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Inverse conduction method using finite difference method

Jami Naga Venkata Sai Jitin[1], Atul Ramesh Bhagat[2]

[1]Dept of Aeronautical and Automobile Engineering, Manipal Institute of Technology, Manipal University, Manipal, Karnataka, India 576104
[2]Scientist 'E', Advanced System Laboratories, Defence Research and Development Organisation, Hyderabad 500058

Email: jitinjami1@gmail.com

Abstract. A numerical method is used to solve an inverse heat conduction problem using finite difference method and one-dimensional Newton-Raphson optimization technique. A thermocouple placed anywhere on the one-dimensional rod will read the temperature at that point, this temperature when fed into the FORTRAN code can predict the heat flux subjected onto the rod. The code has been further modified to predict variable heat flux (with time) as well. Error contribution of distance of thermocouple from source and time is demonstrated. Accurate prediction of heat flux variable with time has also been validated. First, a FORTRAN code was written to simulate and solve a transient model of a rod subjected to constant temperatures on both sides using Finite Difference Method. Next, a FORTRAN code was written to solve a steady state, and consequently a transient, model of a rod subjected to heat flux from one side using FDM and is tested to measure temperature at any node on the rod. Further, this code was modified to predict heat flux based on temperature data provided using Newton-Raphson optimization technique.

1. Introduction

Inverse heat conduction techniques are used when output can be measured, consequently the input is to be calculated. It finds its application in various fields. To predict heat generated inside a furnace by measuring temperature from the outside wall, to tweak the parameters according to the needs. To predict heat flux acting on the head of a re-entry vehicle by measuring temperature from the inside, so that appropriate materials can be used to make the heat shield among various other applications.

Numerical methods for such problems have been developed before, most of them being based on discretization by finite difference scheme. Vogel et al. [1] takes a 'Boundary inverse heat conduction problem' in which a one-dimensional rod has 2 temperature boundary conditions and 2 thermocouples placed at 10% away from each end. The end temperatures are estimated by using inverse conduction technology considering the temperature history from the thermocouples. R.C.Mehta [2] considers a case with heat flux on one side of the rod and the other side open ended. In the inverse conduction technique used, he assumes an initial value for heat flux and reduces the difference between the calculated temperature and the temperature reading using Newton-Raphson method. R.C.Mehta[3], a solution is proposed for inverse heat conduction with a heat flux and a radiation boundary condition using Regula-Falsi iterative method.

In the present paper, an optimisation technique is used to find the heat flux subjected onto a one-dimensional rod using FDM coded on FORTRAN. Firstly, temperature(calculated) is found at the end of the rod for a certain amount of flux and Newton-Raphson optimisation is used to estimate corresponding flux for the (measured) temperature using the same relation.

2. Problem Formulation

Governing equation for a 1D heat conduction is as follows:

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \tag{1}$$

Inverse heat conduction problem to find the heat flux is solved as an initial value problem in the first step. Here we have used Crank Nicholson formation for discretizing the heat equation using finite difference method.

$$-\lambda T_{i+1}^{n+1} + 2(1+\lambda)T_i^{n+1} - \lambda T_{i-1}^{n+1} = \lambda T_{i+1}^{n} + 2(1-\lambda)T_i^{n} + \lambda T_{i-1}^{n} \tag{2}$$

Here, $\lambda = \alpha \frac{\Delta t}{(\Delta x)^2}$

A Neumann boundary condition is used as a boundary condition, which gives us the heat equation in the following form on the boundaries.

$$-k\frac{\partial T}{\partial x} = q - \rho C_p \frac{\partial T}{\partial t}\left(\frac{\Delta x}{2}\right)\bigg|_{x=0} \tag{3}$$

$$-k\frac{\partial T}{\partial x} = - \rho C_p \frac{\partial T}{\partial t}\left(\frac{\Delta x}{2}\right)\bigg|_{x=L} \tag{4}$$

