

Free vibration analysis of functionally graded sandwich flat panel

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Abstract. In this paper, the free vibration behavior of functionally graded ceramic-metal sandwich flat panels is investigated. The sandwich panels are considered to be composed of metal rich face layers with varying composition and a homogenous isotropic ceramic core layer. The kinematics of the panels is modeled using ANSYS commercial package. The convergence and the validity of the present model have been established by comparing the present results with the benchmark results available in published literature. Subsequently, the influence of power index, symmetry type and the support conditions on the non-dimensional fundamental frequency is investigated. The fundamental frequency is found to decrease with increasing power index value and increase with number of constraints at the support. The stiffness of the panels is affected by core thickness and the panels with higher core-to-face thickness ratio are observed to have lower fundamental frequency.

1. Introduction

Now days, laminated composites are mostly used in various engineering structures, aerospace and automotive industries because of their higher material properties, low fatigue life and higher stiffness to weight ratio and strength to weight ratio. The conventional laminated composite structures are made up of different layers of homogeneous lamina which are bonded together so as to gain increased mechanical properties. The major flaw is the debility of laminated materials is that owing to abrupt change in material properties between the interfaces of different materials results high inter-laminar stresses and lead to deamination. This problem of deamination can be solved using materials in which properties vary smoothly and continuously, such kind of materials are known as functionally graded materials. Functionally graded (FG) material is constructed by ceramic & metal, in such a way that properties are varied by changing volume fraction of constituent materials along thickness of plate. Lok and Cheng [1] proposed a closed-form solution for the forced response of an orthotropic thick plate and sandwich panel made of FG material using Rayleigh-Ritz method. The bending, buckling and free vibration analysis of simply supported sandwich plate non symmetric about the mid plane using transverse shear deformation theory have been studied [2,3]. Solutions to bending and free vibrations of laminated composite and sandwich plates have also been presented using layerwise displacement model [4]. Novel hyperbolic shear deformation theory for buckling and vibration of

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functionally graded sandwich plate has also been developed [5]. Natrajan and Manickam [6] investigated the influence of the gradient index and aspect ratios on different FGM plates. Neves et al. [7] derived a higher order shear deformation theory for analysing static, free vibration and buckling behaviour of isotropic and sandwich functionally graded plates using a meshless technique. Zenkour [8] introduced the effects of transverse shear and transverse normal strains for the bending analysis of functionally graded sandwich plates using a simple four-unknown shear and normal deformations theory. Tornabene et al. [9] utilized higher-order equivalent single layer theories for free vibrations of free-form doubly-curved FG shells.

From this brief review of the literature, it is clear that the bending and free vibration characteristics of FG sandwich panels have been extensively studied and several efficient theoretical, numerical/analytical solutions have been developed. The aim of the present investigation is to study the influence of power index, type of symmetry and support conditions on vibration behaviour of FG sandwich flat panels in the framework of the FSDT. The FG sandwich flat panel is modelled in ANSYS commercial ware using ANSYS parametric design language (APDL) code. The aforementioned analysis is carried out and the results are discussed in detail.

2. Theoretical Background

In the present analysis, a flat rectangular FG sandwich panel of length a , width b and thickness h as shown is considered. The geometry and lay-up scheme of core and face of the FG sandwich panel is shown in Fig.1. The FG sandwich flat panels have been modeled in ANSYS using the parametric design language code (APDL). The panels are discretized using Shell281 element that has eight nodes with six degrees of freedom per node. The displacement field of the panels is defined on the basis of FSDT as following:

$$[u \quad v \quad w]^T = [u_0 + z\theta_x \quad v_0 + z\theta_y \quad w_0 + z\theta_z]^T \quad (1)$$

where, u , v and w are the displacements of any point on k^{th} layer at time t along the x , y and z coordinate axes, respectively; u_0 , v_0 and w_0 are the corresponding displacements of a point on the mid-plane; θ_x and θ_y are the rotations of normal to the mid-surface ($z = 0$) about the y and x -axes, respectively; θ_z is the higher order term in the Taylor series expansion which accounts for the linear variation of displacement function along thickness direction.

