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Structural behavior of non-prismatic mono-symmetric beam

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Abstract. This paper attempts to understand the structural behavior of non prismatic channel beams subjected to bending through finite element(FE) analysis. The present study aims at shedding some light on how tapered channel beams behave by studying the effect of taper ratio on structural behavior. As a weight reduction is always desired in aerospace structures beams are tapered in order to obtain highest structural efficiency. FE analysis has been performed to study the effect of taper ratio on linear deflection, lateral torsional buckling, non-linear parameters and dynamic parameters. Taper ratio tends to affect the mechanics of tapered beams innocuously and adversely. Consequently it becomes important to understand and document the mechanics of channel tapered beams. Channel beams generally have low torsional rigidity due to the off-shear loading. The effect of loading type and location of applied load have been studied for flange taper, web taper and symmetric taper for different conditions. Among these, as the taper ratio is increased, the torsional angular deflection increases but begins to decrease when the beam is web tapered and symmetrically tapered for a mid web loaded beam. But when loaded through the shear center, an increase in the torsional angular deflection can be observed with increase in taper ratio. This indicates that a flange taper could cause an increase in torsional angular deflection with an increase in taper ratio.

Keywords: **Tapered beams, channel beams, structural behavior, finite element analysis**

1. Introduction

The utilization of tapered beams (beams with varying cross sections) has been increasing in recent times in aerospace, civil and mechanical structures. This is due to the fact that tapered beams meet the aesthetic and functional requirements of the structure. Tapered beams are also known to have high stiffness to mass ratio, better wind and seismic stability. Tapered beams are generally chosen in order to be able to optimize the load capacity at every cross section. To be able to use tapered beams more often a balance between the fabrication cost and material cost has to be present. A plethora of research has been done on doubly symmetric I-section and the mono symmetric T-section tapered beams over the past three decades. Scanty literature is available on the structural behavior of tapered C-section. This present study aims at understanding the structural response of tapered thin walled C-section as the taper ratio is varied, the shear forces are considered negligible during this analysis. Analytical models to analyze tapered beams have been developed by various authors over the past decades[1–7] and will not be repeated here. There are no available classical methods to analyze tapered beams[8]. This study does not look into developing new analytical models but to study the structural behavior of a tapered channel beam. The results obtained are based on finite element analysis.

C-section beams originally were designed to be used in bridges but now are also used in aerospace, naval and in civil construction. In C-section beams the axis of bending does not coincide with the centroid and the shear center lays behind the web, hence any bending load applied on the web or the flange would induce torsion.

Thin walled beams with open and closed sections are often seen in aerospace applications. Thin walled beams when loaded in bending may fail in a bending-torsion mode coupling as the torsional strength is relatively less when compared to the bending strength.

With increasing taper the major moment of inertia had a linear decrease from root to tip[9]. Kim et al.[2] found that in tapered cantilever beams the location of maximum stress is a function of the loading type and the taper ratio. When a cantilever beam is loaded with a concentrated moment at the free end the location of maximum bending stress depends on the taper ratio and for an UDL loading the location of maximum bending stress is always at the fixed end. The lateral torsional buckling was found to be strongly affected by taper ratio[4]. Tapered beams with tapered flanges can resist stability loss in comparison to beams with tapered webs[10]. It was also deduced by Marques et al.[11] that the location of failure was a function of taper ratio and by varying the taper ratio the location of failure can be estimated. Taper ratio also decreases the amount of distortion, higher the taper of the section better the resistance to distortion and warping[12, 13]. Tapering the beam further minimizes the distance between shear center and centroid, ameliorating the critical load[14]. With the change in flange width while only tapering the web can increase or decrease the critical loads[14]. The moment capacity at each section of tapered beams decreases from the clamped end to free end. The plastic hinge for a prismatic beam is formed at the fixed end, as the taper increases the plastic hinge moves towards the tip[15]. Effect of loading positions were studied by Yeong and Jong[16], loads were applied at the top flange, mid web and bottom flange and it was found that loads applied at the top flange reduces the critical loads in comparison to other loading conditions. The increment in critical loads due to taper are mainly dependent on boundary conditions, cantilevers show a significant improvement whereas in simply supported beams the increment is trivial[17].

Hence it is of utmost importance to study the effect of taper ratio. Taper ratio is defined as the ratio of the tip dimension to the root dimension.

