# Estimation of Stability Derivatives in Newtonian Limit for Oscillating Cone

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**Abstract**. Stability derivatives in Newtonian limit for an oscillating cone are obtained. Stiffness derivative decreases with pivot position for the entire range of cone angles. For cone angles in the range 20 to 30 degrees there is substantial increase in the stiffness derivative. Damping derivative decreases with pivot position for various cone angles and attains a minima at h = 0.75 and then again with increase in pivot position there is non-linear increment in damping derivatives. There is considerable change in the magnitude, for higher cone angles in the range of 20 degrees and above as the centre of pressure has further moved towards the trailing edge of the cone at pivot position around h = 0.88. Stiffness derivative increases with cone angle for various pivot positions. Damping derivative with cone angle for various fixed pivot positions is seen to increase linearly with cone angle, it is also observed that this trend of linear increment tend to become non-linear for cone angles in the range 20 degrees and beyond. Damping derivative decreases with pivot position h = 0.8 for the entire range of cone angles, however, for pivot position h = 1.0, shows almost constant values up to cone angle of 20 degrees. For cone angle 30 degrees value of damping derivative coincides irrespective of pivot position being at h = 0.8 or 1.0 and this trend is attributed to the 3-D effect of the flow field.
**Keywords:** Hypersonic flow, Newtonian limit, Oscillating cone, Mach number, Stability derivative.

## 1. Introduction

The present work evaluates stability derivatives in Newtonian limit for an oscillating cone in hypersonic flow. At hypersonic speeds the “nose cones” are often non-slender and blunt. The reason for such a configuration, is the problem of aerodynamic heating and hence the heat transfer at such speeds. Although the present work is not for blunt bodies with detached shocks, once a theory is developed for the ogives with sharp nose, it can then be extended to more practical shapes to incorporate the bluntness. It is really interesting to note that the study of hypersonic flows, which was restricted to slender bodies and low angles of attack, should attain a stage of non-slender shapes and at

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large angle of attack flows; promising an emphasized development of efficient future hypersonic systems.

Ghosh [1] developed a new hypersonic similitude with the assumptions of attached bow-shock and Mach number behind the shock being greater than 2.5. This similitude was valid for the windward surface of an aero-foil with large flow deflection. His work was extended to oscillating non-planar wedges by Crasta and Khan to calculate the aerodynamic pitching moment derivatives in both Supersonic [2] and Hypersonic flows [7, 8, 9, 10, 11 and 12].

The large deflection similitude of Ghosh [1] has been extended by Ghosh, K., [2] to axisymmetric bodies with attached shock. Equivalence with a new piston-motion with axial symmetry has been established. The cone (or quasi-cone) results from the revolution of a wedge (or quasi-wedge) round the streamwise axis and a similar revolution of the independent fluid slab [1] produces an axially symmetric conico-annular space. It is shown by Ghosh, K., [2] that the flow past a cone or quasi-cone is equivalent to a piston-motion in this conico-annular space, which is called the similitudinal slab. Although Ghosh, K., [2] gives similitude for cones and quasi-cones, he gives a solution based on the similitude for a cone only. This solution gives a constant density shock layer. Hence the constant density form of the unsteady Bernoulli's equation is used to find pressure on the cone surface in terms of the piston Mach number. Results are obtained for hypersonic flow for a perfect gas over oscillating cones and ogives of different Mach numbers and semi- angles.

