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Wave force on the submerged inclined thin plate in intermediate depth of water

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Abstract

The paper presented is a solution of wave force, using small amplitude, linear wave theory on two-dimensional inclined submerged rectangular thin plates. The solution method is confined in a finite domain, which contains both the region of different depth of water and the plate. In this paper, analytical solution is derived and the result is validated with numerical results. Simpson's 1/3 rule is used here for the validation. The variations of horizontal and vertical wave forces on the plate with respect to the wave amplitudes are obtained in the intermediate depth of water and at a different plate angle. It is observed that the wave forces on the plate are high at the free surface for d/L = 0.24 compare to the other relative depth. It is also observed that the wave forces of the two types of plate are gradually converging with the decreasing value of the relative depth. Laplace's equation and boundary value problems are solved in a finite domain, by the method of separation of variables and the small amplitude linear wave theory.

Keywords: Linear wave; Laplace's equation; Horizontal and Vertical Wave force; Inclined Rectangular thin plate, Numerical result

1. Introduction

The interaction between linear wave and maritime structures of the inclined plane is of fundamental importance in coastal and ocean engineering. An attempt is made to design a new type wave energy converter, wherein kinetic energy obtainable from the ocean wave can be easily converted into the form of potential energy and subsequently can be used for any other purpose. A rectangular thin plate is considered, which acts as an energy absorber of wave energy converter and moves in the surge direction due to the action of an ocean wave. The plate is an element of the entire design of wave energy converter. The aim of this paper is a solution of ocean wave force on two-dimensional inclined submerged rectangular thin plate in intermediate depth of water and is the initial step for further research work in the development and design of wave energy converter. In the present paper, the force exerted on a submerged inclined rectangular thin plate due to the normally incident waves is studied by means of linear wave theory and the result is validated with the numerical results. Simpson's 1/3 rule is used here for the validation.

Various types of studies on the wave force were carried out at different time by many authors on different types of submerged structures, like vertical barrier of different geometrical shapes, etc. Hanssen and Torum [1] (1999) experimentally studied the breaking wave forces on tripod concrete structures on shoal using a Morison's equation. The main purpose of their study was to investigate both horizontal and vertical forces as well as overturning moments due to wave acting on the tripod.

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Meylan [2] (2001) presented a variational equation for the wave force on floating thin plates in his article ‘‘A variational equation for the wave forcing of floating thin plates’’. He had used free surface Green's function for the solution. Solutions of the variational equation were presented for some simple thin geometric plates using polynomial-based functions. Maiti and Sen [3] (1999) investigated a numerical time-simulation algorithm for analyzing highly nonlinear solitary waves interacting with plane gentle and steep slopes, employing mixed Eulerian–Lagrangian method and also found out the pressures and forces on the inclined wall by Bernoulli's equation.

Sundaravadivelu et al. [4] (1997) experimentally investigated the measurement of wave force and moments on an intake well due to regular wave using Linear Diffraction Theory.

Tsai and Jeng [5] (1990) studied the forces on vertical walls due to obliquely incident waves employing Fourier Series. For the calculation of the short crested wave system, Prabhakar and Sundar [6] (2001) numerically computed the variations of pressure on vertical walls at a constant depth of water by the Fourier Series approximation method and their results obtained from this method were compared with the experimental results conducted by Nagai [7] (1969). Prabhakar and Sundar also compared their results with the results of Fenton [8] (1985) for the pressure curves at still water level and sea bed. Mallayachari and Sundar [9] (1994) investigated the wave pressure exerted on vertical walls due to regular and random waves, using Fourier Series approach.

Most of the other authors investigated the breaking wave forces exerted on a vertical circular cylinder (e.g., Sawaragi and Nochino, [10] 1984; Apelt and Piorewiez, [11] 1987; Hovden and Torum, [12] 1991). Neelamani et al. [13] (2002) experimentally investigated wave forces on a seawater intake structure consisting of a perforated square caisson in a regular and random wave.

