

Vibration suppression of sisal-epoxy bio-composite laminated beam

Kallol Khan¹, Koustav Roy² and Sayan Batabyal³

¹ Department of Mechanical Engineering, NIT Durgapur, India

² Department of Mechanical Engineering, NIT Durgapur, India

³ Department of Aerospace Engineering and Applied Mechanics, IIST Shibpur, India

E-mail: roy.koustav12@gmail.com, sayanb6292@gmail.com

Abstract: To meet with the market demands of increasing articles made of different polymeric materials a large number of non-biodegradable polymeric composites have been developed without realizing about the ecological havoc it created. Here we discuss about the Vibration Suppression of Sisal-Epoxy bio-composite containing a layer of magnetostrictive material as actuator. The analytical solutions have been developed on the basis of first order beam theory (TBT) and velocity feedback control to obtain a comparison between different lay-ups and different locations of the magnetostrictive layer. The self-vibration damping property of Sisal Fiber is not considered for calculations.

1. Introduction

Over the past few years the application of composite materials has increased manifold, penetrating research labs and conquering new markets relentlessly. The need for light-weight, low cost and seismic resistant materials has emphasised more development in the field of composites and smart materials.

Due to the manufacturing of flexible engineering structures in aerospace/automotive industries and rehabilitation of pre-existing structures, it is essential to control vibrations in the composite structures improving performance and longevity. Vibration control can be achieved in composites by impregnating it with smart materials displaying magnetostrictive property. Magnetostrictive materials as actuators have several advantages such as the ability to generate large strains and easy embeddability into the host materials. Here we use Terfenol-D (alloy of Terbium, Iron and Dysprosium) as magnetostrictive material.

The growing concern about environment protection, ecological balance and biodegradability has led to the growth of natural fibres as reinforcement materials. Our study here is about vibration suppression of a simply supported beam containing Sisal fibre as reinforcement material embedded in epoxy matrix. Sisal fibre is obtained from the leaves of the plant *Agave sisalana* (one of the important fibre crops of India) and displays interesting mechanical properties.



2. Theoretical formulation-

2.1. Objective-

The objective of this study is to represent the formulation of a problem based on the first order beam theory i.e. Timoshenko theory and study the effects of material properties of laminates and the properties of magnetostrictive layer on vibration suppression of the composite. Also, this paper aims to bring out the importance of the location of the magnetostrictive layer in controlling the laminate.

2.2. Displacement and Strain fields-

According to the Timoshenko Theory, the displacement field can be assumed as shown in Equation (1) and corresponding linear strains are shown in Equation (2)^[1].

$$\begin{aligned} u(x, z, t) &= z\phi \\ v(x, z, t) &= 0 \\ w(x, z, t) &= w_0(x, z, t) \end{aligned} \quad (1)$$

$$\begin{aligned} \varepsilon_{xx} &= z \frac{\partial \phi}{\partial x} \equiv z \varepsilon_{xx}^{(1)} \\ \gamma_{xz} &= \phi + \frac{\partial w_0}{\partial x} \equiv \gamma_{xz}^0 \end{aligned} \quad (2)$$

Considering the presence of a velocity proportional closed-loop feedback control^[2], the magnetic field intensity H is proportional to the coil current, $I(x,t)$ (as shown in Equation (3)). The constant of proportionality also called Coil constant, k_c can be calculated as given in Equation (4).

$$H(x, t) = k_c I(x, t) \quad (3)$$

$$k_c = \frac{n_c}{\sqrt{b_c^2 + 4r_c^2}} \quad (4)$$

2.3. Equations of motion-

Using Hamilton's principle, we obtain the integral form of the equations of motion as shown in Equation (5).

$$0 = \int_0^T \int_0^L \int_A [\sigma_{xx} (z \delta \varepsilon_{xx}^{(1)}) + \sigma_{xz} (\delta \gamma_{xz}^{(0)})] dA dx dt - \int_0^T \int_0^L \int_A \rho [(z\phi)(z\delta\phi) + w\delta w_0] dA dx dt - \int_0^T \int_0^L q \delta w_0 dx dt \quad (5)$$

Upon solving the integral form, the analytical solution can be obtained in terms of displacement variables (w_0, ϕ) as shown in Equation (6a) and (6b).

$$-D_{11} \frac{\partial^2 \phi}{\partial x^2} + A_{55} \phi + \beta \frac{\partial^2 w_0}{\partial t \partial x} + I_2 \frac{\partial^2 \phi}{\partial t^2} = 0 \quad (6a)$$

$$-A_{55} \frac{\partial \phi}{\partial x} - q + I_0 \frac{\partial^2 w_0}{\partial t^2} = 0 \quad (6b)$$