## 2. Analysis

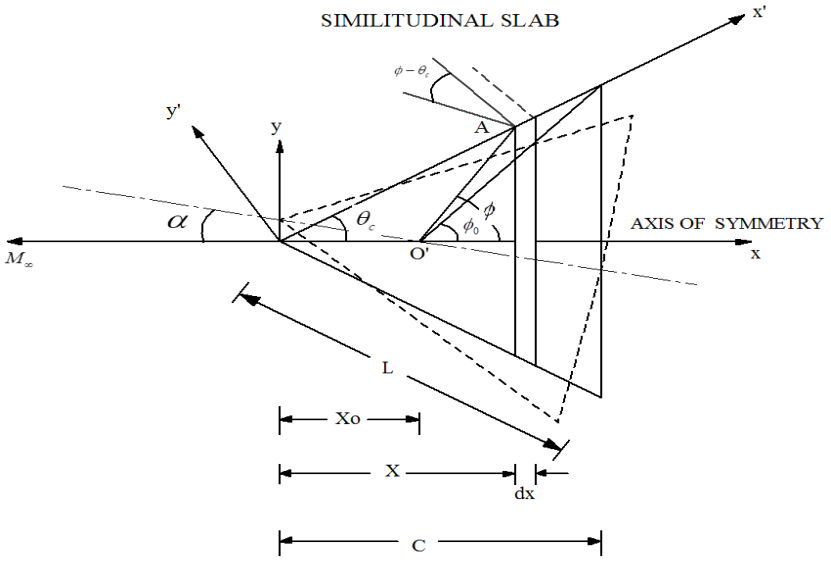



**Figure 1.** Cone Geometry

From the geometry we have

$$\tan\phi = \frac{x\tan\theta_c}{x-x_0};\ \tan\phi_0 = \frac{c\tan\theta}{c-x_0}$$

where $\phi$ is the angle subtended by A at O' with x-axis, and for different location of A, $\phi$ varies from $\pi$ to $\phi_c$, $\theta_c$ is the cone semi-angle and c is the chord length. The definition of the Stiffness coefficient denoted by $C_{m_\alpha}$, is

$$C_{m_\alpha} = \left[\frac{\partial M}{\partial \alpha}\right]_{\alpha,q\to 0} \frac{1}{\frac{1}{2}P_\infty U_\infty^2 S_b c}$$

where $S_b$ = base area of the cone = $\pi(c\tan\theta_c)^2$, $c$ = chord length of the cone. On solving we obtain the following,

The Stiffness derivative for an oscillating cone is given by

$$C_{m_\alpha} = D[h^3(1-2n^2-(1-h)\{H(2+h)+n^2h(1+2h)\})]$$

where

$$D = \frac{2}{3(1+n^2)}\left[1+\frac{1}{4}\left(\varepsilon+\frac{1}{2}K\frac{d\varepsilon}{dM_{po}}\right)\right]$$

$$D = \frac{2}{3(1+n^2)}\left[1+\frac{1}{4}\left\{\left(\frac{2+(\gamma-1)K^2}{2+(\gamma+1)K^2}\right)+\frac{1}{2}K\left(\frac{-8K}{[2+(\gamma+1)K^2]^2}\right)\right\}\right]$$

$$D = \frac{2}{3(1+n^2)}\left[1+\frac{1}{4}\left\{\left(\frac{2+(\gamma-1)M_\infty^2\sin^2\theta_c}{2+(\gamma+1)M_\infty^2\sin^2\theta_c}\right)+\frac{1}{2}M_\infty\sin\theta_c\left(\frac{-8M_\infty\sin\theta_c}{[2+(\gamma+1)M_\infty^2\sin^2\theta_c]^2}\right)\right\}\right]$$

$$D = \frac{2}{3(1+n^2)}\left[1+\frac{1}{4}\left\{\left(\frac{2+(\gamma-1)M_\infty^2\sin^2\theta_c}{2+(\gamma+1)M_\infty^2\sin^2\theta_c}\right)+\frac{8}{2}\left(\frac{M_\infty^2\sin^2\theta_c}{[2+(\gamma+1)M_\infty^2\sin^2\theta_c]^2}\right)\right\}\right]$$

In the Newtonian limit $M_\infty$ tends to infinity and $\gamma$ tends to unity,

$$D = \frac{2}{3(1+n^2)}\lim_{\substack{M_\infty\to\infty\\ \gamma\to1}}\left[1+\frac{1}{4}\left\{\left(\frac{2+(\gamma-1)M_\infty^2\sin^2\theta_c}{2+(\gamma+1)M_\infty^2\sin^2\theta_c}\right)+\frac{8}{2}\left(\frac{M_\infty^2\sin^2\theta_c}{[2+(\gamma+1)M_\infty^2\sin^2\theta_c]^2}\right)\right\}\right]$$