Deb Roy and Ghosh [14] (2006) investigated the wave force on vertical submerged circular thin plate in shallow water by the use of the Morison's equation. The main purpose of their study was to investigate both horizontal and overturning moments due to regular wave acting on the plate.

Deb Roy and Ghosh [15] (2009) investigated the wave force on vertical submerged circular thin plate in shallow water due to oblique wave at different incident angle of the wave.

Deb Roy and Ranjan [16] (2015) theoretically investigated of wave force on vertical submerged rectangular thin plate at shallow water by the use of Fourier series technique.

Teo [17] (2003) theoretically investigated wave pressure on a vertical wall due to short-crested waves: fifth-order approximation.

Jeng [18] (2002) theoretically investigated wave kinematics of partial reflection from a vertical wall at different angle due to short-crested waves: third-order approximation.

2. Mathematical Formulation

Consider a thin rectangular plate Γ_b of dimensions l_1, l_2 and l_3 be inclined at an angle θ ($0 \le \theta \le \pi/2$) with the horizontal and submerged in intermediate depth of water. Here l_1 is the thickness of the plate, l_2 is the width of the plate and l_3 is the height of the plate. The system is idealized as a 2D Cartesian coordinate system and hence l_2 is considered a unity. The plate is subjected to an incoming wave which is assumed to be travelling in the x-direction. The fluid domain occupies the region $-d \le z \le 0$, $-\infty < x < +\infty$ except for the thin plate in the fluid region. The plate Γ_b of negligible draft occupies the following positions, as shown in Fig. 1.

Consider incompressible inviscid irrotational flow without surface tension and with atmospheric pressure $P = 0$. The system is idealized as 2D and Cartesian co-ordinates are employed in the plane x–z, vertical to the fluid surface. The z-axis is directed vertically upward from the still water level, which is measured positive upwards and the *x-axis* is measured along the direction of propagation of waves. The plane x–y is horizontal and coincides with the surface of the fluid when in equilibrium. The geometry is as depicted in Fig. 1.

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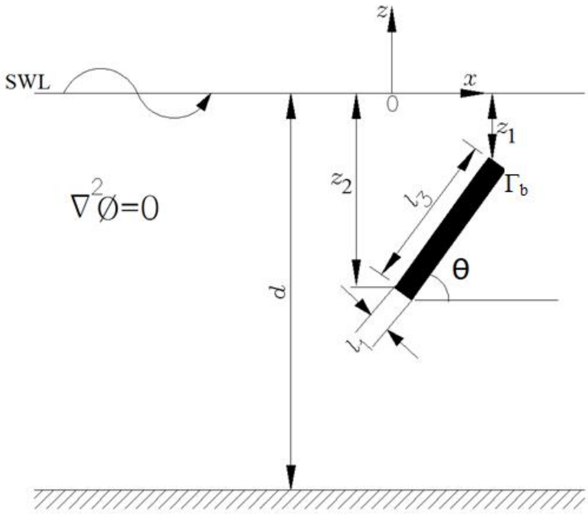

Fig. 1. Definition sketch of the plate positions

2.1 Governing equation

The governing equation and the boundary conditions are non-dimensionalize by the following non-dimensional parameters.

$$\left(x^*, z^*, \eta^*, d^*\right) = k\left(x, z, \eta, d\right),\ \phi^* = \sqrt{\frac{k^3}{g}}\phi,\ t^* = \omega t,\ \omega^* = \frac{\omega}{\sqrt{gk}} \tag{1a}$$

$$P^* = \frac{kP}{\rho g},\ F_x^* = \frac{F_x}{0.5\rho g d^2},\quad F_z^* = \frac{F_z}{0.5\rho g d^2}, \tag{1b}$$

where $k=2\pi/L$ (L=wave length) and $\omega=2\pi/T$ (T=time period) are the wave number and wave frequency respectively. Dimensionless quantities are denoted by the symbol star (*) and will be omitted in the sections 2, 3 and 4 for the sake of simplicity.