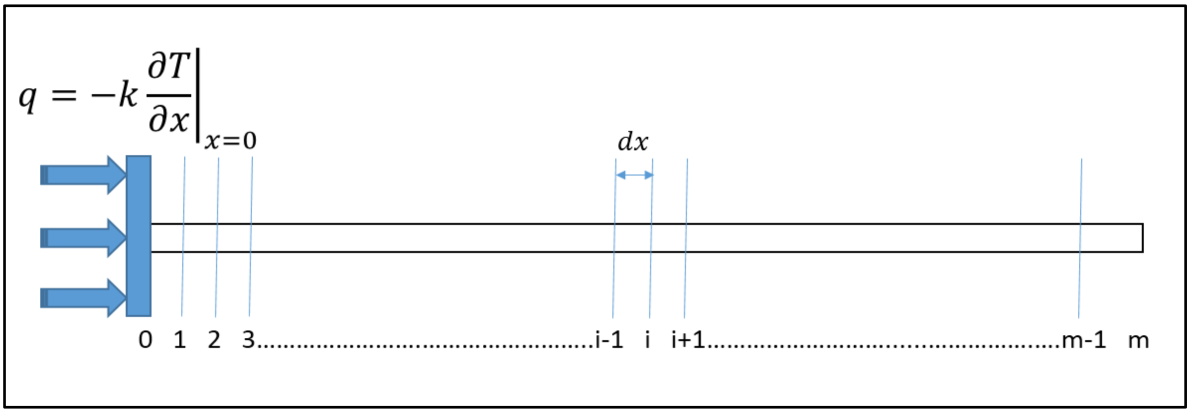

Figure 1. Neumann Condition

Equation (3) and (4) have heat flux and insulated boundary condition, respectively. Rearranging the above formulation in tridiagonal form would give us the following equation (5). Here, $\theta = 0.5$, for Crank-Nicholson Scheme. The LHS in the above equation is in the form of a tridiagonal matrix (TDMA) which is solved by Thomas Algorithm. Using this we can find the temperature at any node in a one-dimensional rod after a certain amount of heat flux is applied for a certain amount of time.

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$$\begin{bmatrix} 1+2\lambda\theta & -2\lambda\theta & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ -\lambda\theta & 1+2\lambda\theta & -\lambda\theta & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & -\lambda\theta & 1+2\lambda\theta & -\lambda\theta & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & \ddots & \ddots & \ddots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \ddots & \ddots & \ddots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \ddots & 1+2\lambda\theta & -\lambda\theta & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & -\lambda\theta & 1+2\lambda\theta & -\lambda\theta \\ 0 & 0 & 0 & 0 & \cdots & 0 & -2\lambda\theta & 1+2\lambda\theta \end{bmatrix} \begin{bmatrix} T_0^{j+1} \\ T_1^{j+1} \\ T_2^{j+1} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ T_m^{j+1} \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ d_m \end{bmatrix} \tag{5}$$

In the inverse heat conduction, we're tackling, the only input is temperature measured at a certain distance from the source flux and after a certain amount of time. Newton-Raphson optimisation is used to extrapolate heat flux from the measured temperature(t_m). The main reason of using this optimisation method is its precision in one variable optimisation. First, a dummy value of heat flux(Q) is given and its corresponding nodal temperatures are found after a small amount of time. The calculated temperature is then subtracted from the measured temperature, the modulus of which is compared to a predetermined error value. If the modulus value doesn't satisfy the inequality, the value of dummy Q is either increased or decreased based on the sign. This is an iterative procedure till we reach a converged value.

$$f(x) = \left|(T_c - T_m)\right| \leq \varepsilon \tag{6}$$

$$q_{i+1} = q_i - \frac{f(x)}{f'(x)} \tag{7}$$

3. Numerical Analysis and Results

Three different prototypes have been considered with different properties in terms of materials, shape and dimensions.

Table 1 - Properties of materials used

Properties	Copper	Steel
Length(m)	0.0025	0.05
Density(kg/m^3)	8960	7800
Thermal Conductivity(W/m.K)	400	50
C_p(kJ/kg.K)	1000	1000

The above 2 test subjects are objected to various types of heat flux inputs. The first is with constant heat flux subjected onto the rod followed by reading temperature at every node on the rod, and predicting the value of the heat flux with temperature at each node as measured temperature (t_m). The second is, heat flux changing with time. There are 3 kinds of inputs that have been considered, namely, step input, linearly varying input, and input in the form of a sine curve. In this case, temperature readings are taken from a single node at certain time steps, the time steps can be varying or constant in size. The full data set of temperature is used to estimate the heat flux the rod was subjected to overtime.