2. Model Development

2.1. Geometric Model

A C-section beam with similar web and flange dimensions were chosen. Figure 1 shows the cross section of the beam, where $W_{f(r,t)}$ denotes the width of the flange and $H_{w(r,t)}$ denotes the height of the web, the suffix r and t denotes root and tip.

$$T.R = \alpha = \frac{W_{f(r)}}{W_{f(t)}} \tag{1}$$

$$T.R = \alpha = \frac{H_{w(r)}}{H_{w(t)}} \tag{2}$$

The taper ratio is calculated as per equation 1 and 2, where equation 1 is the taper ratio for flange taper (figure 2) and equation 2 is the taper ratio for web taper (figure 4).

Locations shown in figure 4 are indicative of the three locations which are of paramount importance in order to understand the behavior of taper channel beams. Concentrated load has been applied at location 2 which is the center of the web, loading at the shear center would not be practical as it is an imaginary point placed behind the web. The shear axis and centroidal

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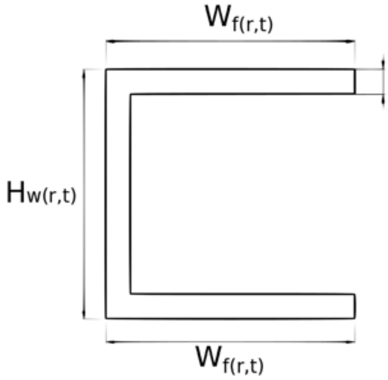

Figure 1: Dimensions to calculate the taper ratio

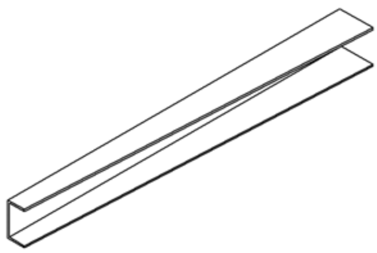

Figure 2: Flange Taper

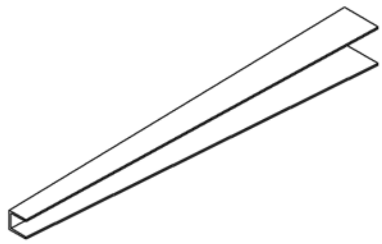

Figure 3: Complete Taper (Web and Flange)

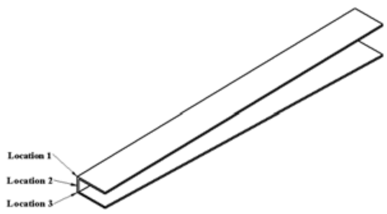

Figure 4: Web Taper

axis is not parallel and not in the same plane, hence the minor axis bending and torsion will always be coupled. It can be modeled using the MPC (multi point constraint) technique or a rigid link in FEA. Taper ratio's varying from 1(prismatic case) to 0.1 have been studied.

In analyzing this beam we assume that the beam is elastic (no material nonlinearity), the beam is composed of thin walled sections. Every section is assumed to be rigid in its own plane. Longitudinal displacements and shearing deformations are neglected. The thickness over the entire span of the wing is constant and does not vary.

2.2. FE Modeling

The present analysis deals with thin walled channel beams and hence can be modeled with shell elements in the commercial Finite Element (FE) software ANSYS. Modeling a thin walled structure with a solid element can result in exhaustive use of computational time and space. Results using the shell elements are more accurate than the beam element due to the fact that shell elements use lesser assumptions than beam elements[1]. Beam elements yield reasonably accurate results for buckling mode shapes and critical loads and as long as the beam is not short. Accuracy increases as the beam length increases[18]. Local effects near the loading points cannot be captured in beam elements[18]. Shell181 was used to model the thin walled C-section beam. Shell181 is a four noded element with each node having six degrees of freedom. This element will not solve if there is zero thickness and the solution is terminated if the thickness at the integration point vanishes. Convergence studies consisting of a simple cantilever beam with a tip load using the Euler-Bernoulli assumptions were performed to evaluate the quality of the finite element model.

The input mechanical properties for linear isotropic materials are, Young's modulus of 200 GPa, Poisson's ratio of 0.3 and density of 8000 kg/m^3. Graphs were plotted using fourier and higher order polynomial models in Matlab.

