

Plastic Deformations Occuring in Shells with Developable Middle Surfaces during Bending

M Rynkovskaya¹

1 Associate Professor, Peoples' Friendship University of Russia (RUDN University), Moscow, Russia

E-mail: rynkovskaya_mi@pfur.ru

Abstract. A huge number of investigations on developable surfaces have been made, but the problem of analysis of plastic deformation during bending was usually out of the path. It is widespread in practice to make these surfaces with the means of bending. In previous papers, there has been found no note of the appearance of plastic deformations in a plate which is being bent into a cylinder. That is why analytical investigation of bending of developable shells is considered to be of great value. In the paper, analytical formulas which give the opportunity to determine the bending moments appearing due to parabolic bending of an elastic slab, as well as formulas for limitations to the slope angle of rectilinear generator of evolvent helicoids during the process of parabolic bending of an annulus into the given helicoids, are presented. The author also derived the formulas for determination of the minimum radius of a cylinder made by bending of a plate without the emergence of plastic deformation in it.

1. Introduction

Developable surfaces are widely used in industrial design [1] and manufacturing [2], shipbuilding [3], engineering [4] and architecture [5], computer graphics for texture mapping, some kinds of surface modeling [6], computer animation, etc. According to [5], a developable surface can be flattened onto a plane without stretching, tearing or creasing, and tangent lines of the edge of regression (cuspidal edge) of this surface generate it.

Planes, cylindrical surfaces, conical surfaces, surfaces of tangents to any curve, surfaces composed of patches of previous types (transition surfaces) are developable surfaces. A few articles [7] can be found about oloid, a rather amazing developable surface which is used for example in the oloid agitator, a device which consists of two oloids on two axes, functions like a paddle and can successfully imitate natural water movements. Considerable amount of information about geometry and stress-strain analysis of thin elastic developable shells can be found in a monograph [8]. An analytical stress-strain analysis of developable helicoids is presented in works [9], [10]. Developable helicoid is a well-known developable surface.

As far as modern architecture is concerned, free form shapes are becoming increasingly popular, but the relationship between shape and fabrication poses new challenges and requires greater sophistication from the underlying geometry. Most materials which are used for dry building enclosures are supplied as sheet goods, making developable surfaces – surfaces foldable from a flat sheet – the geometry of choice [11]. Non-developable curved surfaces are made primarily by casting, stamping, or similar methods that need a die or mold, which lacks economies of scale if the individual components are different from each other. That is why a family of curved three-dimensional shapes that can be fabricated from two-dimensional sheet materials by way of bending or from curved creases is extremely desired either by industry design or by architecture.



Since developable surfaces are mostly used with materials that are not amenable to stretching, they may be of a benefit for manufacturing. In shipbuilding or automobile industry, surfaces are formed using only rollers or presses, and heat treatment is then used only to remove distortion induced by welding or other means. However, doubly curved surfaces must be heat treated after rolling to induce additional curvature. This process is very time-consuming and labour-intensive, because it is mostly done by hand and requires a skilled artisan with years of training to achieve the correct amount of bending [12]. Designing a ship hull or a vehicle entirely of singly curved, or developable, surfaces would reduce manufacturing costs.

Zhao and Wang [13] propose a new method for designing a developable surface by constructing a surface pencil passing through a given curve, which is quite in accord with the practice in industry design and manufacture. By representing the surface by a combination of the given curve, and the three vectors decomposed along the directions of Frenet trihedron frame, the authors derive the necessary and sufficient conditions for a surface to be developable.

In paper [14], the authors simplify the modelling process by introducing an intuitive sketch-based approach for modelling developables, and construct an algorithm that given an arbitrary, user specified 3D polyline boundary, constructed using a sketching interface, generates a smooth discrete developable surface that interpolates this boundary.

According to Williams and Skaggs [15] the process of bending metal to curved shapes by applying bending moments thereto is not new, and the use of heat to relieve stress in metals which have been formed by bending is a well-known, widely commercially used process. For example, each sheet can be formed to desired curvature by means of turnbuckles attached to rigid connecting brackets at the vertical edges of the sheets, thereby effectively producing a bending moment at the edge of the sheet. Despite the mention of providing the proper bending moment on the sheet metal, there is no analytical formula for its calculation.

