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Prediction of Directional Young's Modulus of Particulate Reinforced MMC using Finite Element Methods

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Abstract. The aim of this present study is to predict the Young's Moduli of metal matrix composite (MMC) by three dimensional (3-D) Finite Element Analysis (FEA) and compare the results with Halpin-Tsai (HT) model. In the present work, cubic shaped Representative Volume Element (RVE) has been considered to model the MMC. Particle is taken as cylindrical in shape. The parameters considered in present study are particle aspect ratio (ratio of height and diameter) and volume fraction. Further, a wide range of the modulus ratios (ratios of Young's moduli of particle and matrix) have been considered to investigate their effects on effective Young's modulus of the composite. It is observed from the study that the modulus ratio, aspect ratio and volume fraction of particle are the most dominating factors in directional effective Young's modulus of metal matrix composite.

1. Introduction

Metals are normally alloyed with other elements to improve their physical and mechanical properties and wide ranges of alloy composition are available. Final properties are strongly influenced by thermal and mechanical treatment which determine the microstructure [1-2]. Metal matrix composite (MMC) has been used in industries like aerospace, automotive, electronics and infrastructure, because of their attractive properties such as light weight, stronger, stiffer material properties, high wear resistance and its high specific strength and stiffness at room or at elevated temperature than those of base metal counterparts [1-2]. Generally two approaches are adopted to determine the properties of composites; macro-mechanical analysis where composite material is considered as a homogeneous orthotropic continuum and micro-mechanical analysis where composite material is at the fibre/particle and matrix level. The particles which are used as reinforcement in matrix, play a vital role in controlling such mechanical properties of MMCs. Particle volume fraction, particle size and shape, particle aspect ratio (ratio of height and diameter), orientation of particle in matrix field; interfacial bonding are the key factors of controlling the composite properties [3]. During the last few decades, several attempts have been made to explore the relationship between the structure and the deformation of particulate metal matrix composite (PMMC). Continuum models exhibit some detailed quantitative results about the composite strength revealing the effect of aspect ratio, volume fraction and strain hardening component of matrix [4-5]. Typically, the unit cell technique combined with the known material properties of fibre and matrix is used to determine the overall behaviour of the composite [6]. In this context, Ganesh and Chawla [7] correlated the material microstructures with their macroscopic properties in order to understand the material behaviour of existing materials as well as to develop new materials. Chawla and Chawla [8] reveals about the micro-structural study of reinforcement in metal matrix to understand the material morphology. Numerical simulation methods like Finite Element Analysis (FEA) offer an alternative way to understand the behaviour of composites.

2. Methodology

In the present study, parameters considered for the FE analysis are; modulus ratio, λ_1 (ratio of Young's moduli of particle and matrix); particle aspect ratio, λ_2 (ratio of particle height and diameter); particle height fraction, λ_3 (ratio of particle height and RVE length); particle area fraction, λ_4 (ratio of particle cross-section and RVE cross-section), particle volume fraction (ratio of particle volume and RVE volume, $V_f=\lambda_3\lambda_4$). Generally the effective elastic response of a composite with perfect particle–matrix bonding under load is evaluated by the conventional Rule of Mixtures (ROM) [1], where particles are continuous ($\lambda_3=1$) in matrix field and can be formulated as:

$$E_c = E_p V_p + E_m V_m = E_m(\lambda_1 \lambda_4 + 1 - \lambda_4) \quad (1)$$

where E and V are the Young's modulus and volume fraction for the matrix (m) and the particles (p), respectively. ROM does not account for the effect of aspect ratio. But Young's modulus varies with particle shape and size even for a constant volume fraction in discontinuous ($\lambda_3<1$) particle reinforced composites. Hence, ROM is not effective for estimating effective Young's modulus of composite for discontinuous particle reinforced MMCs. In order to estimate the longitudinal Young's modulus (E_c), the following theoretical semi empirical model has been proposed by Halpin-Tsai (HT) model [9] for a perfectly oriented discontinuous reinforcement in the composite, where the load is applied parallel to the reinforcement orientation.

$$E_c = \frac{E_m(1+2sqV_p)}{1-qV_p} = \frac{E_m(1+2q\,\lambda_2\,\lambda_3\,\lambda_4)}{1-q\,\lambda_3\,\lambda_4} \quad (2)$$

where E_c, E_p and E_m are the Young's moduli of composite, particle and matrix, respectively; s is the aspect ratio (λ_2); V_p is volume fraction of the particle ($\lambda_3\lambda_4$); q is a geometrical parameter that is expressed as,

$$q = \frac{\left({}^{Ep}/_{Em}\right)-1}{\left({}^{Ep}/_{Em}\right)+2s} = \frac{\lambda_1-1}{\lambda_1+2\,\lambda_2} \quad (3)$$