3. Results and Discussion

3.1. Deflection

The stiffness of a bi-symmetric tapered beam is highest if only the flange is tapered and the stiffness is minimum if the beam is tapered completely (web and flange). Figure 5, 6 and 7 show the variation in deflection at the three locations indicated in figure 4 as a function of taper ratio. A large displacement nonlinear analysis was performed to obtain the lateral deflections.

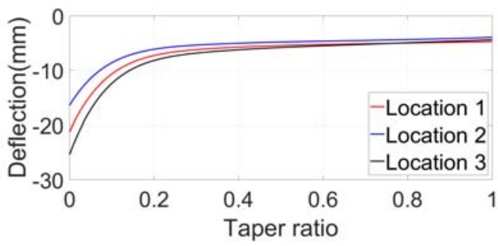

Figure 5: Deflection(Y) of C-section with tapered flange

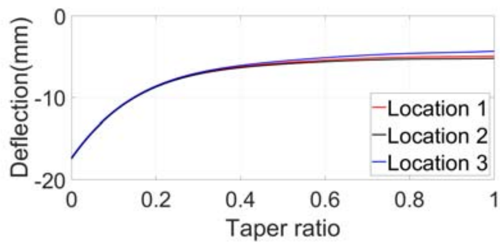

Figure 6: Deflection(Y) of C-section with complete taper

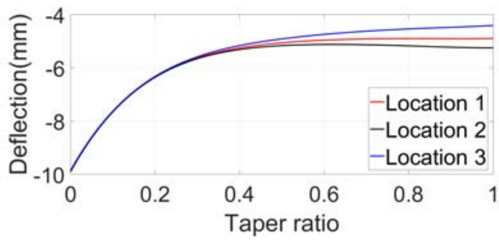

Figure 7: Deflection(Y) of C-section with tapered web

Large displacements account for change in stiffness due to change in shape. It can be seen that the displacements at the three locations are different when compared to the complete taper and web taper. Though the displacements are similar in the no taper condition there is a variation when the beam is tapered completely and also when the web alone is tapered. The upper flange has higher lateral deflection when only the flange is tapered than the mid of the web which indicates induced torsion. The web tapered beam has the highest resistance to bending and the flange tapered beam has the lowest. The distance of the shear center from the centroid could be a factor to cause this behavior. The axis of symmetry could play an important role in the structural behavior, T-section and mono-symmetric I-sections were symmetric about the Y-axis whereas C-section is symmetric about the X-axis. This could lead to variations in the structural behavior.

A similar behavior can be noticed in the minor axis bending. Figures 8, 9 and 10 give the results of minor axis bending as the taper ratio is varied. Tapering the flange resulted in the least resistance to minor axis bending and the web tapered beam has the highest. This is not similar to the results that were obtained for various other mono-symmetric tapered beams available in literature. There is variation in behavior of channel tapered beams in comparison to the other mono-symmetric beams like the T-section or mono-symmetric I-section with varying flange lengths. The distance between the centroid and the shear center begins to decrease with

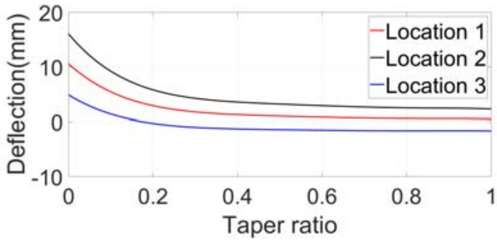

Figure 8: Deflection(X) of C-section with tapered flange

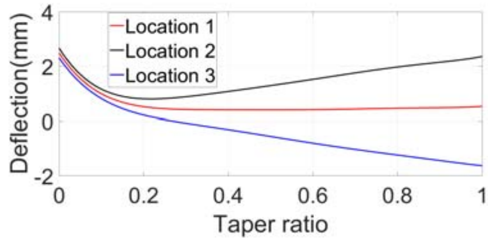

Figure 9: Deflection(X) of C-section with complete taper

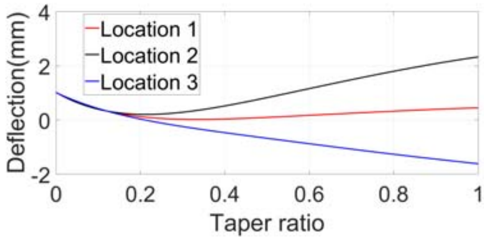

Figure 10: Deflection(X) of C-section with tapered web

increase in taper when the flange is tapered and there is an increase in the distance between centroid and shear center when the web is tapered, this phenomenon could result in a different behavior when compared to other tapered mono-symmetric beams. It is well known fact that to induce symmetric bending without torsion the load has to be applied through the shear center and not the centroid. The distance between centroid and shear could cause instabilities due to unsymmetrical bending.