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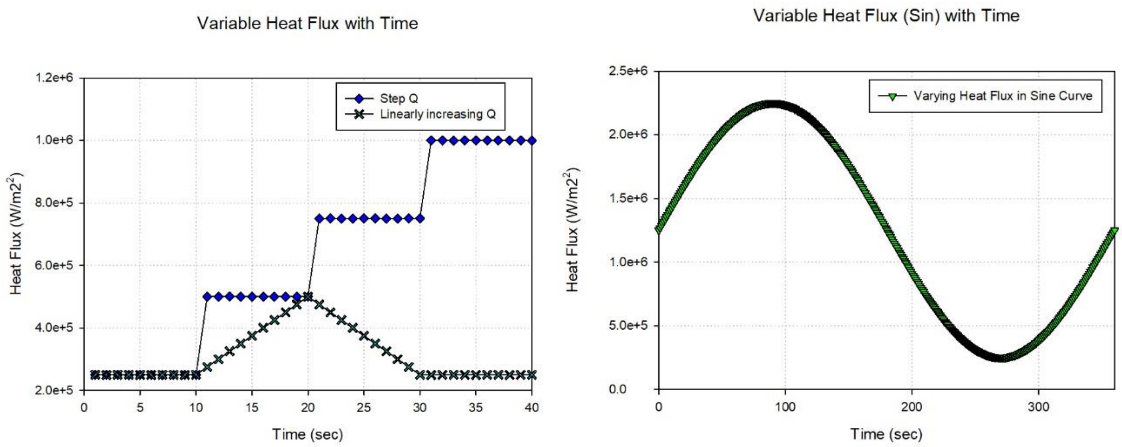

Figure 2. Variable Heat Flux input: Step Input, Linearly Varying and Input in the form of sine curve

Figure 4,5 and 6 describes the solution plots for the test cases shown in figure 2. Inferences have been made based on acquired data.