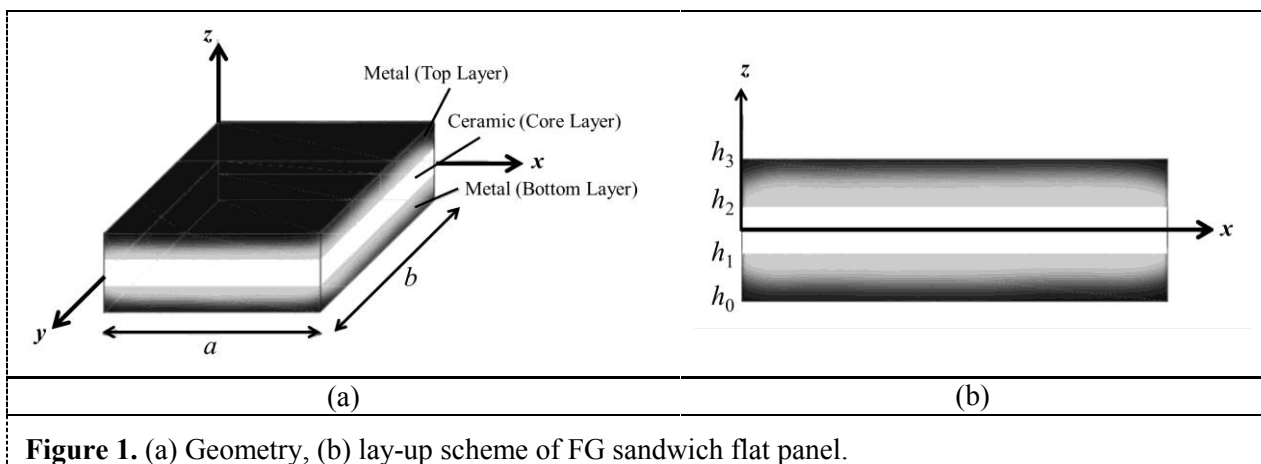


Figure 1. (a) Geometry, (b) lay-up scheme of FG sandwich flat panel.

In the considered sandwich flat panel, the face layers are made up of isotropic material and their properties vary in the direction of thickness (z -direction) and the core layer is completely homogeneous and isotropic. It is assumed that the Poisson's ratio remains constant and the Young's modulus varies as:

$$E(Z) = E_m + (E_c - E_m)V^{(n)} \quad (2)$$

where, E_m and E_c are the Young's modulus of elasticity of metal and ceramic, respectively of FGM sandwich, $V^{(n)}$ is the volume fraction ($n=1,2,3$). The volume fraction varies through the thickness as per the following power law:

$$\left. \begin{aligned} V^{(1)} &= \left(\frac{x_3 - h_0}{h_1 - h_0} \right)^k, Z \in [h_0, h_1] \\ V^{(2)} &= 1, Z \in [h_1, h_2] \\ V^{(3)} &= \left(\frac{x_3 - h_3}{h_2 - h_3} \right)^k, Z \in [h_2, h_3] \end{aligned} \right\} \quad (3)$$

where, k = power law index.

The natural frequency of the vibrating can be obtained by solving the eigenvalue equation:

$$([K] - \omega^2[M])\{\Phi\} = 0 \quad (4)$$

where, $[K]$ and $[M]$ are the stiffness and mass matrices, respectively, ω is the natural frequency of vibration, and $\{\Phi\}$ is the corresponding mode shape vector.

3. Results and Discussion

In this section, the effects of power index, symmetry and support condition on the free vibration behaviour of FG sandwich flat panels is analysed. Two FG materials namely Aluminum-Zirconia (Al/Zr) and Aluminum-Alumina (Al/Al₂O₃) are considered and their properties are listed in Table 1. The sandwich panel symmetries (ratio of face-core-face thickness) used in present problem are: 1-1-1, 1-2-1, 2-1-1, 2-2-1 and 3-1-2. Considering the total thickness of sandwich flat panel as h and the plane of symmetry at the middle of panel, the symmetries are defined as in Table 2.