By applying limit

$$D = \frac{2}{3(1+n^2)}$$

∴ The Stiffness derivative in the Newtonian limit is given by,

$$C_{m_\alpha} = \frac{2}{3(1+n^2)}[h^3(1-2n^2)-(1-h)\{H(2+h)+n^2h(1+2h)\}]$$

∴ The Damping derivative for an oscillating cone is given by

$$C_{m_q} = \frac{D}{2}[h^4(2n^2-3n^4-1)-(1-h)\{H(3H+h(H+2n^2)+2h^2n^2)+n^4h^2(1+3h)\}]$$

where

$$D = \frac{2}{3(1+n^2)}\left[1+\frac{1}{4}\left(\varepsilon+\frac{1}{2}K\frac{d\varepsilon}{dM_{po}}\right)\right]$$

In the Newtonian limit $M_\infty$ tends to infinity and $\gamma$ tends to unity,

Following the same steps as in stiffness derivative we get

$$D = \frac{2}{3(1+n^2)}$$

∴ The Damping derivative in the Newtonian limit is given by,

$$C_{m_q} = \frac{D}{2}[h^4(2n^2 - 3n^4 - 1) - (1-h)\{H(3H + h(H + 2n^2) + 2h^2n^2) + n^4h^2(1 + 3h)\}]$$

Various results have been plotted and discussed.

## 3. Result and Discussion

Before analysing the results, we have to keep in mind that we are evaluating stability derivatives in the Newtonian limit. In the Newtonian limit the specific heat ratio γ of air, normally its value is 1.4 but in this case it will tend to one. Another variable is the Mach number which has a wide range but in view of the Newtonian limit it will tend to infinity. Which implies that the inertia of the flow will no more be a variable. This means that in the present case results will reflect the effect of the geometric parameters alone. Keeping these facts in mind we need to discuss the results.

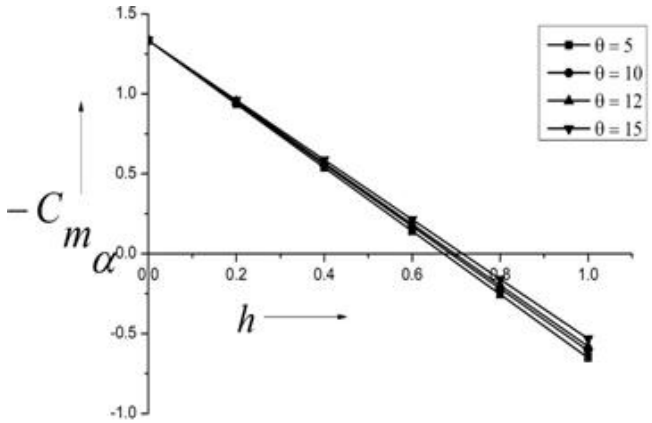



**Figure 2.** Stiffness derivative versus pivot position

Figure 2 shows the dependence of stiffness derivative with pivot position for various cone angles in the range from 5 to 15 degrees. Stiffness derivative decreases with pivot position for various cone angles and further it is noticed that up to 20% from the nose of the cone there is no variation between stiffness derivatives with various cone angles due to the negligible increase in the surface area of the cone and hence the surface pressure value. Slight variation appears when pivot position moves beyond 40% and substantial change at h = 1 which is 100% from the nose of the cone.