Let $\phi(\bar{x}, t)$ denote velocity potential and the governing equations for irrotational wave motion are given by the Laplace equation:

$$\nabla^2\phi\left(\bar{x}, t\right) = 0, \tag{2}$$

2.2 Boundary conditions

To solve the governing Eq. (2) a set of boundary conditions is required. They are summarized as below:

(a) The dynamic boundary condition at the free surface (DFSBC):

$$\omega\phi_t + \eta = 0, \qquad \text{at } z = 0 \tag{3}$$

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(b) The kinetic boundary condition at the free surface (KFSBC):

$$\phi_z = \omega\eta_t, \qquad \text{at } z = 0 \tag{4}$$

(c) Combined free surface boundary condition (CFSBC):

$$\phi_z + \omega^2\phi_{tt} = 0, \qquad \text{at} \quad z = 0, \tag{5}$$

(d) Bottom boundary condition (BBC):

$$\phi_z = 0, \qquad \text{at } z = -d \tag{6}$$

where η is the wave profile elevation, g is the acceleration due to gravity, t is the time, d is the depth of water.

3. Method of Solutions

Employing the Laplace Eq. (2) into the BBC and the linearized DFSBC, the desire velocity potential for small amplitude linear waves can be derived by the method of separation of variables (Ippen, [19] 1966 or Sorensen, [20] 1978). A useful form of this velocity potential is,

$$\phi = \frac{1}{\omega}\frac{\cosh(z+d)}{\cosh(d)}\sin(x-t), \tag{7}$$

Integrating the velocity potential into the linearized DFSBC and letting z=0 yields the equation for the wave surface profile,

$$\eta = \cos(x-t), \tag{8}$$

The dispersion relation,

$$\omega^2 = \tanh d, \tag{9}$$

4. Pressure and force on the inclined plate

Evaluation of pressure induced by the wave on the plate can be determined from Bernoulli's equation:

$$P = -z - \omega\phi_t - \frac{1}{2}\left[\phi_x^2 + \phi_z^2\right], \tag{10}$$

The Horizontal force F_x and the vertical force (F_z) per unit width of the plate can be determined by integrating the above expression over the wetted contour.

$$F_x = \int_{\Gamma_b} P n_x ds, \tag{11}$$

$$F_z = \int_{\Gamma_b} P n_z ds \tag{12}$$

Here ρ is the water density and n_x and n_z are the x-component and the z-component of the exterior unit normal vector $\bar{n}$ on Γ_b. The plate is inclined at an angle θ with respect to the horizontal, $n_x = \sin\theta$ and $n_z = \cos\theta$ (Maiti eta. el. 1999). Determination of the time derivative ϕ_t and velocities ϕ_x and ϕ_z are done by first order difference rules. In the configuration shown there is no variation of pressure in the y-direction, as the waves are assumed to be long crested and propagation in the x-direction. Refer to the Fig. 1 consider first the approximate wave induced pressure on the plate that is in the x-direction and z-direction located at $0 \le x \le l_3\cos\theta$.

5. Result and discussion

The aim of this paper is to investigate the effect of ocean wave force by linear wave theory on inclined submerged rectangular thin plate as shown in Fig. 1. For each type results have been estimated up to six decimal places in order to achieve accuracy. The results have been discussed in intermediate depth of water ($0.05 \le d/L \le 0.5$).

To demonstrate the effects of the first-order wave on the plate, representative sets of analytical results with the numerical results for horizontal force (F_x) for the two types of vertically submerged rectangular thin plate in intermediate depth of water between the ranges $0.1 \le d/L \le 0.24$ are shown in Table 1-2. It is observed from the Table 1-2 that analytical results are closely in agreement with the numerical results and their percentage errors are very less in each case. As shown in the Table 1-2 wave forces on the plate are more of a relative depth of water (d/L = 0.24) comparable to the lower relative depth of water (d/L = 0.16, 0.12 and 0.10). From the Table 1 it is clear that, on a/d (= 0.033, 0.075, 0.116) and d/L (= 0.24), the agreement of analytical results with the numerical results is less than equal to 3.3493% for type I (z/d = 0). This concludes that wave force on the plate is important at a relative depth of water (d/L = 0.24) compared to the other relative depth of water.