Firstly, the convergence of the present model is tested. Sandwich panels with $h=0.01$ m, $a/h=10$ and $a/b=1$ are considered. The first natural frequency of a clamped Al/Zr and simply supported Al/Al₂O₃ FG sandwich panel is obtained and the non-dimensional frequency parameter for different mesh sizes is shown in Fig. 2. It is evident from Fig. 2 that the model converges well for both of the materials and the support conditions. Based on the results of the convergence test, a (14×14) mesh has been utilized throughout for the analysis.

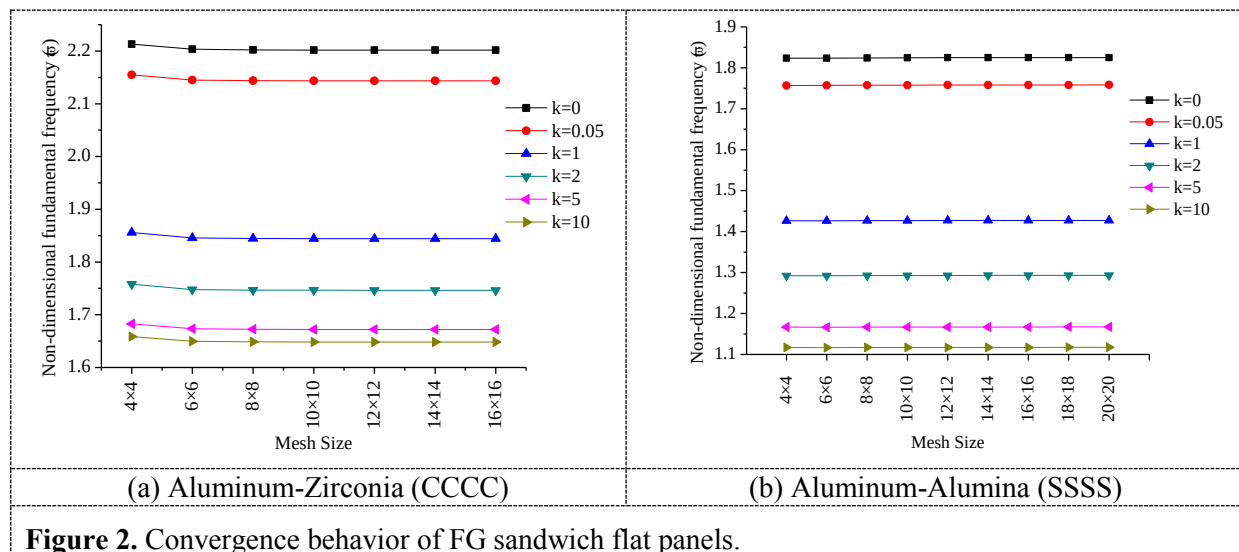
Table 1. Material properties used in the present analysis

	Young's Modulus (GPa)	Poisson's Ratio
Aluminum	70	0.3
Zirconia	151	0.3
Alumina	380	0.3

Table 2. Configuration in different symmetry

	h_0	h_1	h_2	h_3
1-1-1	$-h/2$	$-h/6$	$h/6$	$h/2$
1-2-1	$-h/2$	$-h/4$	$h/4$	$h/2$
2-1-1	$-h/2$	0	$h/4$	$h/2$
2-2-1	$-h/2$	$-h/10$	$3h/10$	$h/2$
3-1-2	$-h/2$	0	$h/6$	$h/2$

Further, to establish the efficacy of the current formulation the non-dimensional frequency parameter $\left(\varpi = \omega \frac{a^2}{h} \sqrt{\frac{\rho}{E}}, \rho = 1 \text{ kg/m}^3, E = 1 \text{ GPa} \right)$ for a simply supported square Al/Al₂O₃ sandwich panel is obtained for various symmetries and compared with the results of Zenkour [3] as depicted in Table 3. It can be clearly observed that the present values are in excellent agreement with the reference values for all symmetries corresponding to each power index value.