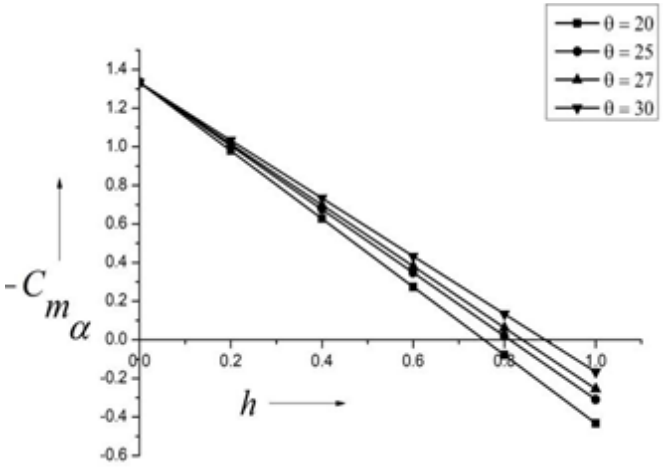



**Figure 3.** Stiffness derivative versus pivot position

Figure 3 shows the dependence of stiffness derivative with pivot position for various cone angles in the range from 20 to 30 degrees. Stiffness derivative decreases with pivot position for various cone angles as was seen in the previous case in figure 2, further it is seen that up till 10% from the nose of the cone there is no variation between stiffness derivative with cone angles for the entire range of the cone angle. As the cone angle increases the difference in stiffness derivative values for pivot position 20% onwards from the nose of the cone is clearly visible. This may be due to increase in the cone angle and hence the surface area and the resultant surface pressure for the range of the geometrical parameters of the present study. From the figure it is found that the centre of pressure is at h = 0.75, 0.82, 0.85, and 0.9. This shift in the centre of pressure due to the progressive increase in the cone angle will result in increased static margin as the centre of gravity remains fixed. However, we have to take final call on the maximum value of the static margin depending on the mission requirement and the type of application. For instance in case of fighter planes we are interested to have very low static margin as it will be helpful for Pilot in manoeuvring the fighter plane, whereas, in case of spinning shells there is a limiting value of the static margin for the operational requirement to enable the shell to avoid the base landing. For shells the fuzzes attached to the nose they function only due to the impact hence, base landing of the shells from the Guns must be avoided.