Fig. 2 shows the plot of the computed results of horizontal force (F_x) on the plate which increase with the dimensionless wave amplitude (a/d = 0.033 to 0.116) for various relative depths of water d/L = 0.24, 0.16, 0.12, 0.10. It is observed that the horizontal force on the plate is very high for higher relative depth of water (d/L = 0.24) compared to the lower relative depth of water (d/L = 0.16, 0.12, 0.10) for an (a/d) ratio. Also, it is seen that the horizontal force gradually decreases with the

Table 1
Comparison of horizontal force between analytical values (F_x) and the numerical values (F_n) in various relative depths of water for d = 3m, l_1 = 1mm, l_3 = 0.5m.

	T = 3s	d/L = 0.24		T = 4s	d/L = 0.16	
a/d	$F_x \times 10^{-9}$ Analytical	$F_n \times 10^{-9}$ Numerical	Error (%)	$F_x \times 10^{-9}$ Analytical	$F_n \times 10^{-9}$ Numerical	Error (%)
Type I (z/d=0)						
0.033	1.215921	1.175209	3.3482	0.561357	0.542608	3.3398
0.075	2.697233	2.606919	3.3484	1.231161	1.190041	3.3400
0.116	4.135670	3.997182	3.3486	1.865530	1.803218	3.3402
Type II (z/d=-1)						
0.033	0.576171	0.556878	3.3484	0.388511	0.375535	3.3399
0.075	1.257797	1.215676	3.3488	0.842258	0.814125	3.3401
0.116	1.896547	1.833026	3.3493	1.260570	1.218461	3.3404

decreasing *z/d* (= 0 to -1) for an (*a/d*) ratio. Fig. 2 also shows that the horizontal force of two types, which gradually converges to the decreasing value of the relative depth of water and when *d/L* = 0.10, the convergence is very close. Fig. 2 also shows that the results obtained from analytical solutions are in good agreement with the results obtained from numerical solutions.

Fig. 3 shows the plots of horizontal force (F_x) and vertical force (F_z) versus non-dimensional wave amplitude (*a/d)* on the inclined plate for θ = 90^0 60^0 45^0 30^0 at a relative depth d/L=0.24. It is observed that F_x decreases with the decrease of plate angle and the vertical force increases with the decrease of plate angle. F_x and F_z are approximately same at θ = 45^0. It is also observed that the vertical force (F_z) is high at θ=30^0 compared to the horizontal force (F_x).

Table 2
Comparison of horizontal force between analytical values (F_x) and the numerical values (F_n) in various relative depths of water for *d* = 3m, l_1 = 1mm, l_3 = 0.5m.

a/d	*T* = 5s *d/L* = 0.12 $F_x\times10^{-9}$ Analytical	$F_n\times10^{-9}$ Numerical	Error (%)	*T* = 6s *d/L* = 0.10 $F_x\times10^{-9}$ Analytical	$F_n\times10^{-9}$ Numerical	Error (%)
Type I (z/d=0)						
0.033	0.330093	0.319077	3.3371	0.219537	0.212213	3.3358
0.075	0.719403	0.695395	3.3372	0.476745	0.460841	3.3359
0.116	1.082817	1.046680	3.3373	0.714827	0.690980	3.3360
Type II (z/d=-1)						
0.033	0.264637	0.255806	3.3371	0.189167	0.182857	3.3358
0.075	0.572129	0.553035	3.3372	0.408413	0.394789	3.3359
0.116	0.853724	0.825232	3.3374	0.608534	0.588233	3.3360

Fig.4 shows the plots of horizontal force (F_x) and vertical force (F_z) versus non-dimensional wave amplitude (a/d) on the inclined plate for θ = 900 600 450 300 at a relative depth d/L = 0.12. It is observed that F_x decreases with the decrease of plate angle and the vertical force F_z increases with the decrease of plate angle. It is also observed from the Figures 3 and 4 that F_x is more at time T=3s and at a relative depth d/L = 0.24 for θ = 900 compared to the other relative depth of water.