**Figure 2.** Convergence behavior of FG sandwich flat panels.**Table 3.** Validation of non-dimensional fundamental frequency parameter

Power index (k)		<i>1-0-1</i>	<i>2-1-2</i>	<i>2-1-1</i>	<i>1-1-1</i>	<i>2-2-1</i>	<i>1-2-1</i>
0	CLPT [3]	1.87359	1.87359	1.87359	1.87359	1.87359	1.87359
	TSDPT [3]	1.82445	1.82445	1.82445	1.82445	1.82445	1.82445
	Present	1.82484	1.82484	1.82484	1.82484	1.82484	1.82484
1	CLPT [3]	1.26238	1.32023	1.37150	1.37521	1.43247	1.46497
	TSDPT [3]	1.24320	1.30011	1.34888	1.35333	1.40789	1.43934
	Present	1.23475	1.29057	1.32316	1.34278	1.38435	1.42720
10	CLPT [3]	0.94321	0.95244	1.05185	1.00524	1.11883	1.13614
	TSDPT [3]	0.92839	0.94297	1.03862	0.99551	1.10533	1.12314
	Present	0.92473	0.93828	0.98543	0.99080	1.05516	1.11724

Now, the influence of power index on the non-dimensional fundamental frequency parameter of the sandwich panels is investigated. A simply supported $\text{Al}/\text{Al}_2\text{O}_3$ FG sandwich flat panel of uniform thickness $h=0.01\text{m}$, thickness ratio $a/h=10$, aspect ratio $a/b=1$ and five symmetry types (*1-1-1*, *1-2-1*, *2-1-1*, *2-2-1*, *3-1-2*) is considered. The power index (k) is varied to take the values: $k=0, 0.5, 1, 2, 5$ and 10 . The non-dimensional frequency parameter for the considered symmetries is obtained for varying power index value and shown in Fig. 3. It is evident that the frequency decreases for increasing power index values for all of the symmetries. This indicates that the stiffness of the panels decreases with increasing power index (k) and as a result the non-dimensional frequency parameter also decreases.

Further, the influence of symmetry on the free vibration responses of the FG sandwich panels is studied. Simply-supported square sandwich panels of materials $\text{Al}/\text{Al}_2\text{O}_3$ and Al/Zr with $h=0.01\text{m}$ and $a/h=20$ are considered. The non-dimensional frequency parameter is plotted for increasing power index (k) and depicted in Fig. 4. It is observed that the *3-1-2* scheme has the least frequency of all the symmetries for all values of k , followed by *2-1-1*, *1-1-1*, *2-2-1* and *1-2-1* in the increasing order. This reflects on the fact that the panels with a thicker core (higher core-to-face thickness ratio) are stiffer as compared to the panels with a thinner core thereby having higher fundamental frequency. Additionally, the corresponding natural frequencies for each power index are lower for Al/Zr as compared to $\text{Al}/\text{Al}_2\text{O}_3$ which is an indicative of higher stiffness of $\text{Al}/\text{Al}_2\text{O}_3$ panels compared to Al/Zr .

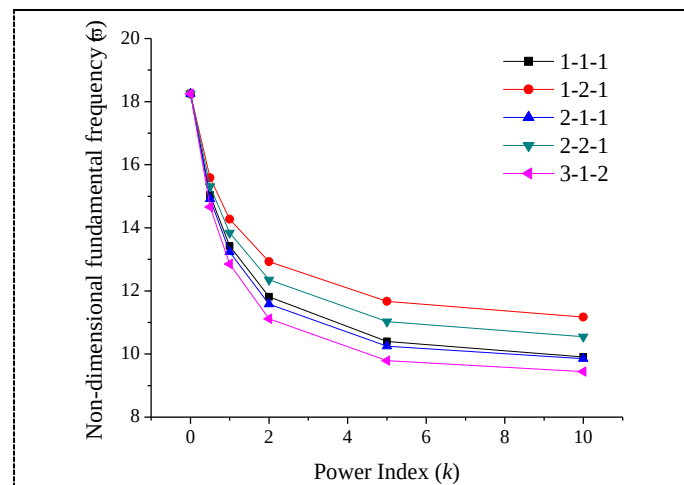


Figure 3. Variation of non-dimensional frequency parameter with power index.