It is shown in the paper [16] that multistage bending technique is an alternative way to produce cylindrical tubes. The authors use one pair of punch and die to experimentally investigate pure bending of a blank sheet into a cylindrical tube by multistage process, and numerical simulation was conducted on bending and spring back with LS-DYNA solver. The sequence is such that a blank sheet is bent into C-tube and then squeezed with welding into O-tube, while gap width and roundness of C-tube were the two dominant parameters to evaluate the bending performance. The method is considered to help reducing severe thinning and shape defects in the finished tubes.

Obviously, there are a lot of articles on application of developable surfaces in the fields of architecture, shipbuilding and design [17-21]. But the most of the papers are about geometry and modeling, some articles are about static strength analysis of shells with developable middle surfaces, very few works about dynamic analysis and almost all of the papers consider that the whole shell works in elastic stage. In general, a variety of investigations on developable surfaces has been made, [22-26] – the only few examples of the XXI century, but according to the investigation the problem of mathematical analysis of plastic deformation during bending is still avoided. Sometimes it is mentioned in articles, but most of the research relies on experimental data [16].

Folding elastically toward desired angels can cause plastic deformation near the folding line. The author has found out only a few works where plastic deformation is mentioned, furthermore, there has not been found any analytical methods for calculation or determination. At the same time some analytical issues have been developed at the Department of Strength of Materials and Structures in Peoples' Friendship University of Russia (Moscow, Russia). The paper written by Prof. S.N. Krivoschapko [27] was published in 2003, but it seems like it has not become widely known; in there the problem of determination of the shell stress-strain state in view of very large displacements u_u , u_s , u_z , but very small normal strains ε_u , ε_s and shear strains ε_{us} is investigated.

However, this issue cannot be ignored, because in most cases in industry design of ship hulls, automobile elements, building enclosures, etc., produced by bending of thin metal plates, developable surfaces are used. The most desired property of a developable surface for industry is that it can be obtained by bending from flat manufacture product. As far as the author's investigation is concerned,

it is extremely hard to find any useful practical or analytical methods on stress analysis of developable shells when plastic strain occurs.

Rather poor accuracy may develop in calculation of a cylindrical shell and obtaining strains from external loads in the case when the plastic stresses which arise in the process of manufacture of the product are not taken into account.

2. Materials and methods

The problem to discuss below mostly belongs to metal products with the yield point σ_y which are produced by bending.

Analytical formulas giving the opportunity to determine the bending and twisting moments emerging in the process of parabolic bending of an elastic slab [27]

$$M_1 = D(\kappa_1 + \nu\kappa_2); \quad (1)$$

$$M_2 = D(\kappa_2 + \nu\kappa_1); \quad (2)$$

$$H = M_{su} = 0, \quad (3)$$

where M_1, M_2 are bending moments; H, M_{su} are twisting moments; $D = Eh^3/[12(1-\nu^2)]$ is flexural rigidity of a shell; ν is a Poisson's ratio; h is shell thickness; κ_1, κ_2 are parameters of curvature variations,

can be obtained from formulas of Hooke's law for shells in lines of principal curvatures by substituting the last expressions in them:

$$\begin{aligned} \kappa_1 &= \frac{1}{R_1'} - \frac{1}{R_1} + \frac{\varepsilon_1}{R_1}; \\ \kappa_2 &= \frac{1}{R_2'} - \frac{1}{R_2} + \frac{\varepsilon_2}{R_2}, \end{aligned} \quad (4)$$

where R_1 and R_2 are the principal radii of curvatures of a surface before bending; R_1' and R_2' are the principal radii of curvatures of a surface after bending.

Since the Lamé coefficients of a developable surface under parabolic bending do not change, linear strains $\varepsilon_1, \varepsilon_2$ and angular displacement ε_{12} are equal to zero, as well as the force resultants.

Furthermore, it is predictable that $\frac{1}{R_1'} = \frac{1}{R_1} = 0$, because rectilinear generatrices of the developable

middle surface are considered to be unchanged when bending is parabolic. Besides, $\frac{1}{R_2} = 0$ if the

bending process begins from a plane slab. Thereby, when parabolic bending from a plane slab is assumed, Equations (1-3) become

$\kappa_1 = 0$; $\kappa_2 = \frac{1}{R_2'}$, and the formulas for bending and twisting moments (Equations (4)) will be:

$$\begin{aligned} M_1 &= D\nu\kappa_2 = \nu M_2 = M_u; \\ M_2 &= D\kappa_2 = M_s; \\ H &= M_{su} = 0. \end{aligned} \quad (5)$$

It is also possible to find formulas for moments M_u, M_s, M_{us} for curvilinear non-orthogonal coordinates u, s according to formulas of transition from internal moments in lines of principal curvatures:

$$\begin{aligned} M_u &= M_2 \cos^2 \chi + M_1 \sin^2 \chi; \\ M_s &= M_2; \\ M_{us} &= (M_1 - M_2) \sin \chi \cos \chi, \end{aligned} \quad (6)$$

where χ is an angle between two intersecting curvilinear coordinate lines u and s .

