

# Evaluation of adhesive scarf joint strength by using singular stress field of small interface crack

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**Abstract.** In the present study, the adhesive strength of scarf joint is examined by using the stress intensity factor of the fictitious small interface crack. The stress intensity factor of small crack near the interface edge is dominated by the singular stress field of the interface corner. In this study, to evaluate the joint strength, small crack is assumed at the interface corner of the scarf joint. The stress intensity factor of the interfacial crack is calculated by changing the thickness of the adhesive layer and the scarf angle. By using the experimental fracture strength of the scarf adhesive joint specimens under tension, the values of the critical stress intensity factor are calculated. From the analysis result, when the combination of adhesive materials and the scarf angle are fixed, the critical stress intensity factors of the small interface crack are constant value irrespective of the adhesive layer thickness. Therefore, the adhesive joint strength can be evaluated as the constant stress intensity factor of small interface crack. In addition, it is possible to evaluate easily the stress intensity factor by using the dimensionless coefficients depending only on the material combination when the crack length is sufficiently smaller than the thickness of the adhesive layer. The effects of adhesive layer thickness and adhesion angle of scarf joint specimen were discussed and the effectiveness of the proposed method was indicated.

## 1. Introduction

In recent years, bonded dissimilar materials are used as structural materials in various industrial fields. It is generally known that the adhesive strength of a scarf joint is higher than that of a butt joint structure. In these bonded joint components, due to the mismatch of the material elastic properties, singular stresses occur at the interface corner. These stresses are especially high and may result in the initiation of failure from the interface-edge, which is a problem in terms of adhesive strength. Then, many studies have been conducted to evaluate the adhesive strength focusing on the singular stress field at the interface corner. Recently, in order to estimate the fracture stress of bonded plates, the small crack approach has been proposed [1,2]. In this method, by assuming a small crack at the interface edge, the stress intensity factor of small interface crack is used to evaluate the intensity of corner singular stress and the criterion of interface fracture can be expressed as the constant critical stress intensity factor [1,2]. Oda *et al* [3,4] have analyzed the singular stress of butt joint specimens with small crack in detail, and they have reported that when the interface crack is sufficiently smaller than the adhesive layer thickness, the stress intensity factor of the small interface crack is equivalent to the intensity of the singular stress field at the interface corner and it can be converted to each other.



However, it has not been studied whether the small crack model is appropriate for the scarf joint.

In this study, a scarf joint model assuming a small crack will be analyzed. The effect of the scarf angle and the adhesive layer thickness on the stress intensity factor of small interface crack will be discussed. By using experimental fracture loads [5], the evaluation method of the fracture strength for the scarf adhesive joint will be examined.

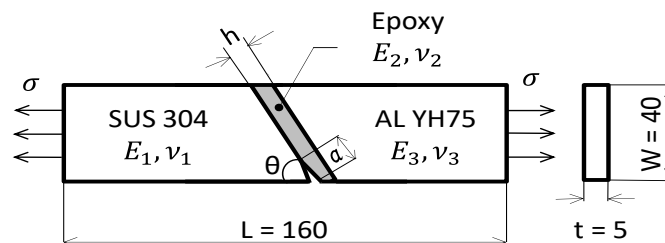
## 2. Analysis method

Figure 1 shows an adhesive joint model under tension analyzed in this study. The scarf joint consists of a thin adhesive layer sandwiched between two different plates. A small crack is assumed at the interface corner where fracture is expected. The subscripts 1, 2 and 3 of  $E$  and  $\nu$  in figure 1 refer to SUS304 stainless steel, Epoxy resin and AL YH5 aluminum alloy, respectively. The material constants used in this analysis are shown in table 1. The detailed material properties are described in [5]. In addition, Dundurs composite parameters  $\alpha$  and  $\beta$  defined by equation (1) are shown in table 2. The parameters represent the material combination of the interface.

$$\alpha = \frac{G_n(\kappa_m + 1) - G_m(\kappa_n + 1)}{G_n(\kappa_m + 1) + G_m(\kappa_n + 1)}, \quad \beta = \frac{G_n(\kappa_m - 1) - G_m(\kappa_n - 1)}{G_n(\kappa_m + 1) + G_m(\kappa_n + 1)}, \quad (1)$$

$$G_m = \frac{E_m}{2(1 + \nu_m)}, \quad \kappa_m = \begin{cases} \frac{3 - \nu_m}{1 + \nu_m} & \text{(Plane stress)} \\ 3 - 4\nu_m & \text{(Plane strain)} \end{cases} \quad (m, n = 1, 2, 3)$$