The constitutive relations and other notations used in the above equations are shown below-

$$\begin{Bmatrix} I_0 \\ I_2 \end{Bmatrix} = \int \rho \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} dz \quad (7)$$

$$D_{11} = \int \bar{Q}_{11}^{(k)} z^2 dz; \quad A_{55} = \int \bar{Q}_{55}^{(k)} dz \quad (8)$$

$$\beta = k_c \cdot c(t) \int Q_{11}^{(m)} d^{(m)} z dz \quad (9)$$

Here we discuss the simply supported boundary conditions. Assuming solution of the form-

$$w_0(x, t) = W(t) \sin \frac{n\pi x}{a} \text{ and } \phi(x, t) = X(t) \cos \frac{n\pi x}{a}$$

We can take external load $q=0$ for vibration control and for solving the differential equation we take- $W(t) = W_0 e^{\lambda t}$ and $X(t) = X_0 e^{\lambda t}$. The eigenvalue λ [2] can be written as $\lambda = \alpha \pm i\omega_d$, so that-

$$w_0(x, t) = \frac{1}{\omega_d} e^{-\alpha t} \sin \omega_d t \sin \frac{n\pi x}{a} \quad (10)$$

3. Tables-

Table 1 contains the details of material properties of Sisal-Epoxy composite. Similarly, Table 2 specifies the material properties of the magnetostrictive layer i.e. Terfenol-D. Table 3 and table 4 give us the value of the inertial coefficients and the stiffness coefficients for different lamination schemes respectively. In table 5, the vibration parameters α and ω_d are listed for different lamination schemes.

Table 1: Material Properties of Sisal-Epoxy composite [3,4]

$E_{11}(\text{GPa})$	$E_{22}(\text{GPa})$	$G_{12}(\text{GPa})$	$G_{13}(\text{GPa})$	$G_{23}(\text{GPa})$	ν_{12}	$\rho(\text{kg/m}^3)$
12.1	9.17	4.59	4.59	1.83	0.7	1220

Table 2: Material Properties of magnetostrictive layer [5]

$E_m(\text{GPa})$	ν_m	$P(\text{kg/m}^3)$
26.5	0	9250

Table 3: Inertial coefficients for different laminates

Laminate	I_0	$I_2(\times 10^{-4})$	$I_4(\times 10^{-9})$	$I_6(\times 10^{-14})$	$-\beta$	$-\epsilon(\times 10^{-4})$
[45/m/45/0/90] _s	28.2600	2.9974	4.0336	5.9804	30.9785	3.872
[m/90 ₄] _s	28.2600	4.2822	8.2734	1.6888	39.8295	8.1650
[m/0 ₄] _s	28.2600	4.2822	8.2734	1.6888	39.8295	8.1650

Table 4: Stiffness Coefficients for different laminates

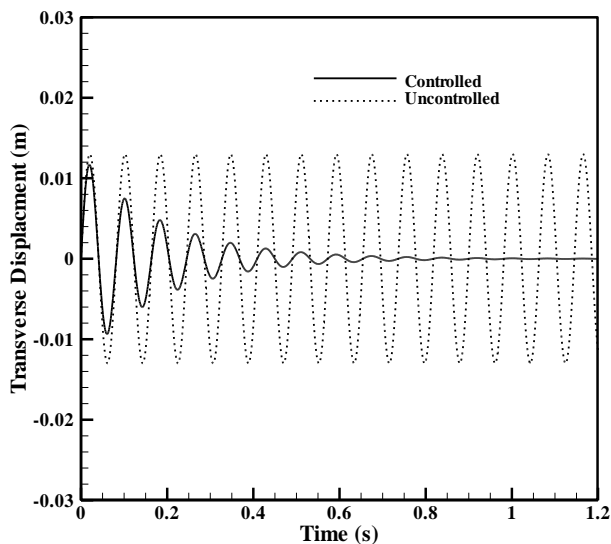
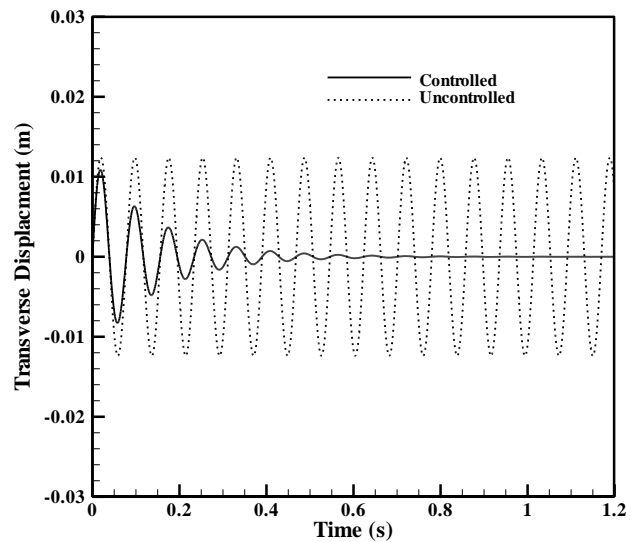
Laminate	$D_{11}(\times 10^3)$	$F_{11}(\times 10^{-2})$	$H_{11}(\times 10^{-7})$	$A_{55}(\times 10^7)$	$D_{55}(\times 10^2)$	$F_{55}(\times 10^{-3})$
$[45/m/-45/0/90]_s$	1.7214	2.53	4.3911	5.2180	5.206733	7.2
$[\pm 45/m/0/90]_s$	1.6213	2.34	4.1017	5.2180	4.001933	4.9
$[m/90_4]_s$	1.7	2.82	5.3575	4.114	6.169133	11.9
$[m/0_4]_s$	1.8989	3.02	5.5757	6.322	7.346733	13