Fig. 5 shows the plots of horizontal force (F_x) versus wave amplitude (*a/d)* on the inclined plate for θ = 60^0 45^0 30^0 at 0.10 ≤ d/L ≤ 0.24. It is observed that the horizontal force on the plate decreases with the decrease plate angle. It is also observed from the figures that horizontal force (F_x) is more at a relative depth d/L = 0.24 and this force decreases at a relative depth of water d/L = 0.10. This concludes that the horizontal wave force on the plate is important at a relative depth of water (*d/L* = 0.24) compared to the other relative depth of water.

Fig. 6 shows the plots of vertical force (F_z) versus wave amplitude (*a/d)* on the inclined plate for θ=60^0 45^0 30^0 at 0.10 ≤ d/L ≤ 0.24. It is observed that the vertical force at the plate is more at a plate angle θ = 45^0 compared to another angle of the plate. And this vertical force is more of a relative depth d/L=0.24 and is very low at a relative depth d/L=0.10.

Fig. 7 shows that the time histories of horizontal forces (F_x) on the inclined plate at different plate angles θ = 90^0 60^0 45^0 and at different wave stiffness H/L = 0.06 to 0.02 by keeping the depth of water *d* = 3m is constant. The wave forces are obtained from first order solution, which included in the figures to demonstrate the effects of linear wave theory.

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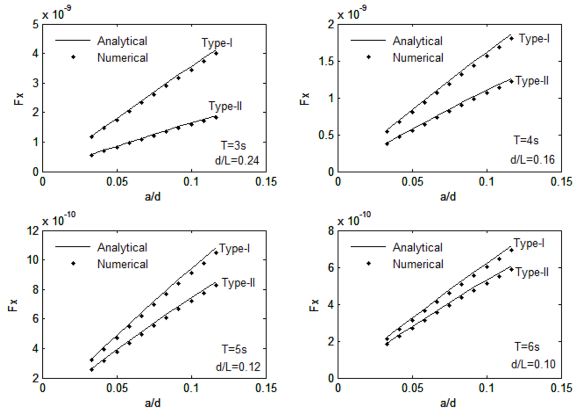

Fig. 2. Horizontal force (F_x) versus wave amplitude (a/d) for z/d=0 (type I) & z/d=-1 (type II). Here, d = 3m, l_1 = 1mm and l_3 = 0.5m.

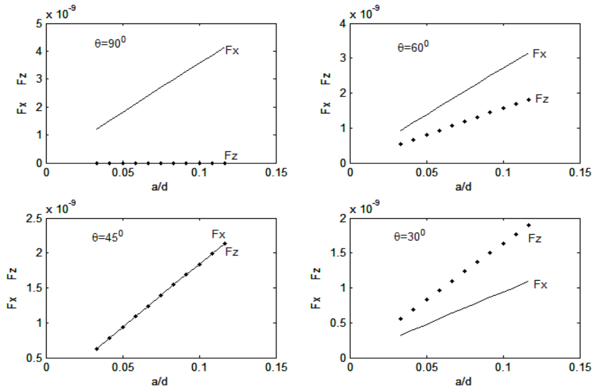

Fig. 3. F_x and F_z versus wave amplitude (a/d) at the free surface (z/d=0). Here, T=3s, d = 3m.

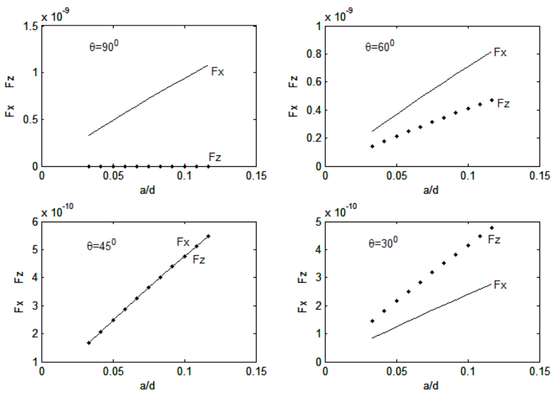

Fig. 4. F_x and F_z versus wave amplitude (a/d) at the free surface (z/d=0). Here, T=5s, d = 3m.