Let us illustrate the application of the formulas given above with an example.

3. Numerical experiments

A thin steel rectangular plate of dimensions $l \times b \times h$ with the following properties: Poisson's ratio is $\nu=0.3$; Young's modulus is $E=2.06 \cdot 10^5 \text{ MPa}$; thickness is $h=0.001 \text{ m}$; yield stress is $\sigma_y = 245 \text{ MPa}$; is bent into an L-shaped one (figure 1). Let us calculate the minimum value of the radius (R_{\min}) of bending when the plate will remain in the elastic stage (may turn back to its original flat state). In other words, the plate is bent without plastic deformations ($\sigma_{s,\max} \leq \sigma_y$).

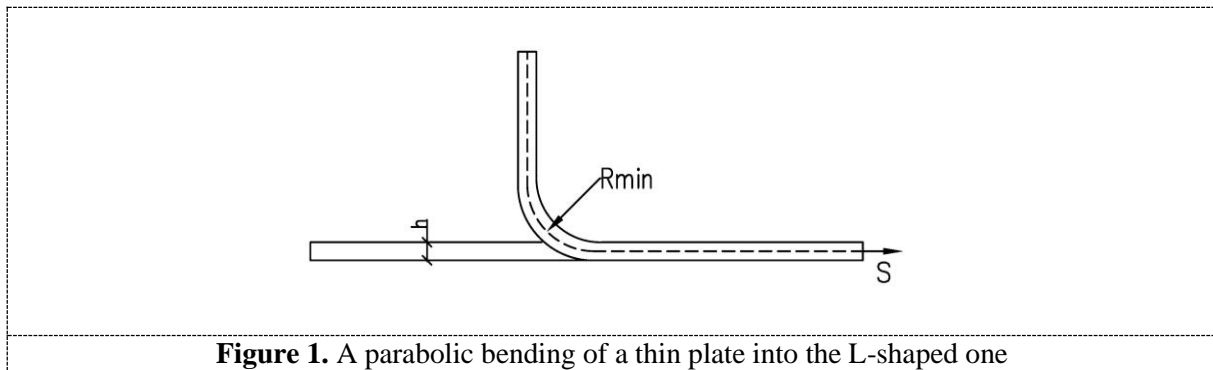


Figure 1. A parabolic bending of a thin plate into the L-shaped one

According to the Equations (1-4), the curvature variations and bending moments:

$$\kappa_1 = 0; \quad \kappa_2 = \frac{1}{R_{\min}}; \quad M_1 = \frac{D\nu}{R_{\min}} = M_u; \quad M_2 = \frac{D}{R_{\min}} = M_s. \quad (7)$$