When the web is tapered the major moment of inertia (I_{xx}) decreases at faster rates in comparison to the minor moment of inertia(I_{yy}), as a result with increase in degree of taper the value of I_{yy} becomes larger than I_{xx}. This results in reduced stiffness in the axis of loading.

3.2. Modal & Transient

It is important to understand the dynamic response of the structure to time dependent loading. Natural frequencies and mode shapes help in understanding the structural behavior in order to be able to design better optimized structure. The first three modal frequencies have been extracted as shown in figures 11, 12 and 13. Modes were extracted with the block lanczos method using the sparse matrix solver. First modal frequency increases as the degree of taper increases for all three cases of taper. The second modal frequency increases in the case of web taper and complete taper but decreases when the flange is tapered. The third modal frequency increases when the degree of taper increases (taper ratio decreases) when the flange and web+flange is tapered, decreases at quick rate when the web is tapered.

Transient analysis of the tapered channel beam was also performed as time based nonlinear analysis where a sinusoidal load was applied as a base excitation to the tip. Results are presented to characterize the dynamic displacement response to sinusoidal loading. Minimum and maximum displacement for all three cases (flange taper, web taper and complete taper) are presented in figure 14. The displacements are indicative of the damping present in the structure as the degree of taper is increased. The flange taper shows highest minimum and maximum displacement which indicates lowest damping. The complete taper and web taper displacements are very close with the web taper having the highest damping and hence the lowest minimum

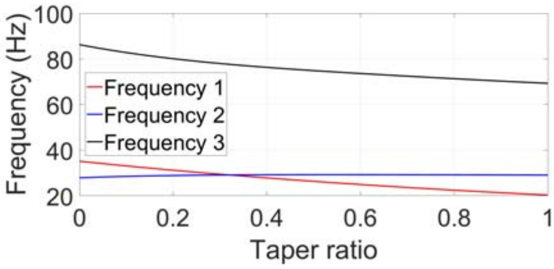

Figure 11: Modal frequencies of C-section with tapered flange

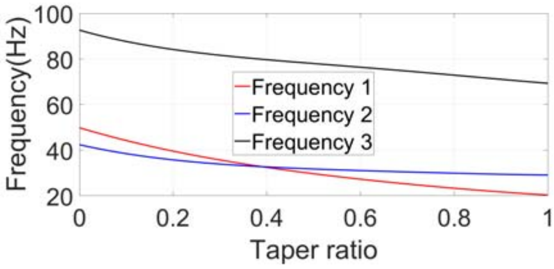

Figure 12: Modal frequencies of C-section with complete taper

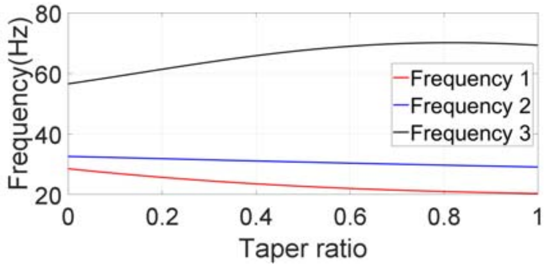

Figure 13: Modal frequencies of C-section with tapered web

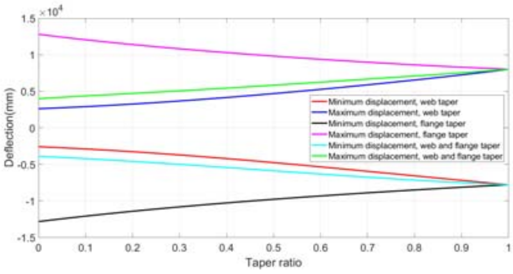

Figure 14: Deflection due to sinusoidal excitation for three cases of taper

and maximum displacements. The maximum and minimum displacements increase as the degree of taper increases in case of flange taper and in the other two cases of web and complete taper the minimum and maximum displacements are decreases indicating the change in damping as function of taper ratio.