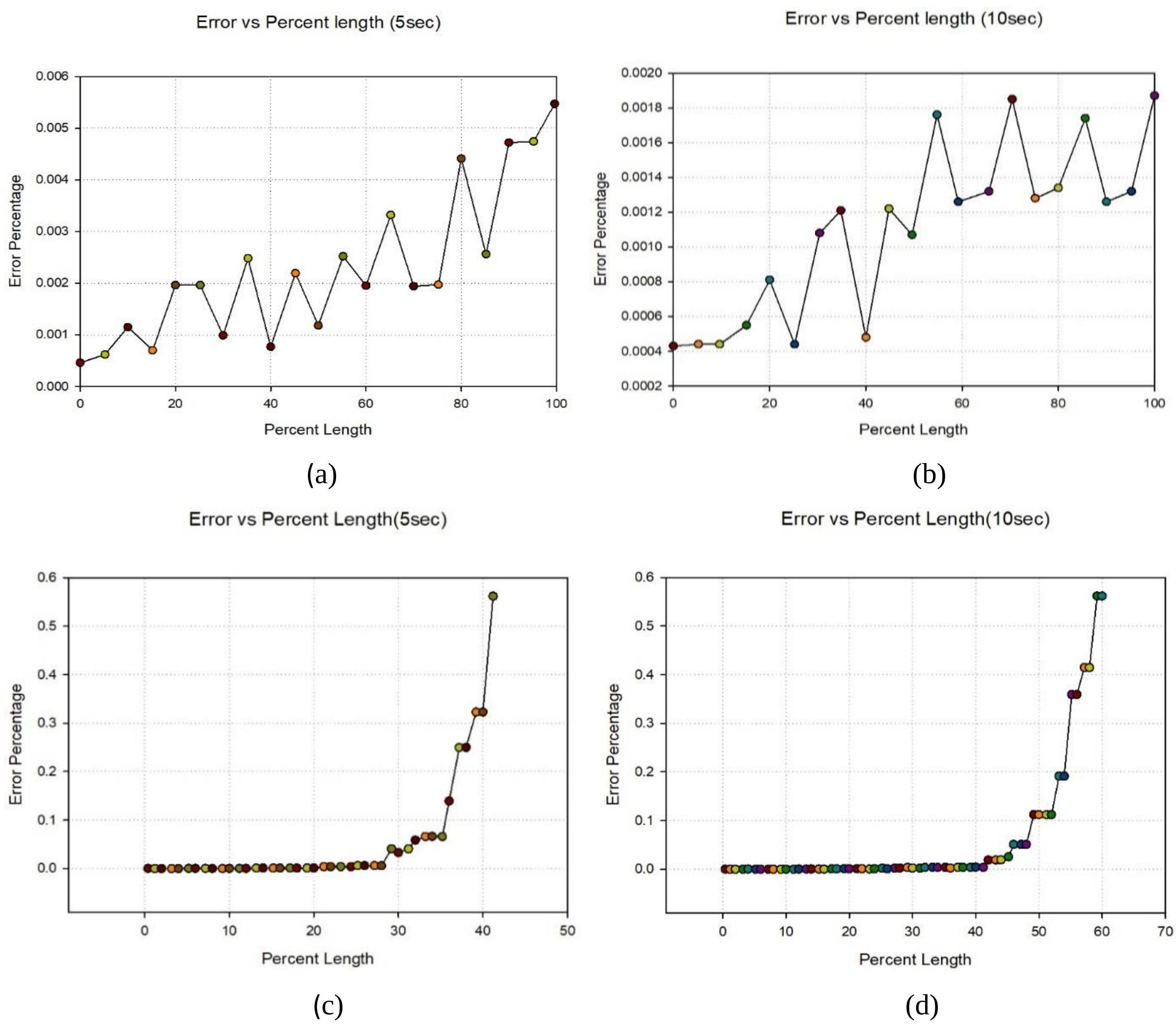

Figure 3. Error vs Percent Length: a) Copper at t = 5sec, b) Copper at t = 10sec, c) Steel at t = 5sec, and d) Steel at t = 10sec

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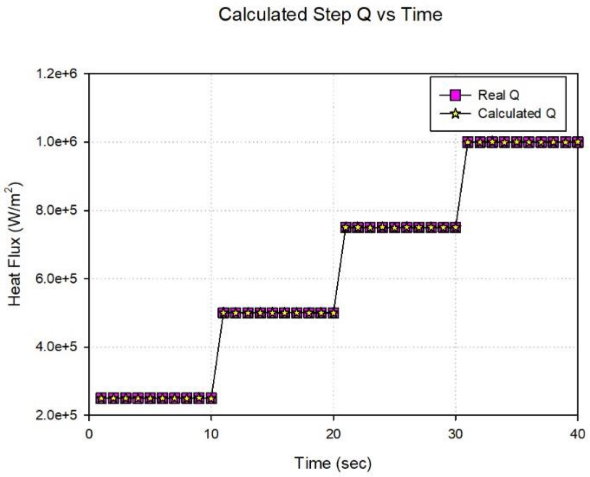
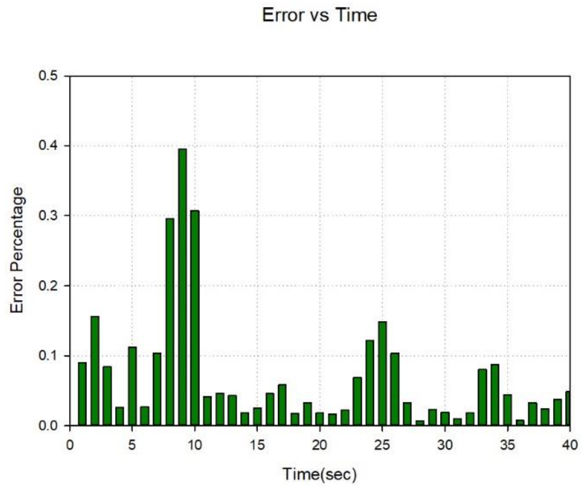
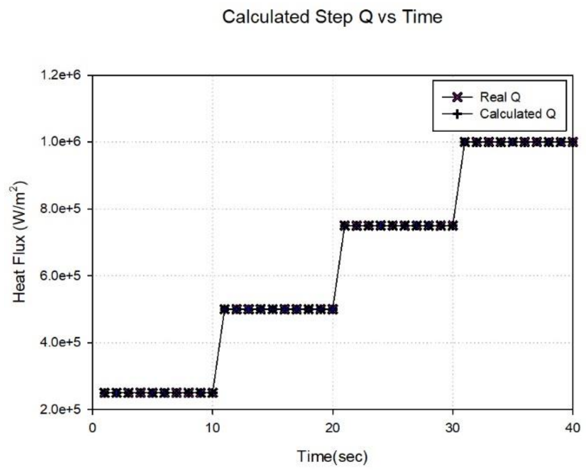