The joint has the length of the plate  $L$ , the plate width  $W$ , the plate thickness  $t$ , the adhesive layer thickness  $h$  and the scarf angle  $\theta$ . The same joint specimen has been used in the tensile fracture strength test conducted by Afendi *et al* [5]. The scarf angles are set to be  $\theta = 45^\circ, 60^\circ$  and  $75^\circ$ .



**Figure 1.** Analysis model.

**Table 1.** Mechanical properties used in the analysis.

Material	Young's modulus $E$ (GPa)	Poisson's Ratio $\nu$
SUS304	206	0.3
Epoxy	3.4	0.396
AL YH5	71	0.33

**Table 2.** Dundurs composite parameters  $\alpha, \beta$ .

Material combination	Parameter $\alpha$	Parameter $\beta$
SUS / Epoxy	0.9650	0.1642
Epoxy / YH5	-0.9037	-0.1517

The singular stress occurs at the interface free edge in the adhesive joint without the crack. The order of stress singularity in the vicinity of the interface corner is different from that of crack, and it

varies depending on the material combination and the edge configuration. The exact solution of  $1-\lambda$  is obtained by the following eigen-equation [6]:

$$A\beta^2 + 2B\alpha\beta + C\alpha^2 + 2D\beta + 2E\alpha + F = 0 \quad (2)$$

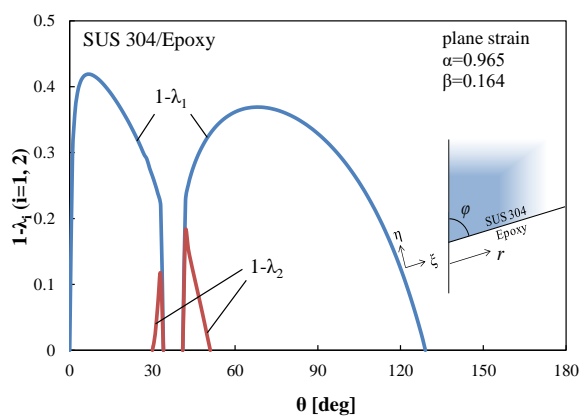
Here,

$$\begin{aligned} A &= 4K(\lambda, \theta_1)K(\lambda, \theta_2 - \theta_1), \\ B &= 2\lambda^2 \sin^2 \theta_1 K(\lambda, \theta_2 - \theta_1) + 2\lambda^2 \sin^2(\theta_2 - \theta_1) K(\lambda, \theta_1), \\ C &= 4\lambda^2(\lambda^2 - 1) \sin^2 \theta_1 \sin^2(\theta_2 - \theta_1) + K(\lambda, \theta_2 - 2\theta_1), \\ D &= 2\lambda^2 [\sin^2(\theta_2 - \theta_1) \sin^2(\lambda \theta_1) - \sin^2 \theta_1 \sin^2\{\lambda(\theta_2 - \theta_1)\}], \\ E &= -D + K(\lambda, \theta_1) - K(\lambda, \theta_2 - \theta_1), \\ F &= K(\lambda, \theta_2), \\ K(\lambda, x) &= \sin^2(\lambda x) - \lambda^2 \sin^2(x). \end{aligned} \quad (3)$$

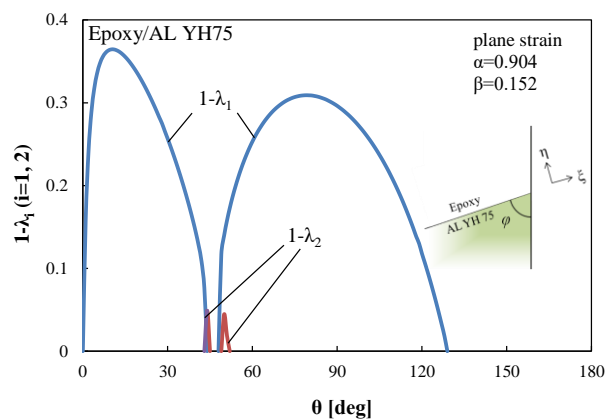
Table 3 shows the values of the order of singularity  $1-\lambda$  at SUS/Epoxy and Epoxy/AL YH75 interface corners when the scarf angle is  $45^\circ$ ,  $60^\circ$  and  $75^\circ$ . Figures 2 and 3 indicate the order of stress singularities  $1-\lambda$  at the interface corner as a function of scarf angle. From these figures, we assume the small interfacial crack along the SUS/Epoxy interface because the singularity index  $1-\lambda$  of SUS 304 / Epoxy interface is always larger than that of Epoxy/YH5 interface.

