

# Effect of hygrothermal environment on free vibration characteristics of FGM plates by finite element approach

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**Abstract.** In this work the numerical investigations are carried out on free vibrations of FGM plates in hygrothermal environment. FGM plate form based on third order shear deformation theory is considered. FGM plate material properties vary the in the thickness direction using simple power law. A four noded rectangular element with 7 DOF per node is used in this analysis. The present method results are compared with results available in literature to assess its efficiency. The investigation is performed on natural frequencies of FGM plates under temperature and moisture conditions. The outcomes show that the vibration characteristics are greatly influenced with the hygrothermal parameters concerned in the study.

## 1. Introduction

Functionally graded material plates are broadly utilized as a part of aviation, gas turbine vanes, vehicle parts and typical zones. Amongst their use, they might be presented to moisture and temperature condition. The absorption of moisture and temperature change influences the mechanical properties of the FGMs. Increment in moisture preoccupation may cause weakening, it impacts the vibration characteristics. Thus, the vibration of FGM plate subjected to hygrothermal loads are essential to consider. Zhang and Zhou [1] considered the neutral surface of the FGM plates using thin plate theory. Reddy and Chin [2] examined temperature dependent material properties for the thermo-mechanical investigation of functionally graded cylinders and plates. Senthil and Batra [3] exhibited a 3-dimensional method for vibration analysis of FG plates with simply supported boundary conditions. Kim [4] presented vibration analysis of FG rectangular plates in thermal fields. He assumed the temperature variation along the thickness direction and consistent in the plane of the plate.

Croce et al. [5] analyzed functionally graded Reissner–Mindlin plates by finite element method. Ramu and Mohanty [6], Ramu [7] used finite element method to determine the free vibration characteristics of FGM plates. The vibration attributes of composite plates under differing temperature and moisture was exhibited by Rath and Sahu [8]. Panda et al. [9] investigated numerical and experimental examination to study the influence of raised temperature and moisture absorption on free vibration behavior of fiber delaminated composite plates. Upadhyay et al. [10] researched the hygrothermal impact on fiber reinforced composites under uniaxial pressure. Lee and kim [11] investigated post buckling characteristics of FG plates in hygrothermal environment.



There are some works related to the vibration analysis of a FGM plate in hygrothermal condition. Based on the study's author has chosen the influence of hygrothermal condition for free vibration analysis of plates. Plate basic kinematics are adopted from third order shear deformation and vibration analysis have been carried out by finite element approach. The proposed work examines the influences of the hygrothermal field and power law index value on plate like structure vibration characteristics. In hygrothermal environment the increase of temperature with different distribution conditions and moisture percentage impacts the vibration characteristics of FGM plate.

## 2. Mathematical Modelling

### 2.1 Constitutive law for FGM plate

Metal and ceramic combinations forms a FGM. For this analysis temperature dependent material properties considered from Reddy and Chin [2].

$$P(z) = P_c(z)Z_c(z) + P_m(z)Z_m(z) \quad (1)$$

where  $P(z)$  represents effective material properties. Thermal expansion coefficient  $\psi$ , moisture expansion coefficient  $\phi$ , Modulus of elasticity  $E$ , mass density  $\rho$  and Poisson's ratio  $\nu$  of the FGM plate.  $P_m(z)$  and  $P_c(z)$  are the metal and ceramic material properties of phases correspondingly at any surface  $z$  from the mid-surface of the FGM plate.

The constituent materials fraction of volume, metal  $Z_m(z)$  and ceramic  $Z_c(z)$  at point  $z$  from mid-surface is expressed as expressed as:

$$Z_c(z) + Z_m(z) = 1 \quad (2)$$

The ceramic constituent volume fraction as per simple power law can be related as

$$Z_c(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^k, 0 \leq k \leq \infty \quad (3)$$

where  $k$  is the index value of power law.