3.3. Lateral Torsional Buckling

Lateral torsional buckling is the twisting of the beam accompanied with lateral bending when the beam is loaded in the major axis plane. There is a similarity in behavior for three cases of taper as shown in figures 15, 16 and 17. Graphs in figures show a exponential decrement in the lateral bending along the plane of major axis with an increase in taper, the curves can be represented by a 5th order polynomial. In the plane of minor axis the deflection due to induced torsion show a different variation. Figures 18, 19 and 20 indicate the variation in deflection in the plane of minor axis as taper increases. There is decrease in the deflection in plane of minor axis as the taper increase (decrease in taper ratio) when a complete taper happens. Flange taper has a relatively unstable behavior in tapered channel beams. There is also a relatively drastic decrease in the warping constant when the flange is tapered keeping the web constant.

4. Summary & Conclusion

This paper reports the structural behavior of tapered channel beams. Tapering does provide structural advantage by increasing stiffness, stability and resistance to warping. Taper also reduces the amount of material used making it more economical. Results and discussions indicate that tapering the flange causes instabilities in comparison to the web taper and flange taper. There is a reduction in the distance between shear center and centroid when the flange is tapered and an increase in the distance between shear center and centroid in the other two cases. Flange taper also has the least stiffness hence the lowest resistance to bending with the web taper having the highest resistance to bending. Web tapered beams also have the highest damping and show a reduction in displacement as the degree of taper increases. The axis of

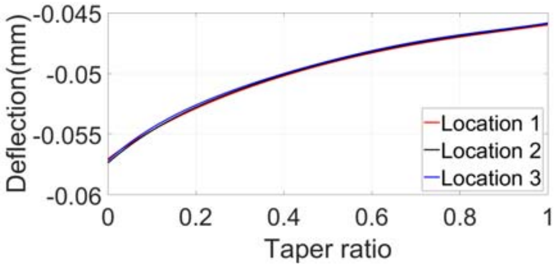

Figure 15: Deflection(Y) of C-section with tapered flange

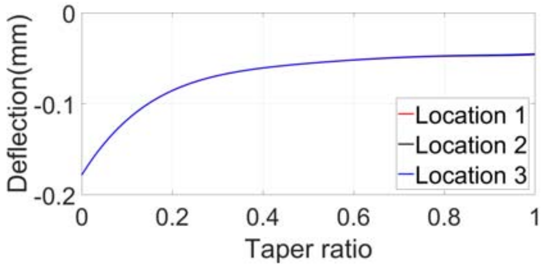

Figure 16: Deflection(Y) of C-section with complete taper

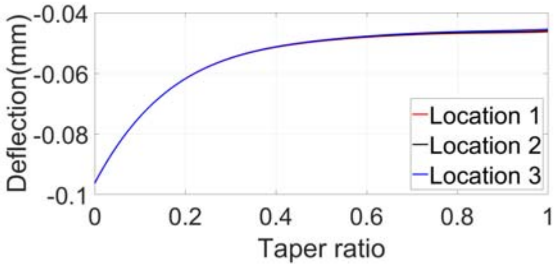

Figure 17: Deflection(Y) of C-section with tapered web

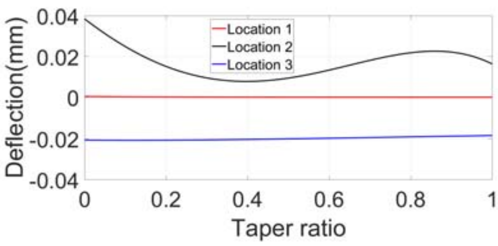

Figure 18: Deflection(X) of C-section with tapered flange

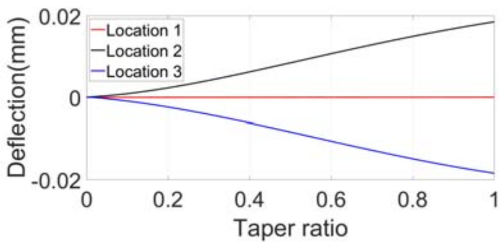

Figure 19: Deflection(X) of C-section with complete taper

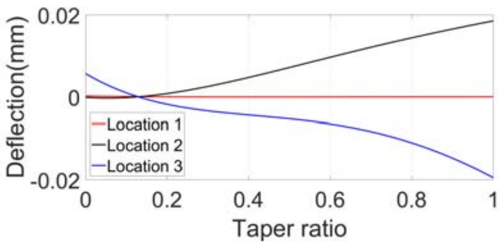

Figure 20: Deflection(X) of C-section with tapered web

symmetry is an important parameter, as the centroid and shear center are positioned along the axis of symmetry. Conclusions show that there is an advantage if web tapered beams are used than flange tapered beams for tapered C-section beams. Unlike I-section and T-section tapered beams, where flange taper has better structural efficiency than web tapered beams, web taper has better structural efficiency in tapered C-section beams. The structural behavior of mono-symmetric C-section beams are not similar to the mono-symmetric I-section or T-section.