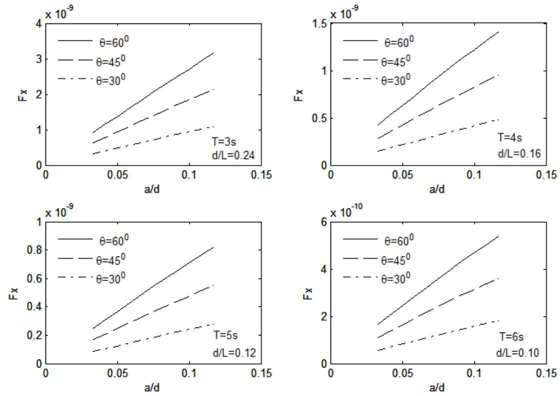

Fig. 5. Horizontal force (F_x) versus wave amplitude (a/d) at the free surface (z/d=0) for different plate angles θ = 60^0 45^0 30^0. Here, d = 3m, l_1 = 1mm and l_3 = 0.5m

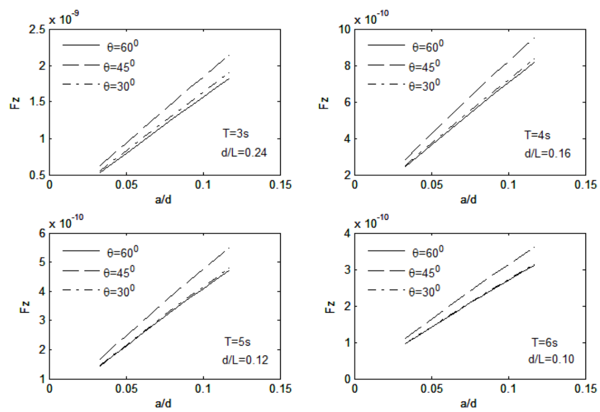

Fig. 6. Vertical force (F_x) versus wave amplitude (a/d) at the free surface (z/d=0) for different plate angles θ = 60^0 45^0 30^0. Here, d = 3m, l_1 = 1mm and l_3 = 0.5m

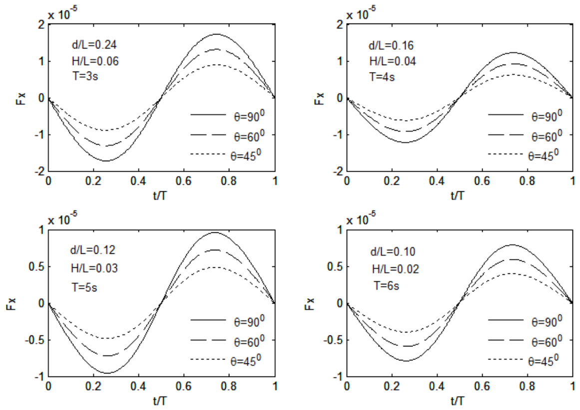

Fig. 7. Horizontal force (F_x) versus time (t/T) for the type I (z/d = 0). Here, a = 0.35m, d = 3m.

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6. Concluding remarks

The present work is studied to analyze the wave forces on submerged inclined rectangular thin plate in intermediate depth of water ($0.1 \leq d/L \leq 0.24$) under different configurations. It has been observed that the wave force on the plate at the mean free surface ($z/d = 0$) is maximum for $d/L = 0.24$ and at $H/L = 0.06$ compared to the other relative depth of water. It has also shown that the horizontal force of two types, which gradually converges to the decreasing value of the relative depth of water and when $d/L = 0.10$, the convergence is very close. F_x and F_z are both inversely proportional to the plate angle. That is when plate angle decreases F_x decreases and F_z increases and vice versa. F_x and F_z are approximately same at $\theta = 45^0$.

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