**Table 3.** The order of singularity  $1-\lambda$  at interface corner.

Interface	$75^\circ$	$60^\circ$	$45^\circ$
SUS/Epoxy	0.3648	0.3619	0.2796
Epoxy/YH75	0.2369	0.1179	0.0000



**Figure 2.** The order of stress singularity  $\lambda$  at SUS/Epoxy vs scarf angle  $\theta$ .



**Figure 3.** The order of stress singularity  $\lambda$  at Epoxy/Al vs scarf angle  $\theta$ .

In this analysis, we use the versatile FEM program MSC.Marc/Mentat. The stress intensity factor of the small interface crack is calculated by the crack tip stress method [7,8]. In this method, the stress value at the crack tip node obtained by FEM is compared with the crack tip stress value of a single

interface crack in an infinite bonded plate. When two analysis models have the same mesh pattern around the crack tip, the numerical error due to the mesh division can be canceled. By using this method, it is possible to easily determine the stress intensity factor with high accuracy in FEM. In the finite element calculation, the plane strain 8-node quadrilateral element is selected and the minimum element size  $e$  around the crack tip is set to be  $e/a = 8.14 \times 10^{-4}$  with respect to the crack length  $a$ .

### 3. Numerical results and discussion

#### 3.1. Effect of crack length on stress intensity factor of small interface crack

We investigate the effect of crack length on the stress intensity factor of the assumed small interfacial crack. The stress intensity factors  $K_1$  and  $K_2$  under the fixed adhesive thickness  $h=1$  mm ( $h/W=0.025$ ) are calculated by the crack tip stress method when the crack length is changed. The dimensionless stress intensity factors  $F_1$  and  $F_2$  are determined as follows:

$$K_1 + iK_2 = (F_1 + iF_2)\sigma\sqrt{\pi a}(1 + 2i\varepsilon), \quad \varepsilon = \frac{1}{2\pi} \ln\left(\frac{1-\beta}{1+\beta}\right) \quad (4)$$

Here,  $a$  is the crack length,  $F_1$  and  $F_2$  are the dimensionless stress intensity factors,  $\sigma$  is the tensile stress, and  $\varepsilon$  is the bi-material constant according to the combination of materials. The relations between the relative crack length  $a/h$  and the dimensionless stress intensity factor are presented in figures 4 and 5.

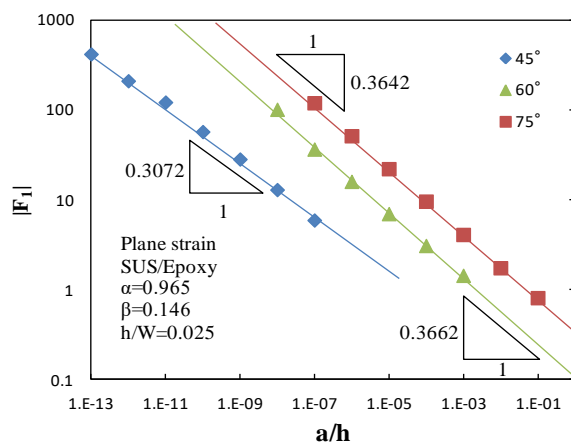


Figure 4. Relationship between  $F_1$  and  $a/h$ .