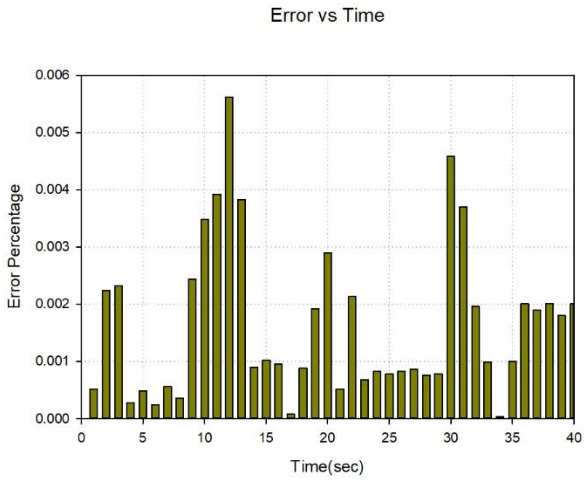
(a)



(b)

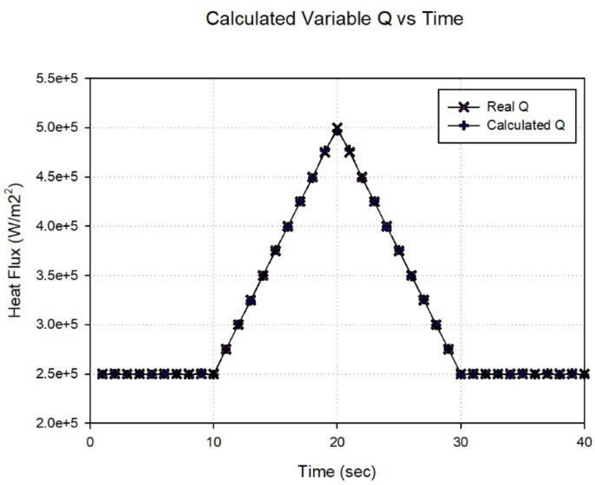


(c)

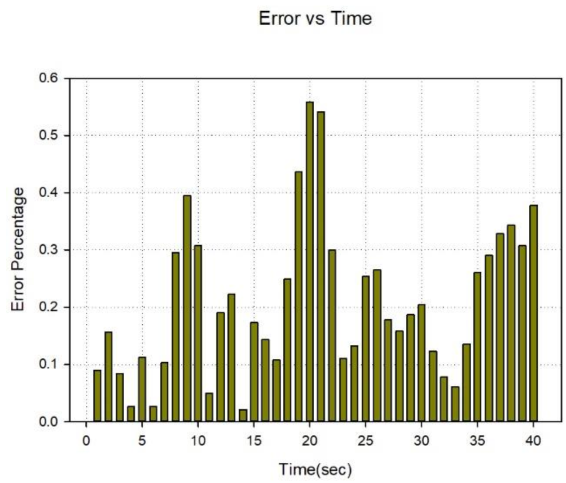


(d)