3. Finite Element Analysis

In this present study, three dimensional (3-D) Finite Element Analysis (FEA) are being carried out with Representative Volume Element (RVE) as cubic shape to represent major features of the microstructure. RVE is considered as high symmetry in nature where single particle in cylindrical shape is located at centre of the RVE. Total volume of the RVE is taken as 5080 μm^3 [10] for all FE analyses. Due to symmetry of RVE, only one eight volume of the RVE is being modelled in this analysis. Three symmetry boundary conditions are considered for three respective faces of RVE where out of plane translational degrees of freedom (DOF) are being restricted. RVE is under tensile load in terms of 1% strain in longitudinal direction in which particle longitudinal axis is being oriented. Wide range of modulus ratios (λ_1) is being considered (0, 0.1, 5.54, 10, 20, 100) to investigate their effects on effective Young's modulus of composite. Alfonso et al. [10], investigated the effect of directional strength of composite considering particle as SiC (Young's modulus and Poisson's ratio of 401.4 GPa and 0.18) and matrix as Aluminium Al356 (Young's modulus and Poisson's ratio of 72.4 GPa and 0.33) with modulus ratio λ_1=5.54. However, in the present study in addition to SiC/Al356 composite, different modulus ratios (λ_1) are considered with particle and matrix Poisson's ratio kept as 0.3. Both particle and matrix are considered as isotropic in nature. To investigate the effect of aspect ratio of particle (λ_2), particle volume is taken as 610 μm^3 (V_f =12%) [10] and particle dimensions are shown in Table 1. Figure 1 shows schematic representation of full RVE and $1/8^{th}$ RVE. Second order brick elements of ANSYS (solid186) of approximate size of 0.2 μm with reduced integration are used. Particle aspect ratio (λ_2) varies from 0.2 to 1.8. To investigate effect of volume fraction, different particle geometries are considered with aspect ratios (λ_2) of 0.8 and 1.4 as shown in Table 2. Table 3 shows particle geometries for different particle diameters with particle height of 6.53 μm (λ_3=0.380) and 11.50 μm (λ_3=0.669). Further, different particle geometries (shown in Table 4) are also considered with constant diameter of 8.21 μm (λ_4=0.179) and 10.89 μm (λ_4=0.315) to understand the effect of

particle height in estimating Young's modulus of the composite. Directional Young's modulus of composite under axial load can be calculated using equation, $E_c = \frac{R}{A\varepsilon}$, where R is the axial reaction force at support extracted from FEA, A is cross-sectional area of RVE and ε is the average strain of 1% along loading direction.

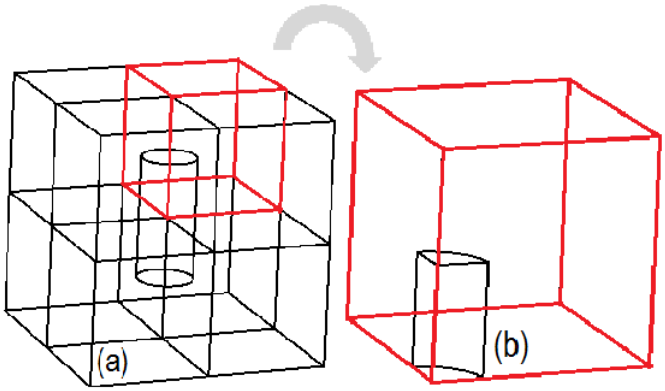

Figure 1. Schematic representation of (a) Full RVE and (b) 1/8th RVE.

Table 1. Dimensions of Particle for different Aspect Ratios (λ_2) [10]

Aspect Ratio (λ_2)	Diameter, d_p (μm)	Height, h_p (μm)
0.2	15.71	3.14
0.6	10.89	6.53
1.0	9.19	9.19
1.4	8.21	11.50
1.8	7.55	13.59

Table 2. Particle height for different Aspect Ratios

Aspect Ratio		Volume Fraction, % V_f (λ_3 λ_4)							
		2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
λ_2=0.8	h_p (μm)	4.695	5.915	6.771	7.453	8.028	8.531	8.981	9.389
λ_2=1.4		6.818	8.590	9.833	10.823	11.658	12.389	13.042	13.636

Table 3. Particle Aspect Ratios for different diameter

Height Fraction		Volume Fraction, % V_f (λ_3 λ_4)							
		2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
λ_3=0.380	λ_2	1.31	0.93	0.76	0.66	0.59	0.54	0.50	0.46
λ_3=0.669		3.07	2.17	1.77	1.53	1.37	1.25	1.16	1.08