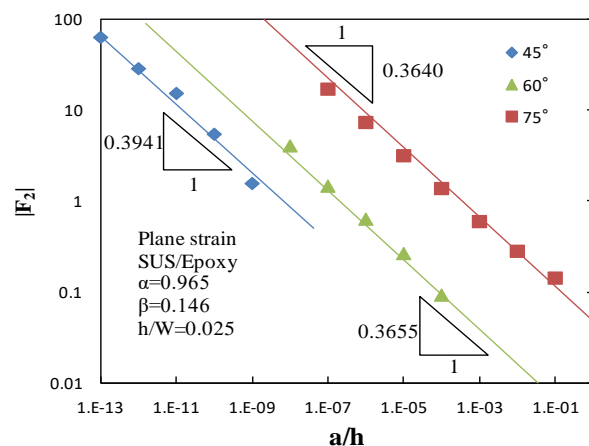
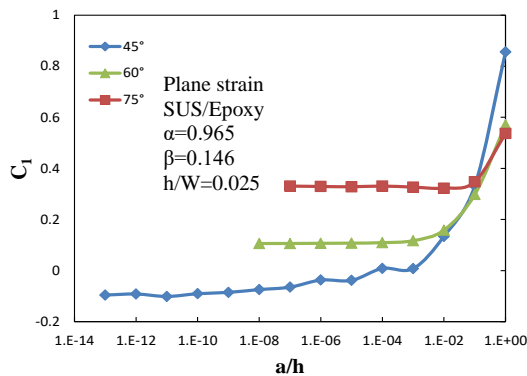


Figure 5. Relationship between  $F_2$  and  $a/h$ .

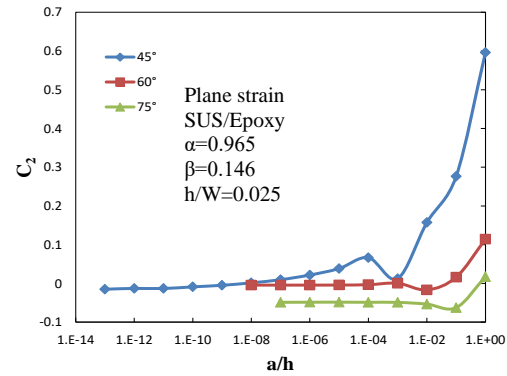
From figures 4 and 5, it is found that  $F_1$  and  $F_2$  show a constant slope on the double logarithmic plot for each scarf angle. When the scarf angle is  $75^\circ$  or  $60^\circ$ , the slope almost coincides with the order of singularity  $1-\lambda$  in table 3. In the case that the scarf angle is  $45^\circ$ , however, the slopes for  $F_1$  and  $F_2$  are different from the value of  $1-\lambda$ . The reason is that when the scarf angle is  $45^\circ$ , two different orders of singularity are obtained (figure 2), and the singular stress field is not dominated by only one singularity. Therefore, it is found that the dimensionless factors  $F_1$  and  $F_2$  are related to the relative crack length  $a/h$  and the singularity index  $1-\lambda$  and  $F_1$  and  $F_2$  can be defined as a function of the relative crack length  $a/h$  as follows [3].

$$F_1 = C_1(h/a)^{1-\lambda}, \quad F_2 = C_2(h/a)^{1-\lambda} \quad (5)$$

In figures 6 and 7, the relations between the coefficients  $C_1$ ,  $C_2$  obtained from equation (5) and the relative crack length  $a/h$  are shown. For  $\theta = 45^\circ$ , the strong singularity index  $1-\lambda_1$  is used for calculation. As shown in figures 6 and 7, it can be seen that the coefficients  $C_1$  and  $C_2$  in equation (5) converge to the constant values when the relative crack length  $a/h < 10^{-3}$  for  $\theta = 60^\circ$  and  $75^\circ$ . However, for  $\theta = 45^\circ$ , we cannot find the converged values of  $C_1$  and  $C_2$  even when  $a/h = 10^{-12}$ .



