temperature. Assumption of considering a linear variation of conductivity between two neighbouring grid points at the interface of two different materials, does not affect much on the variation of temperature at single point with time.

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