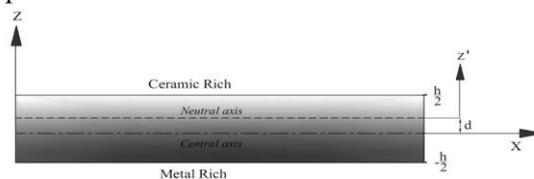


Fig. 1 FGM plate geometry.

### 2.2 Temperature dependent material properties

The following expression can be attained for temperature-dependent properties of material

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (4)$$

Coefficients of material  $P_0$ ,  $P_{-1}$ ,  $P_2$ , and  $P_3$ .

$T = T_0 + T(z)$ , here  $T_0$  is the room temperature and  $T(z)$  is the temperature rise through the thickness direction.

Using equation (4) FGM plates vibrant properties can be coupled as follows

$$E(z, T) = E_m(T) + [E_c(T) - E_m(T)] \left(\frac{1}{2} + \frac{z}{h}\right)^k, \quad \rho(z, T) = \rho_m(T) + [\rho_c(T) - \rho_m(T)] \left(\frac{1}{2} + \frac{z}{h}\right)^k \quad (5)$$

$$\phi(z, T) = \phi_m(T) + [\phi_c(T) - \phi_m(T)] \left(\frac{1}{2} + \frac{z}{h}\right)^k, \quad \psi(z, T) = \psi_m(T) + [\psi_c(T) - \psi_m(T)] \left(\frac{1}{2} + \frac{z}{h}\right)^k$$

The subscript  $c$  and  $m$  represents ceramic and metal constituents, correspondingly.

### 2.3 Constitutive relations

In assessment of the third order shear deformation plate hypothesis [2], the in-plane displacement fields  $u$ ,  $v$  and the transverse displacement  $w$  of the plate can be expressed with respective to the neutral surface.

$$\begin{aligned}
u &= u_n + z' \theta_x - c_1 z'^3 (\theta_x + w_{n,x}), \\
v &= v_n + z' \theta_y - c_1 z'^3 (\theta_y + w_{n,y}) \\
w &= w_n
\end{aligned} \tag{6}$$

where  $u_n$ ,  $v_n$  and  $w_n$  are represents the displacement fields with respect to the neutral plane of the plate.  $\theta_x$  and  $\theta_y$  are the rotations of normal transverse about the y and x axis, correspondingly. The distance of the considered surface from the neutral layer is given by  $z'$  as shown in figs. (1).

The constituent stress-strain relation of the FGM plate is

$$\begin{Bmatrix} \sigma_{xx}^{HT} \\ \sigma_{yy}^{HT} \\ \tau_{xy}^{HT} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \left\{ \psi(z', T) \Delta T(z') + \varphi(z', T) \Delta C(z') \right\} \tag{7}$$

where

$$Q_{11} = Q_{22} = \frac{E(z', T)}{(1-\nu^2(z', T))}, Q_{12} = Q_{21} = \frac{\nu(z', T)E(z', T)}{(1-\nu^2(z', T))}, Q_{66} = \frac{E(z', T)}{2(1+\nu(z', T))}$$

$\Delta T = T_c - T_m$  and  $\Delta C = C_c - C_m$ , here  $T_m$  and  $C_m$  are reference temperature and the reference moisture concentration at metal surface and ceramic surface, respectively.

#### 2.4 Thermal investigation

The conduct of FGM plate in thermal condition is considered for this investigation. The one dimensional temperature appropriation through the thickness manner is expected. For this condition three thermal conditions are viewed as: uniform, linear and nonlinear distributions.

##### 2.4.1 Temperature distribution uniformly

In uniform temperature environment, the temperature rise along the thickness is given as

$$T(z') = T_0 + \Delta T(z') \tag{8}$$

where  $\Delta T(z') = T_c - T_m$ , represents the temperature gradient and  $T_0 = 300$  K is ambient temperature.  $T_m$  and  $T_c$  are temperature at metal surface and at ceramic surface respectively.