Table 4. Particle Aspect Ratios for different height

Area Fraction		Volume Fraction, % V_f (λ_3 λ_4)					
		2.5	5.0	7.5	10.0	12.5	15.0
λ_4=0.179	λ_2	0.29	0.58	0.88	1.17	1.46	1.75
λ_4=0.315		0.13	0.25	0.38	0.50	0.63	0.75

4. Results and Discussions

4.1. Effect of particle Aspect Ratio for constant particle Volume Fraction

Variations of normalised Young's modulus (λ, defined as the ratio of Young's moduli of composite and matrix) with aspect ratio (λ_2) are shown in Figure 2. Volume fraction of particle V_f ($\lambda_3\lambda_4$) is taken as 12%. Results show that the composite exhibits higher value of λ with increasing λ_2. This is applicable for all values of λ_1 (0-100). When V_f remains constant and λ_2 is higher there is an increase in height (increase of λ_3) and reduction of area (decrease of λ_4) of the particle. Higher value of λ_3 indicates long particle or fiber. If particle and matrix are considered as springs connected together, in case of λ_3=1 (i.e. continuous fiber) and λ_4<1, system is equivalent to two springs are connected in parallel and ROM is applicable and effective stiffness of composite is defined as $K_m + K_p$, where K, m and p, represent stiffness, matrix and particle, respectively. But in case of λ_4=1 and λ_3<1, springs are connected in series, effective stiffness of composite is $\frac{K_m K_p}{K_m + k_p}$. Hence, parallel connection is better in enhancement of Young's modulus of composite. From Figure 2, it can be noticed that rate of increase of λ is more in case of composite having higher value of λ_1. This behaviour is like composites with higher value of λ_1 and λ_2 (long stiffer particle as fiber like carbon nanotube) are more effective in enhancement of stiffness of composite [1]. Normalised Young's modulus; λ, extracted from FEA and HT model shows good agreement for λ_2 between 0.6-1.4 along with λ_1 range of 0-20. However, FEA results are higher than HT model with λ_1 =100. In the case when particle is void and λ_2=0.2, it acts like a sharp crack. As void does not carry any load, there will be redistribution of load path along the loading direction due to presence of sharp crack and effective stiffness of the composite is less. When λ_1=0 and the MMC has high value of λ_2 and less value of λ_4, this results in less severity of crack due to less deviation of load path and loads are being carried out by surrounding matrix material and FEA results matches well with that of HT model.

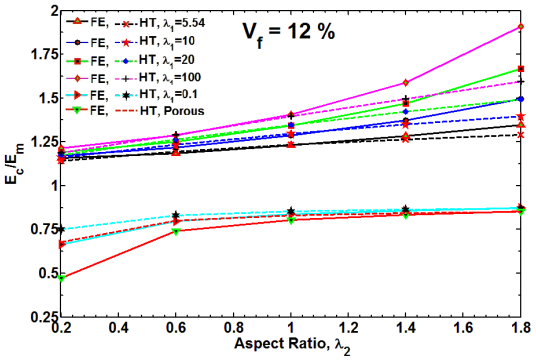

Figure 2. Young's modulus versus aspect ratio with volume fraction (V_f) of 12%.

4.2. Effect of particle Volume Fraction for constant particle Aspect Ratio

The effect of volume fraction in overall modulus of composite with aspect ratios (λ_2) of 0.8 and 1.4 are shown in Figures 3(a) and 3(b), respectively. Figures show that with the increase in volume fraction, strength of composite in longitudinal direction increases for λ_1>1. Further, rate of change of λ is more in case of λ_2=1.4 than 0.8 for λ_1>1 (maximum value of λ are 1.65 and 2.45, respectively). HT model is well suited for all MMC composite of λ_2=0.8 and λ_1>1. When aspect ratio of particle λ_2=1.4 and 1<λ_1<20, HT and FEA results match up to V_f of 12.5%. But in case of λ_2=0.8 and λ_1<1, HT model predicts higher value than that of FEA. This may be due to non-accounting of crack severity in HT model. For material having very low strength ratio, most of the load is being carried out by the matrix and as volume fraction of particle increases overall stiffness of composite decreases and curve shows a downward slope with volume fraction. It can be concluded that both material strength ratio and aspect ratio have a big role as volume fraction increases. Halpin and Tsai also predicted a geometric parameter (q) depends on λ_1 and λ_2 for short particle/fiber [9]. Results also show that the limitation of HT model with higher values of λ_1, λ_2 and V_f ($\lambda_3\lambda_4$).