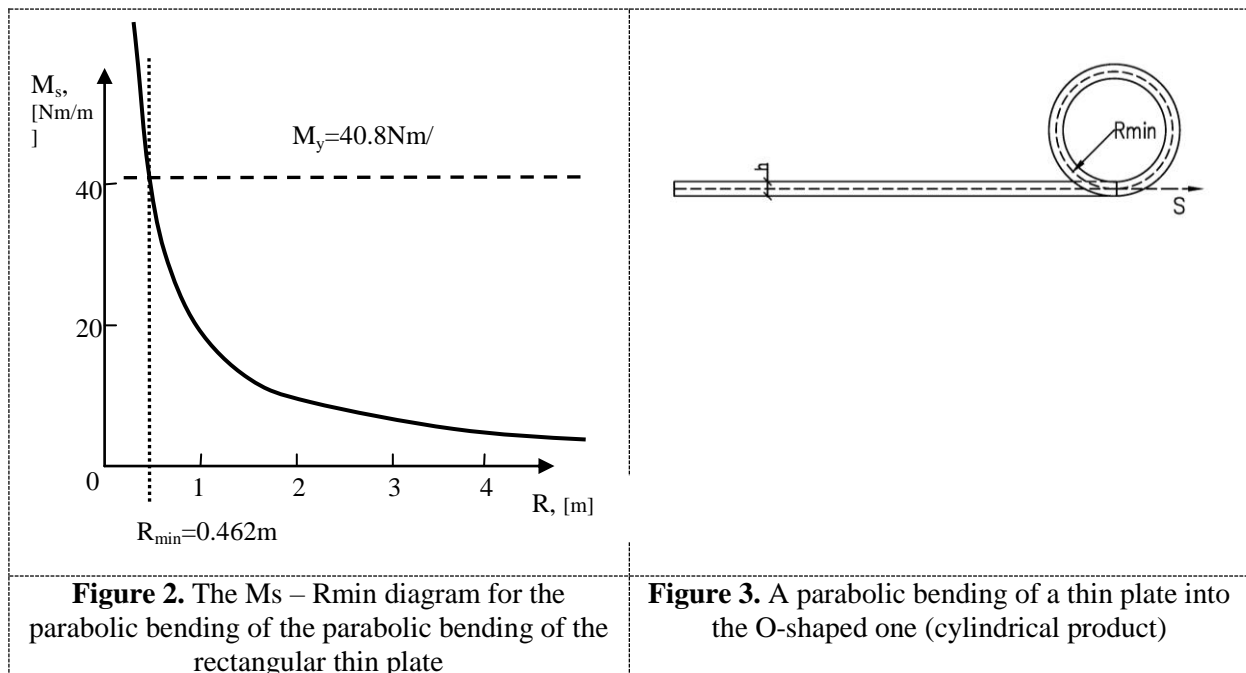
Thus, the minimum radius of the developable surface to be manufactured without plastic deformations can be calculated by the formula $R_{\min} = \frac{D}{M_y}$, where M_y is plastic moment.

The flexural rigidity of the shell is $D=18.86 \text{ N} \cdot \text{m}$, while the bending moment M_y , created by normal stresses, occurs when maximum normal stresses rise to the yield stresses ($\sigma_{s,\max} = \sigma_y$) is

$$M_y = \frac{\sigma_y h^2}{6} = 40.8 \text{ N} \cdot \text{m} / \text{m}.$$

Bending moments M_s and the radii correspondence is presented in figure 2. The point of intersection of the horizontal line $M_y = 40.8 \text{ N} \cdot \text{m} / \text{m}$ and the $M_s - R$ graph shows the minimal bending radius for the plate without plastic deformations ($R_{\min} = 0.462 \text{ m}$ in this example).

A similar calculation can be made for a radius of a cylinder (figure 3) or helicoids made by parabolic bending of a thin flat plate.



4. Results and Discussion

Generally, most engineers consider the task of bending of a thin plate as a task of bending of the middle surface, and do not take into account normal strains, because normal strains do not occur on the middle surface. In practice, since the factor of safety is usually very high for such products as steel tubes, such approach does not lead to any problems. However, it can be useful to consider plastic deformations when a plate is being bent into a product.

The main result of the paper is the formula for determination of the minimum radius which allows the plate to remain in elastic region. This formula is discussed in relation with the L-shaped plate, but can be applied to an O-shaped plate (cylindrical product).

The computer program based on the presented formulas was written with the tools of MathCAD for easy, rapid and accurate design of folded or unfolded surfaces. The similar approach was applied to a beam in [28], and numerical results obtained are satisfactory.

Besides, there are some other points for analytical investigation of bending of developable shells. One of them is a question about the range of the inclination angle that makes a thin helicoidal shell be under plastic stresses. The analytical formulas for limitations to the slope angle of rectilinear generator of evolvent helicoid during the process of parabolic bending of the annulus into the given helicoid may also be easily obtained. The author also gives an example for determination of the smallest allowable radius of L-shaped plate made by bending of a flat plate without the emergence of plastic deformation in it.

In the paper, the bending technology for manufacturing a product from a plate is not considered, as the issue does not relate to the topic of this study and can be selected as a subject for another article. It may be also interesting to get experimental data that will confirm the suggested theoretical results.

5. Conclusion

When calculating, the designer should be aware that the material of the product is already under tension which was caused in the manufacturing process; otherwise it may lead to the destruction of products in the process of loading and/or operation.

In the paper, the analytical formulas for bending moments appearing due to parabolic bending of an elastic slab made of metals with yield plateau are presented. The same formulas may be used for limitations to the slope angle of rectilinear generator of evolvent helicoids during the process of parabolic bending of an annulus into the given helicoid; and determination of the minimum radius of cylinder made by bending of a flat plate without the emergence of plastic deformation in it.

6. References

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