##### 2.4.2 Temperature distribution linearly

The linear variation of temperature distribution along the thickness can be viewed as follows

$$T(z') = T_m + \Delta T(z') \left( \frac{z'}{h} + \frac{1}{2} \right) \tag{9}$$

##### 2.4.3 Temperature distribution nonlinearly

Along the thickness direction with 1-D temperature distribution is considered as

$T = T(z')$ . The equation for a steady-state heat transfer can be represented as

$$-\frac{d}{dz} \left[ R(z') \frac{dT}{dz} \right] = 0 \tag{10}$$

where  $R(z')$  is the thermal conductivity effective material.

This condition is solved by proposing temperature at upper and base surfaces, for example,

$T = T_c$  at  $z' = h/2 - d$  and  $T = T_m$  at  $z' = -h/2 - d$ .

The thickness direction temperature nonlinear distribution can be indicated as

$$T(z') = T_m + (T_c - T_m) \frac{\int_{-h/2-d}^{z'} \frac{1}{K(z')} dz'}{\int_{-h/2-d}^{h/2-d} \frac{1}{K(z')} dz'} \tag{11}$$

### 3. Finite Element Analysis

Finite element modelling of FGM plate is carried out by a rectangular element with four node. The rectangular component has one node at each corner and seven degrees of freedom for every node. The in-plane displacements are  $u$ ,  $v$  and  $w$  is the transverse displacement,  $\theta_x$ ,  $\theta_y$ ,  $\frac{\partial w}{\partial x}$  and  $\frac{\partial w}{\partial y}$  represent the rotations and slopes about  $x$  and  $y$  axes, respectively.

$$u = \sum_{i=1}^4 N_i u_i, \quad v = \sum_{i=1}^4 N_i v_i, \quad w = \sum_{i=1}^4 N_i w_i, \quad \theta_x = \sum_{i=1}^4 N_i \theta_x^i, \quad \theta_y = \sum_{i=1}^4 N_i \theta_y^i, \quad \frac{\partial w}{\partial x} = \sum_{i=1}^4 N_i \frac{\partial w_i}{\partial x}, \quad \frac{\partial w}{\partial y} = \sum_{i=1}^4 N_i \frac{\partial w_i}{\partial y} \quad (12)$$

$N_i, i=1,2,3,4$  represent the element nodal shape functions.

Strain energy of the element ( $U^{(e)}$ )

$$\begin{aligned} U^{(e)} &= U_e^{(e)} - U_{HR}^{(e)} \\ &= \frac{1}{2} \left[ \{q^{(e)}\}^T [K^{(e)}] \{q^{(e)}\} \right] - \frac{1}{2} \left[ \{q^{(e)}\}^T [K_{HR}^{(e)}] \{q^{(e)}\} \right] \\ &= \frac{1}{2} \{q^{(e)}\}^T [K_{ef}^{(e)}] \{q^{(e)}\} \end{aligned} \quad (13)$$

where,  $[K_{ef}^{(e)}] = [K^{(e)}] - [K_{HR}^{(e)}]$ ,  $[K_{ef}^{(e)}]$  is the effective stiffness matrix of the plate element and  $\{q^{(e)}\}$  is displacement vector

Plate element kinetic energy

$$T^{(e)} = \frac{1}{2} \left[ \{q^{(e)}\}^T [M^{(e)}] \{q^{(e)}\} \right] \quad (14)$$

where  $[M^{(e)}]$  is the mass matrix of the plate element.