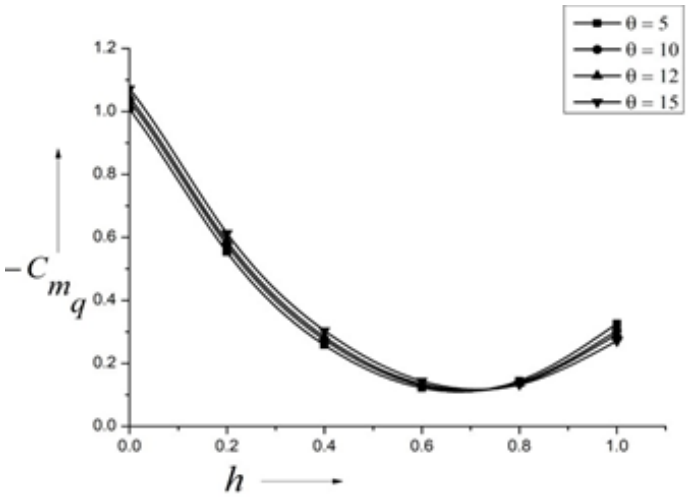



**Figure 4**. Damping derivative versus pivot position

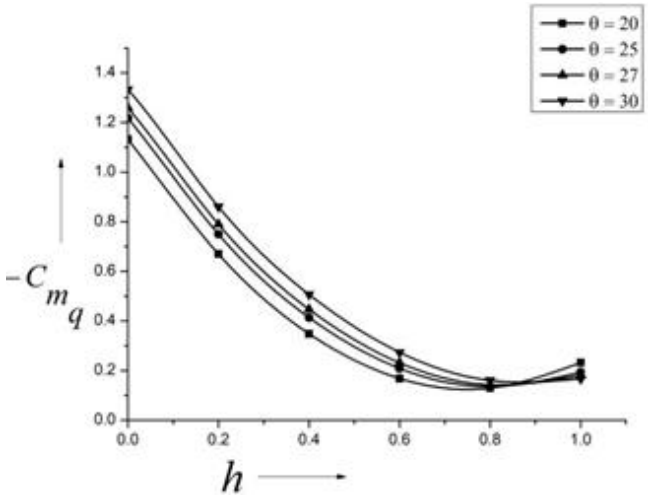



**Figure 5.** Damping derivative versus pivot position

Figure 4 and 5 show the dependence of damping derivative with pivot position for various cone angles. From the figure it is seen that damping derivative decreases with pivot position with cone angles attains a minima at h = 0.75 and then increases. There is no much variation in the values of damping derivative for cone angles from 5 to 15 degrees (Fig. 4). However, figure 5 shows considerable change in the magnitude, for higher cone angles the centre of pressure has further moved towards the trailing edge of the cone at around h = 0.88. As observed in case of stiffness derivative the trends are on the similar lines and the reasons for this trend has been discussed earlier and the same discussion holds here as well.

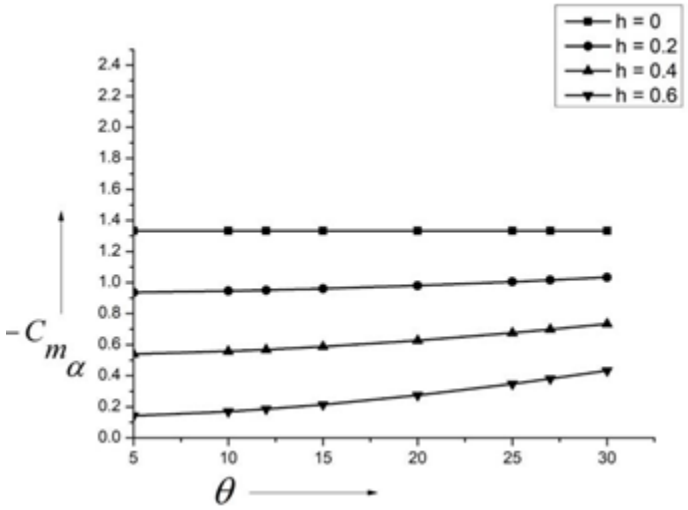



**Figure 6.** Stiffness derivative versus cone angle

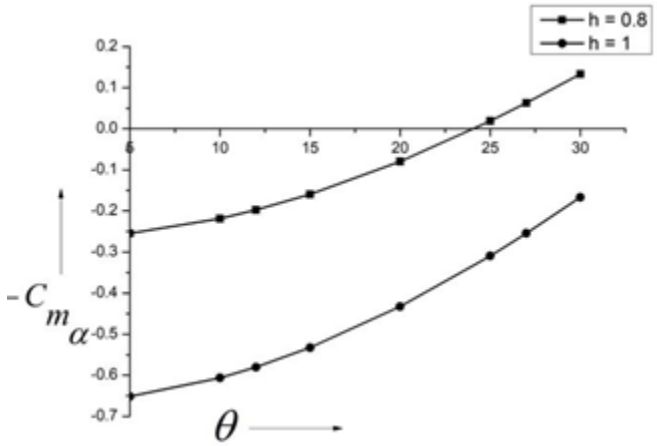



**Figure 7.** Stiffness derivative versus cone angle

Figs. 6 and 7 show the variation of stiffness derivative with cone angle for various pivot positions (i. e. h = 0, 0.2, 0.4, 0.6, 0.8, and 1.0). As expected the stiffness derivative decreases with pivot position for a given cone angle as discussed earlier (Figs. 2 and 3). Further, it is seen that with the increase in the cone angles the stiffness derivative increases linearly for entire range of cone angles and all the pivot positions except for h = 0.6, 0.8, and 1.0. It is also seen that the non-linearity has crept in for h = 0.6, 0.8 and 1.0. This non-linear trend is also seen for cone angle 20 degrees and above for lower pivot positions namely 0.2 and above. This trend could be attributed to the pressure distribution on the surface of the cone.