Detailed research is needed to understand the structural behavior of tapered channel beams.

References

[1] Zhang Lei and Tong Geng Shu. Lateral buckling of web-tapered i-beams: A new theory. *Journal of Constructional Steel Research*, 64:1379–1393, 2008.

[2] Boksun Kim, Andrew Oliver, and Joshua Vyse. Bending Stresses of Steel Web Tapered Tee Section Cantilevers. *Journal of Civil Engineering and Architecture*, 7(11):1329–1342, 2013.

[3] B. Asgarian, M. Soltani, and F. Mohri. Lateral-torsional buckling of tapered thin-walled beams with arbitrary cross-sections. *Thin-Walled Structures*, 62:96–108, 2013.

[4] Ioannis G. Raftoyiannis and Theodore Adamakos. Critical Lateral-Torsional Buckling Moments of Steel Web-Tapered I-beams. *The Open Construction and Building Technology Journal*, 4:105–112, 2010.

[5] Abdelrahmane Bekaddour Benyamina, Sid Ahmed Meftah, Foudil Mohri, and El Mostafa Daya. Analytical solutions attempt for lateral torsional buckling of doubly symmetric web-tapered I-beams. *Engineering Structures*, 3:1207–1219, 1999.

[6] Noel Challamel, Ansio Andrade, and Dinar Camotin. An analytical study on the lateral torsional buckling of linearly tapered cantilever strip beams. *International Journal of Structural Stability and Dynamics*, 7(3):441–456, 2007.

[7] Jong-Dar Yau. Stability of tapered I-Beams under torsional moments. *Finite Elements in Analysis and Design*, 42:914–927, 2006.

[8] M R Pajand and M Moayedian. Explicit stiffness of tapered and monosymmetric I-beam columns. *International Journal of Engineering*, 13(2), 2000.

[9] D. A. Nethercot. Lateral buckling of tapered beams . *IABSE publications, Mmoires AIPC, IVBH Abhandlungen*, 33, 1973.

[10] Juliusz Kus. Lateral-torsional buckling steel beams with simultaneously tapered flanges and web. *Steel and Composite Structures*, 19(4):897–916, 2015.

[11] Liliana Marques, Luis Simoes da Silva, Richard Greiner, Carlos Rebelo, and Andreas Taras. Development of a consistent design procedure for lateraltorsional buckling of tapered beams. *Journal of Constructional Steel Research*, 89:213–235, 2013.

[12] Hamid Reza Ronagh. Parameters Affecting Distortional Buckling of Tapered Steel Members. *Journal of Structural Engineering*, 120(11):3137–3155, 1994.

[13] A. Andrade and D. Camotim. LateralTorsional Buckling of Singly Symmetric Tapered Beams: Theory and Applications. *Journal of Engineering Mechanics*, 131(6), 2005.

[14] Wei bin Yuan, Boksun Kim, and Chang yi Chen. Lateraltorsional buckling of steel web tapered tee-section cantilevers. *Journal of Constructional Steel Research*, 87:31–37, 2013.

[15] P. Buffel, G. Lagae, R. Van Impe, W. Vanlaere, and J. Belis. Design Curve to use for Lateral Torsional Buckling of Tapered Cantilever Beams. *Key Engineering Materials*, 274-276:981–986, 2004.

[16] Yeong-Bin Yang and Jong-Dar Yau. Stability of beams with tapered I-sections. *Journal of Engineering Mechanics*, 113(9), 1987.

[17] C. M. Wang, V. Thevendran, K. L. Teo, and S. Kitipornchai. Optimal design of tapered beams for maximum buckling strength. *Eng. Struct*, 8:276–284, 1986.

[18] Anisio Andrade, Dinar Camotim, and P. Borges Dinis. Lateral-torsional buckling of singly symmetric web-tapered thin-walled I-beams: 1D model vs. shell FEA. *Computers and Structures*, 85:1343–1359, 2006.