**Figure 6.** Relationship between  $C_1$  and  $a/h$ .

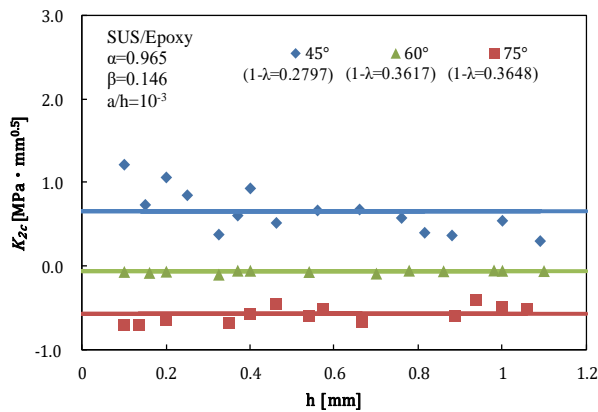


**Figure 7.** Relationship between  $C_2$  and  $a/h$ .

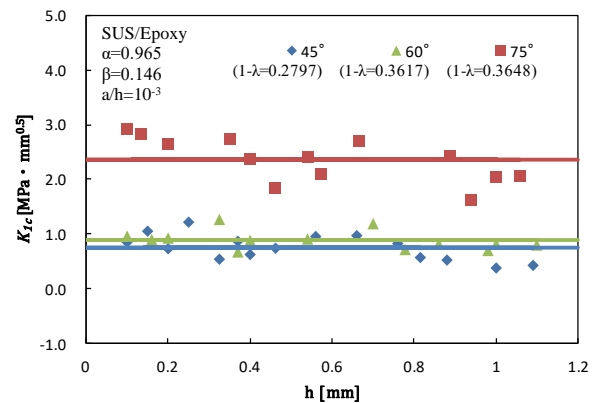
From these results, in the case that the singular stress field near the interface corner is dominated by the single singularity, the values of the coefficients  $C_1$  and  $C_2$  are constant and the dimensionless factors  $F_1$  and  $F_2$  can be represented by equation (5) when  $a/h < 10^{-3}$ . The coefficients  $C_1$  and  $C_2$  depend only on Dundurs composite parameters.

### 3.2. Consideration of interface fracture criteria

Afendi *et al* [5] have investigated experimentally the fracture strength by using the scarf joint specimens shown in figure 1. The fracture load of the scarf joint varies by changing the adhesive thickness and the scarf angle. In this study, the stress intensity factors of the small interface crack are calculated by using the fracture loads in order to consider the fracture criteria.



**Figure 8.** Relationship between  $K_{IC}$  and  $h$ .



**Figure 9.** Relationship between  $K_{2C}$  and  $h$ .

Figures 8 and 9 show the critical stress intensity factors  $K_{IC}$  and  $K_{2C}$  for each adhesive layer thickness at each scarf angle. From figures 8 and 9, it can be seen that the critical stress intensity factors are almost constant for each scarf angle even when the adhesive layer thickness  $h$  changes widely. Here, it is noted that the assumed crack length should be set to less than  $a/h=10^{-3}$ . The fracture criterion can be represented as  $K_{IC}=\text{constant}$  by using the small crack model when the combination of adhesive materials and the scarf angle are the same. Consequently, the application of small crack model is useful for evaluating the fracture strength for the scarf joint for any thickness of adhesive layer.

### 4. Conclusions

In this study, the adhesive joint strength evaluation based on the small interface crack approach was considered. The stress intensity factors of the small interface crack were analyzed by changing the

adhesive layer thickness, the scarf angle and the crack length of the scarf joint. The conclusions can be summarized as follows:

- The dimensionless stress intensity factors  $F_1$  and  $F_2$  were calculated by using the crack tip stress method and examined for the relation between the dimensionless factors and the relative crack length  $a/h$ . The dimensionless factors can be presented as following expressions when the singular stress filed at the interface corner has a single singularity index  $1-\lambda$ .

$$K_1 + iK_2 = (F_1 + iF_2)\sigma\sqrt{\pi a}(1 + 2i\varepsilon),$$

$$F_1 = C_1(h/a)^{1-\lambda}, F_2 = C_2(h/a)^{1-\lambda}.$$

- When the relative crack length is sufficiently small, that is  $a/h < 10^{-3}$ , the dimensionless coefficients  $C_1$  and  $C_2$  become constant. The coefficients  $C_1$  and  $C_2$  depend only on the material combination. If the coefficients  $C_1$  and  $C_2$  can be known in advance, the stress intensity factor of the small interface crack can be evaluate easily without numerical analysis.
- The fracture criterion can be represented as  $K_{IC} = \text{constant}$  by using the small crack model and the experimental fracture loads when the combination of adhesive materials and the scarf angle are the same. By using the  $K_{IC}$  criterion, the fracture strength for any thickness of adhesive layer can be evaluated for the scarf joint.

Therefore, it is concluded that the use of the stress intensity factor of the small interface crack is effective for evaluating the bonding strength of the interface.

## References

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