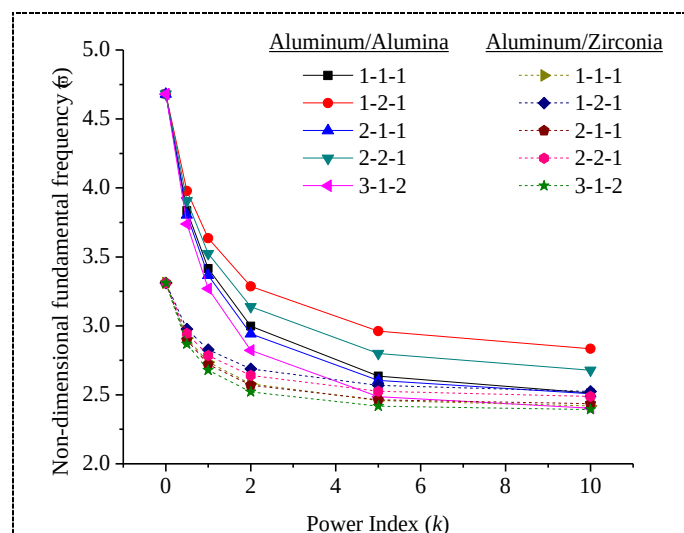
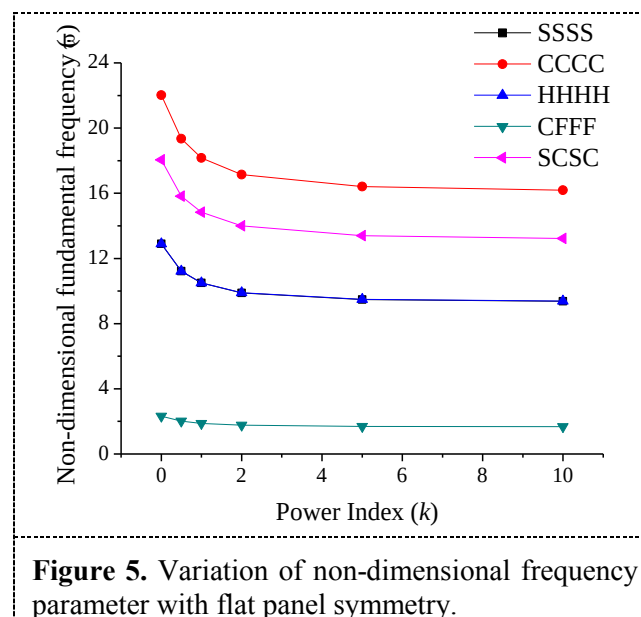


Figure 4. Variation of non-dimensional frequency parameter with flat panel symmetry.

Finally, the influence of support conditions on the fundamental frequencies of FG sandwich panels is considered. Al/Zr square sandwich panels having 3-1-2 symmetry, thickness $h=0.01\text{m}$ and $a/h=10$ subjected to SSSS (all edges simply supported), CCCC (all edges clamped), HHHH (all edges hinged), CFFF (one edge clamped, other edges free) and SCSC (alternate edges simply supported and clamped) support conditions are considered. Fig. 5 shows the variation of non-dimensional frequency parameter with support conditions for increasing power index value. It can clearly be observed that the frequency parameter decreases with increasing number of constraints at the support. The frequency is the highest for the clamped panels and the lowest for the cantilever panels. Decreasing number of constraints at the support causes the panels to be less stiff, thereby leading to lower natural frequency values. Also, the frequency follows a decreasing trend with increasing power index values for all of the support conditions. It is to be noted that the SSSS condition has the behavior similar to HHHH condition for all of the power index values.



4. Conclusion

The free vibration behaviour of functionally graded sandwich panels with uniform thickness has been analysed in the framework of the FSDT using commercial software package ANSYS. The convergence of the present model is established and the non-dimensional fundamental frequency computed using the present scheme is validated with the available benchmark results. Subsequently, the influence of power index, symmetry, and support condition on free vibration behaviour is investigated. It is observed that the stiffness of the panels decreases with increasing power index (k) leading to decreasing frequency with increasing power index. Further, the panels with a thicker core (higher core-to-face thickness ratio) are found to be stiffer as compared to the panels with a thinner core, thereby having higher fundamental frequency. The decreasing number of constraints at the support causes the panels to be less stiff, thereby leading to lower natural frequency values.

5. References

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