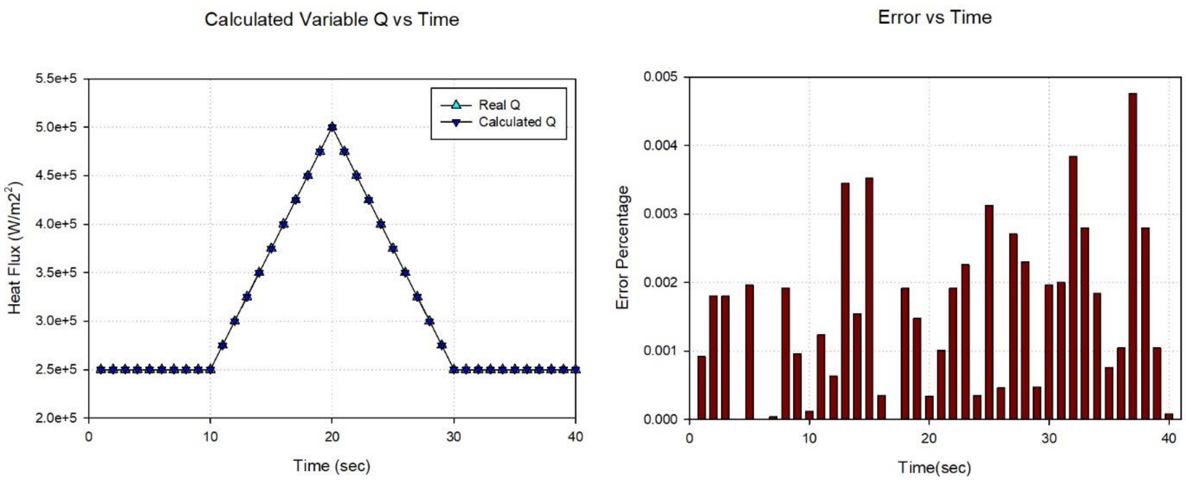
Figure 4. Heat Flux input in step form for t = 40 sec, a) Input and Output graph(Copper), b) Error vs Time (Copper), c) Input and Output graph(Steel) and, d) Error vs Time (Steel)



(a)

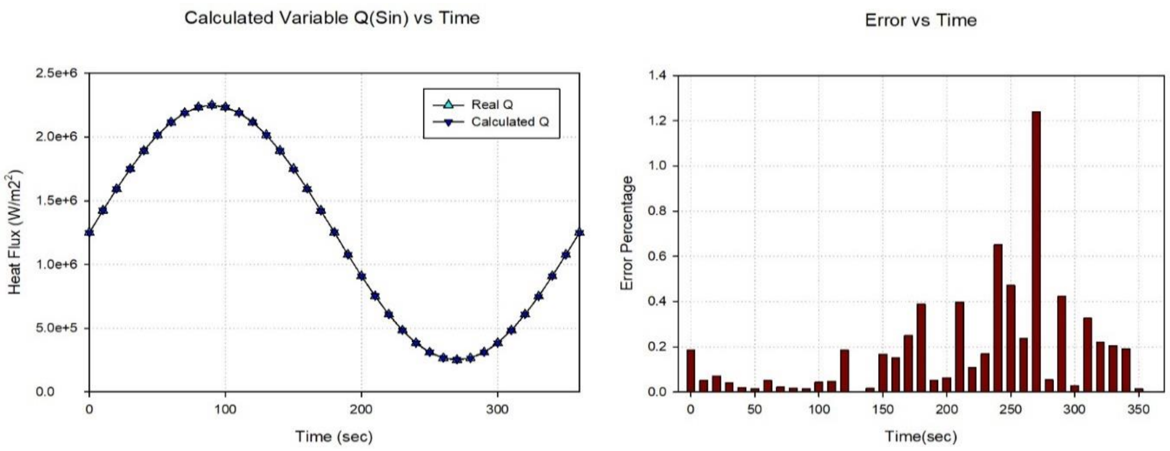


(b)