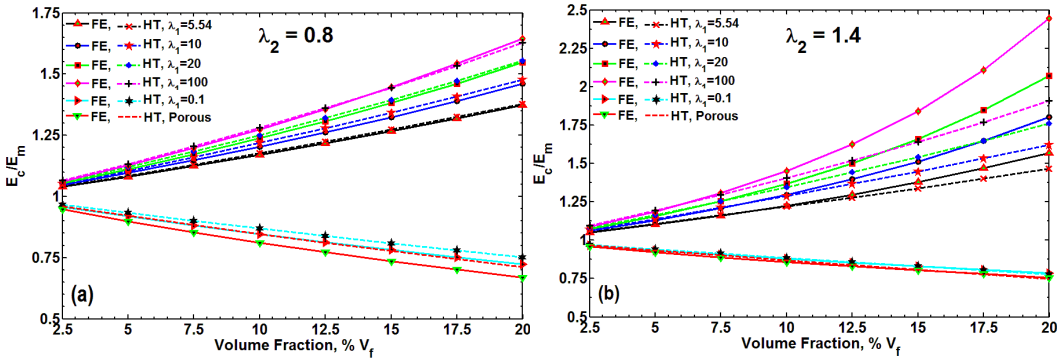

Figure 3. Young's modulus versus volume fraction with particle aspect ratio (λ_2) of (a) 0.8 and (b) 1.4.

4.3. Effect of particle Diameter for constant particle Height

Effect of particle diameter are being investigated considering particle height as 6.53 µm (λ_3=0.380) and 11.5 µm (λ_3=0.669). The only way to increase volume fraction is to increase diameter. If diameter increases with constant height (λ_3 as constant), λ_4 increases, λ_2 decreases and volume fraction ($V_f = \lambda_3\lambda_4$) increases. For λ_1>1, higher value of V_f results more amount of particle with stiffer material in composite and hence higher value of λ. But decrease of aspect ratio (λ_2) exhibits adverse effect in rate of increase of λ. For same volume fraction, λ is less in case of Figure 4(a) compared to Figure 4(b) due to lesser value of λ_2 in former case for λ_1 >1 (maximum value of λ are 1.45 and 1.91, respectively for V_f of 20%). Aspect ratio decreases from 1.31 to 0.46 in case of particle height of 6.53 µm whereas it varies from 3.07 to 1.08 for 11.5 µm (see Table 3). When λ_1 >1, HT model under predicts the FEA results as shown in Figure 4(b) due to higher aspect ratio (λ_2). When λ_1=0, i.e. particle as void, value of λ is less for particle height of 6.53 µm (Figure 4a) as compared to the same with particle height of 11.5 µm (Figure 4b). This is due to the effect of porosity in the MMC. The similar trend is also observed for λ_1=0.1.

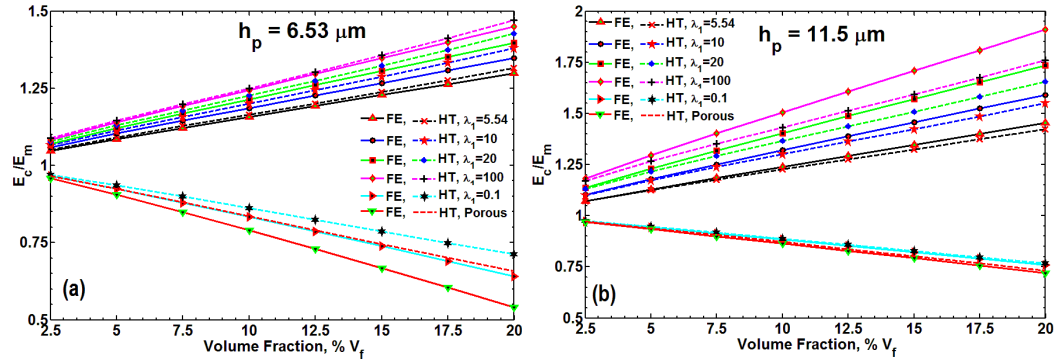

Figure 4. Young's modulus versus volume fraction with particle height (h_p) of (a) 6.53 µm and (b) 11.5 µm.