For the free vibration, the finite element technique gives the governing equations of motion which is solved as an eigenvalue/eigenvector issue can be written as

$$\left[ [K_{ef}^{(e)}] - \omega^2 [M^{(e)}] \right] \{q^{(e)}\} = 0 \quad (15)$$

here  $\omega$  is natural frequency

## 4. Results and Discussion

### 4.1 Comparison studies

The numerical results of frequency parameters of initial six modes of FGM (Si<sub>3</sub>N<sub>4</sub>/SUS304) rectangular plate are calculated by applying third order shear deformation hypothesis. Ref. [3] results are validated with present proposed method. The non-dimensional frequencies showed in table 1. It can be observed that there is good convenient between the outcomes of present approach and the results of Ref. [3]. Natural frequency parameter is expressed as:  $\omega = \frac{\omega W^2}{\pi^2} \sqrt{\frac{I_0}{D}}$

Table 1 All sides clamped (Si<sub>3</sub>N<sub>4</sub>/SUS304) FGM plates subjected to uniform temperature distribution (L=0.2m, h/W=0.1, T<sub>m</sub>= 300K,  $\Delta T = 300K$ ).

L/W	k	Source	Frequency parameters				
			$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_5$
2		Ref. [3]	3.7202	7.3010	7.3010	10.3348	12.2256
		Present	3.6618	7.2832	7.2832	10.2549	12.5202
10		Ref. [3]	3.1398	6.1857	6.1857	8.7653	10.3727
		Present	3.1032	6.2780	6.2780	8.8216	10.5657

### 4.2 Free vibration analysis of FGM plates in hygrothermal environment

Numerical investigation has been performed for FGM (Al<sub>2</sub>O<sub>3</sub>/SUS305) plate of with geometrical dimensions length is 0.2m and thickness is 0.02m. The fundamental natural frequencies of FGM plate in hygrothermal fields are carried out for numerical analysis. Power law indices k=1 and 5 with uniform, linear and nonlinear temperature conditions are demonstrated in figures 2, 3 and 4 shown

fundamental frequency parameter variation verses temperature. The observations from those plots, exposed that the increase of temperature difference reduces the fundamental natural frequency parameters of FGM plates in hygrothermal environment. This is due to the increase of temperature reduces the FGM plate properties strength, so it reduces the frequency parameters.

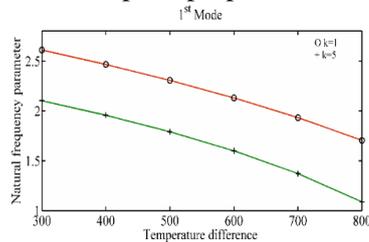


Figure 2 Variation first mode frequency parameter with temperature rise. UTD ( $\Delta C = 1\%$ )

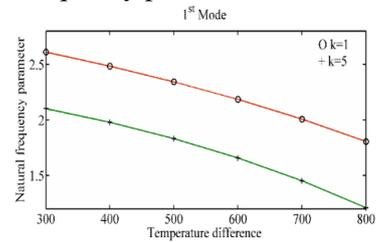


Figure 3 Variation first mode frequency parameter with temperature rise. LTD ( $\Delta C = 1\%$ )

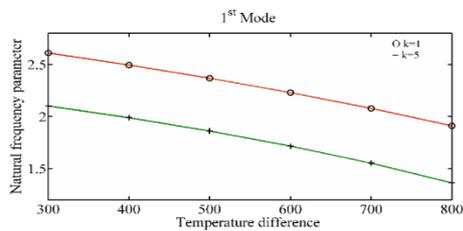


Figure 4 Variation first mode frequency parameter with temperature rise. NTD ( $\Delta C = 1\%$ )

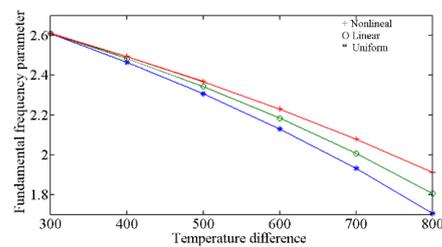


Figure 5 Fundamental frequency parameter verses temperature change with UTD, LTD and NTD. ( $k = 1, \Delta C = 1\%$ ).