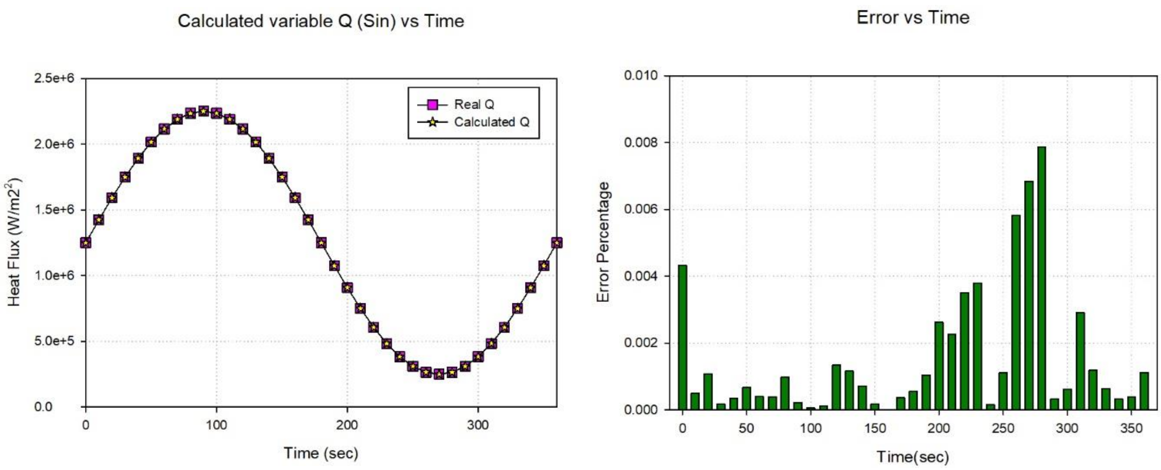


(c) (d)

Figure 5. Heat Flux input in linearly varying form for t = 40 sec, a) Input and Output graph(Copper), b) Error vs Time (Copper), c) Input and Output graph(Steel) and, d) Error vs Time (Steel)



(a) (b)



(c) (d)

Figure 6. Heat Flux input in Sine form for t = 360 sec, a) Input and Output graph(Copper), b) Error vs Time (Copper), c) Input and Output graph(Steel) and, d) Error vs Time (Steel)

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It can be clearly observed from the graphs and results that the error percentage of predicting the heat flux is very low. From figure 3, as the temperature reading sensor is moved away from the source, the error increases and increases rapidly after some distance. This rapid increase gets delayed along the distance as time *t*, is increased. Also, the effect of total length can be clearly scene, the longer the rod has a higher error, this could be since the effect of the heat flux takes some time to propagate through the rod, and for this reason for the upcoming cases, we have considered the end node readings for copper and readings at around quarter distance from the source for stainless steel. In figures 4, 5, and 6, there are few similar patterns that can be observed. The error percentage is fairly low in both the cases. However, it can be observed that in each case separately, the error is comparatively high when the heat flux applied is low.

4. Conclusion

Using Newton-Raphson Optimisation to solve an inverse conduction initial value problem has yielded fruitful results, under a set of circumstances. The circumstance for the best result is when the thermocouple is as close to the source as possible for however long its being heated before taking temperature readings. The trade-off will be between speed and precision. The advantage of this method is the low error percentage even for a large domain.

5. References

[1] J.Vogel, L.Sara and L.Krejc, "A simple inverse heat conduction method with optimization," *Int. J. Heat Mass Transfer*, **Vol. 36**, No. 17, pp.4215 – 20, 1993.
[2] R.C.Mehta, "Estimation of Heat Transfer Coefficient in a Rocket Nozzle," *AIAA*, **Vol. 19**, pp.1085-6, August 1981.
[3] R.C.Mehta, "Numerical Solution of nonlinear inverse heat conduction problem with radiation boundary condition," *Int. J. for Numerical Methods in Engineering*, **Vol. 20**, pp. 1057-66, 1984.
[4] Steven C. Chapra, Raymond P. Canale, *Numerical Methods for Engineers*, Sixth Edition.
[5] John C. Tannehill, Dale A. Anderson, Richard H. Pletcher, *Computational Fluid Mechanics and Heat Transfer* Second Edition.
[6] Suhas V. Patankar, *Numerical Heat Transfer and Fluid Flow*, First Edition.