Table 5: Vibration Parameters for different mode shapes and laminates

Laminate	$-\alpha \pm \omega_d$		
	n=1	n=2	n=3
$[45/m/-45/0/90]_s$	-5.4072 ± 76.8229	-21.6009 ± 307.0947	-48.4980 ± 690.2267
$[\pm 45/m/0/90]_s$	-3.8625 ± 74.6412	-15.4324 ± 298.3963	-34.6577 ± 670.7618
$[m/90_4]_s$	-6.9512 ± 76.2119	-27.7585 ± 304.5953	-62.2839 ± 684.3976
$[m/0_4]_s$	-6.9520 ± 80.5855	-27.7708 ± 322.1285	-62.3459 ± 723.9907

4. Results –

In the following figures, the variation of transverse displacement has been presented for different modes of vibration in different vibration schemes for both controlled and uncontrolled motion.

**Figure 1:** Controlled and Uncontrolled motion at mid-point for layup $[45/m/-45/0/90]_s$ for mode shape $n=1$ **Figure 2:** Controlled and Uncontrolled motion at half of mid-point for layup $[45/m/-45/0/90]_s$ for mode shape $n=2$

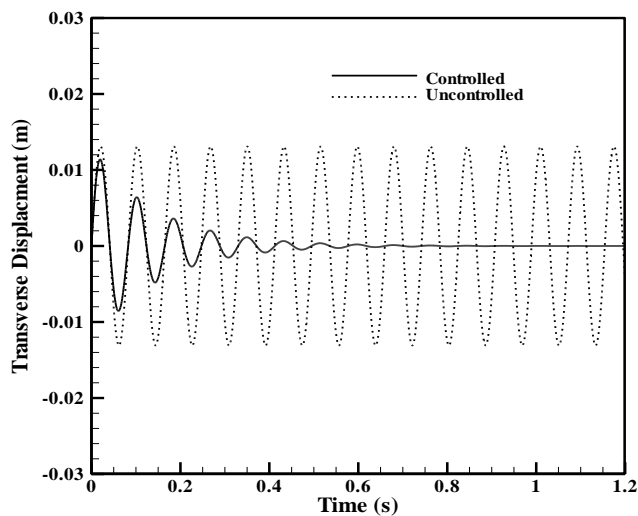


Figure 3: Controlled and Uncontrolled motion at mid-point for layup $[m/90_4]_s$ for mode shape $n=1$