4.4. Effect of particle Height for constant particle Diameter

Variations of overall response in terms of Young's modulus of composite are shown in Figures 5(a) and 5(b) with particle diameter of 8.21 µm (λ_4=0.179) and 10.89 µm (λ_4=0.315), respectively. Due to the increase in height of the particle, volume fraction as well as strength of composite increases for λ_1 >1. Rate of change of λ is more in case of Figure 5(a) than Figure 5(b) due to higher aspect ratio (λ_2) of former case (variation of λ_2 is 0.29 to 1.75 and 0.13 to 0.75, respectively, Table 4). Maximum values of λ obtained from FEA are 2.39 and 1.43, respectively for V_f of 15%. It is observed that the deviation in the prediction of HT model as compared with the FE model is more for higher aspect ratio

of the particle when λ_1 >1. If height of the particle is increased with constant diameter, effect of volume fraction and aspect ratio together enhance the overall stiffness of the composite due to parallel distribution of stiffness between particle and matrix. FE results are matching well with HT model for λ_2<1 and λ_1 >1 but more deviation has been found for λ_2>1 and λ_1>1. However, FE analysis results with soft particle (λ_1=0, 0.1) are in close agreement with HT model prediction for λ_4=0.179 (Figure 5a) and deviates for λ_4=0.315 (Figure 5b).

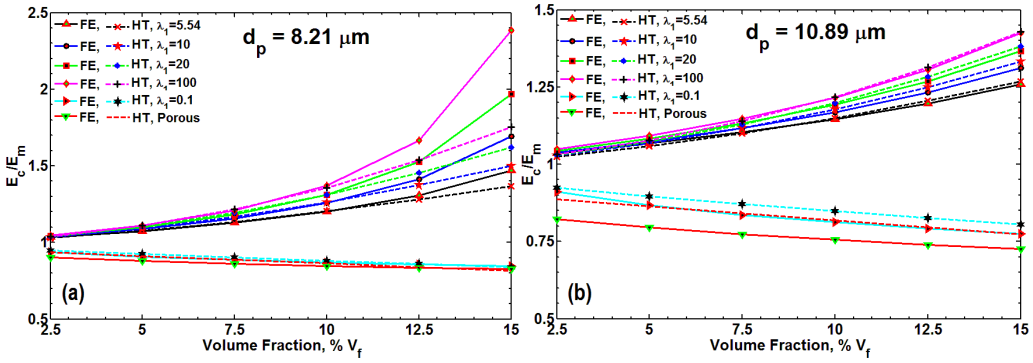

Figure 5. Young's modulus versus volume fraction with particle diameter (d_p) of (a) 8.21 μm and (b) 10.89 μm.

5. Conclusions

Present study shows that the modulus ratio (ratio of Young's moduli of particle and matrix) and aspect ratio of particle (ratio of height and diameter) are the most dominating factors in directional effective Young's modulus of metal matrix composite (MMC) which is further influenced by volume fraction of the particle. It is also demonstrated by the current study that HT model prediction for MMC matches well with FE analysis results for limited range of modulus ratios, aspect ratio and volume fraction of the particle.

References

[1] Chawla N, Chawla K K 2013 Metal matrix composites, 2nd ed., *New York: Springer*
[2] Hull D, Clyne T W 1996 An introduction to composite materials, 2nd ed., *New York: Cambridge University Press*
[3] Lei M and Ledbetter H 1994 Communications: elastic constants of SiC/Al: measurements and modelling, *Metall. Mater. Trans.* **25A** pp 2832-2835
[4] Christman T, Needleman A, Suresh S 1989 An experimental and numerical study of deformation in metal-ceramic composites, *Acta Metall.* **37** pp 3029-3050
[5] Bao G, Hutchinson J W, McMeeking R M 1991 Particle reinforcement of ductile matrices against plastic flow and creep, *Acta Metall. Mater.* **39** pp 1871 – 1882
[6] Williams J J, Segurado J, LLorca J, Chawla N 2012 Three dimensional (3D) microstructure-based modelling of interfacial decohesion in particle reinforced metal matrix composites, *Mater. Sci. and Engg.* **557A** pp 113–118
[7] Ganesh V V, Chawla N 2005 Effect of particle orientation anisotropy on the tensile behavior of metal matrix composites: experiments and microstructure-based simulation, *Mater. Sci. and Engg.* **391A** pp 342-353
[8] Chawla N, Chawla K K 2006 Microstructure-based modelling of the deformation behaviour of particle reinforced metal matrix composites, *J. Master. Sci.* **41** pp 425-430
[9] Halpin J C, Tsai S W 1967 Environmental factors in composite materials design, *Air Force Mater. Lab. Tech. Report* pp 67-423
[10] Alfonso I, Figueroa I A, Sierra J M, Abatal M, Gonzalez G, Rodriguez-Iglesias V, Medina-Flores A, Flores J E 2013 Young's modulus estimation based on high symmetry 3-D finite element model for metal matrix composites, *Comput. Mater. Sci.* **69** pp 304-310