The figure 5 describes the increase of temperature in hygrothermal environment of FGM plate with various thermal fields like uniform linear and nonlinear temperature conditions. The effect of fundamental frequency parameter is more prominent in uniform temperature field compared with linear and nonlinear fields.

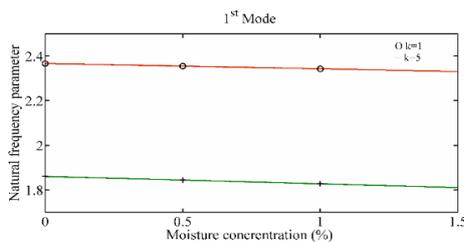


Figure 6(a) Variation of first mode natural frequency parameter verses moisture. ( $\Delta T = 200K$ )

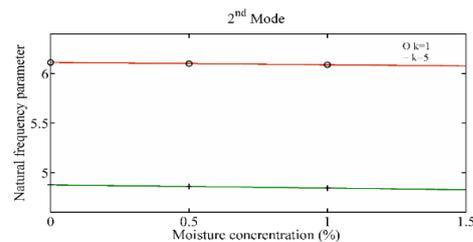


Figure 6(b) Variation of first mode natural frequency parameter verses moisture. ( $\Delta T = 200K$ )

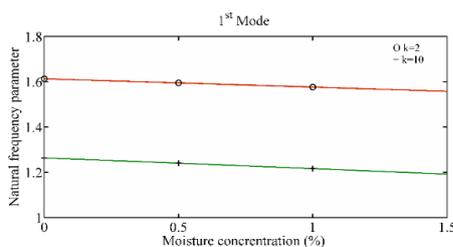


Figure 7(a) Variation of first mode natural frequency parameter verses moisture. ( $\Delta T = 500K$ )

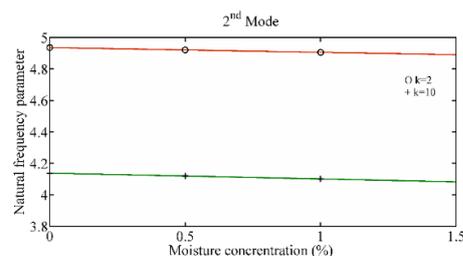


Figure 7(b) Variation of first mode natural frequency parameter verses moisture. ( $\Delta T = 500K$ )

The fundamental frequencies of the simply supported FGM plate in hygrothermal fields with different index values ( $k=1, k=5, \Delta T = 200 K$ ) are shown in figures 6 (a) and (b). The increase of moisture percentage decreases the natural frequency parameter. Increase of power law index causes increases metal volume fraction, so the increased volume fraction of metal absorbs more moisture concentration compared with lower index values. The increased moisture content reduces the frequency parameter. Figures 7 (a) and (b) describe natural frequency parameter variation with raise in moisture content (%) of FGM plate in hygrothermal fields with indices  $k=2$  and  $10$  with  $\Delta T = 500K$ . The increase of moisture concentration from  $0\%$  to  $1.5\%$  reduces the natural frequency parameter. So, with rise of moisture content the fundamental frequency parameter of plate in hygrothermal environment is reduced.

## 5. Conclusion

A mathematical formulation has been derived third order shear deformation theory for FGM plate vibration analysis by finite element approach. A power law is used to form FGM plate by varying material composition along its thickness direction. Studied literature results of FGM plate natural frequencies are compared with the proposed methodology. The present formulated finite element approach results are good agreement with the published results. The formulated methodology has been used for simply supported FGM plate in hygrothermal field with different thermal distribution and index values. The increase index value influences the natural frequencies are decreased. Natural frequencies of FGM plates lower with increase of temperature and moisture consideration. Similarly, in high thermal environments the increase of moisture concentration reduces the natural frequencies of FGM plate. The moisture concentration is also influences at higher values of power law index cases, it causes reduces the natural frequencies.

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