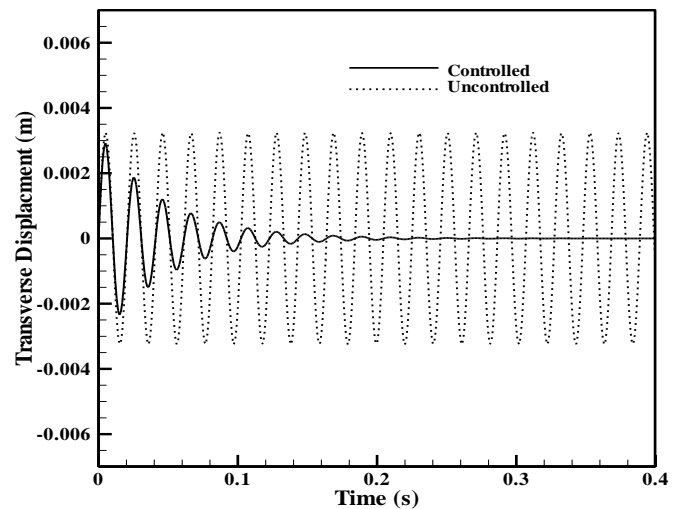


Figure 4: Controlled and Uncontrolled motion at mid-point for layup $[m/0_4]_s$ for mode shape $n=1$

The figure 5 shows a comparative study on the various lay-ups and the transverse displacement at the mid-point for the first mode shape is plotted.

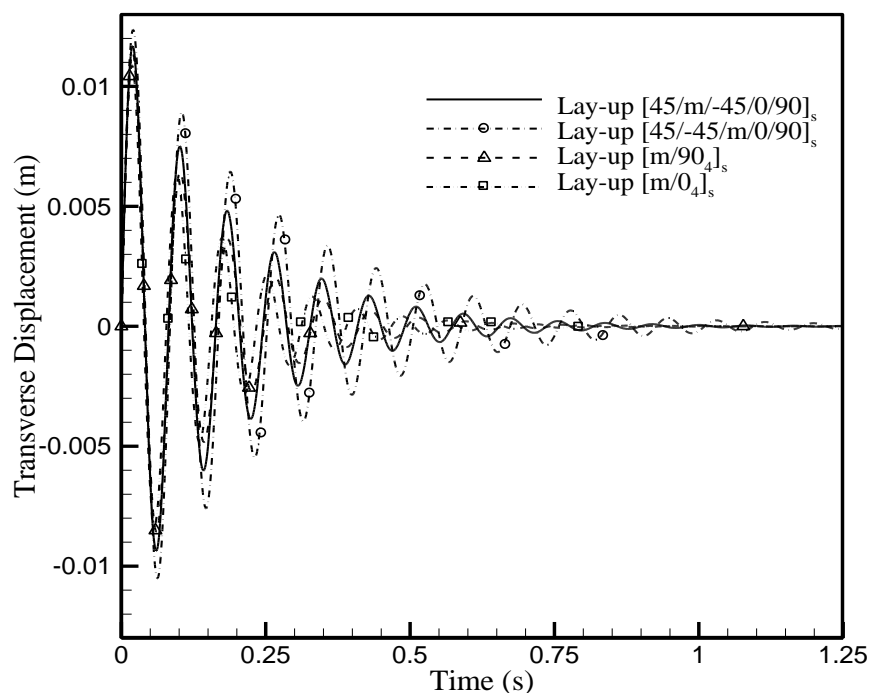


Figure 5: Controlled motion of Sisal fiber based composite laminated beam (at $x=a/2$) for different lamination scheme

It is also observed that the vibration suppression depends upon the position of the magnetostrictive layer in the lamination scheme. A new parameter Suppression Time^[2] can be defined as the time needed to reduce the uncontrolled vibration amplitude to $(1/10)^{\text{th}}$ of its value.

Table 6: Vibration Suppression time ratio for different laminates

Laminate	z(mm)	t at $\frac{w_{0,max}}{10}$	$T_s = \frac{t}{t_{max}}$
$[m/\pm 45/0/90]_s$	4.5	0.080	0.111
$[45/m/-45/0/90]_s$	3.5	0.200	0.143
$[\pm 45/m/0/90]_s$	2.5	0.140	0.200
$[\pm 45/0/m/90]_s$	1.5	0.240	0.333
$[\pm 45/0/90/m]_s$	0.5	0.720	1.000

5. Conclusion:

We observe from the above plots that the vibration suppression is always faster in higher modes of vibration as compared to the first mode of vibration for different lamination schemes.

We also observe that the vibration suppression depends upon the position as well as the repetition of the layer with 0° fiber orientation angles. So as the layer with 0° fiber orientation is placed close to the mid-plane, the vibration suppression occurs faster.

From Table 6 we can conclude that the maximum suppression time occurs for the lay-up $[\pm 45/0/90/m]_s$ in which the magnetostrictive layer is close to the neutral axis. This result also satisfies for other composite materials like CFRP.

The Following results can be extended further for cylindrical beams and shells and the solutions can be analyzed for higher order beam theories as well.

6. References-

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