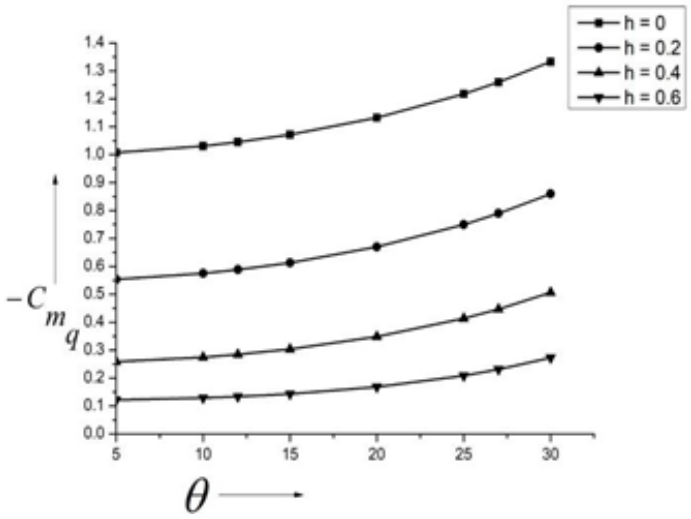



**Figure 8.** Damping derivative versus cone angle

Figure 8 shows the dependence of damping derivative with cone angle for various fixed pivot positions. Damping derivative increases linearly with cone angle, it is also observed that this trend of linear increment tend to become non-linear for cone angles in the range 20 degrees and beyond.

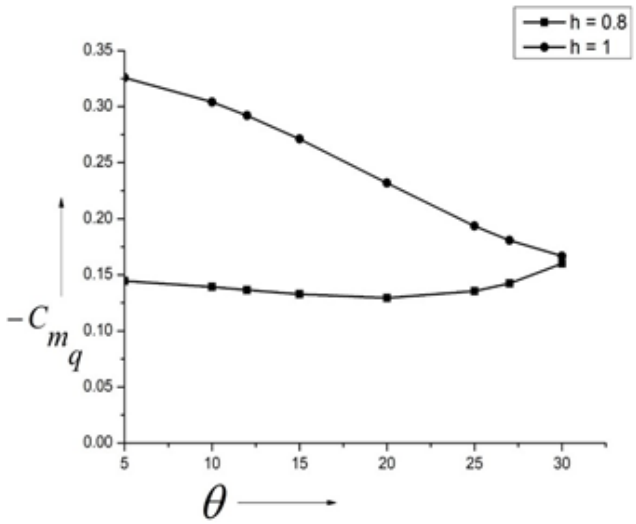



**Figure 9.** Damping derivative versus cone angle

Fig. 9 shows the variation of damping derivative with cone angle for pivot position h = 0.8 and 1.0. Damping derivative decreases with pivot position h = 0.8 for the entire range of cone angles. When we consider the results for pivot position h = 1.0, shows almost constant values up to cone angle of 20 degrees. For cone angle more than 20 degrees the damping derivative starts increasing. It is observed that at cone angle of 30 degrees value of damping derivative coincides irrespective of pivot position being at h = 0.8 or 1.0. This may be due to the 3-D effect of the flow field, cone surface pressure and the effect of body of revolution.

## 4. Conclusion

Based on the above discussion we can draw the following conclusions:

- Stiffness derivative decreases with pivot position for the entire range of cone angles. For lower range of pivot positions and cone angle up to 15 degrees this increment is not seen.

- For cone angles in the range 20 to 30 degrees there is substantial increase in the stiffness derivative and the trend of decrement continues.
- Damping derivative decreases with pivot position for various cone angles and attains a minima at h = 0.75 and then again with increase in pivot position there is non-linear increment in damping derivatives. There is considerable change in the magnitude, for higher cone angles in the range of 20 degrees and above as the centre of pressure has further moved towards the trailing edge of the cone at pivot position around h = 0.88.
- Stiffness derivative increases with cone angle for various pivot positions. Further, it is seen that with the increase in the cone angles the stiffness derivative increases linearly for entire range of cone angles. Non-linearity is observed for h = 0.6, 0.8 and 1.0.
- Damping derivative with cone angle for various fixed pivot positions is seen to increase linearly with cone angle, it is also observed that this trend of linear increment tend to become non-linear for cone angles in the range 20 degrees and beyond.
- Damping derivative decreases with pivot position h = 0.8 for the entire range of cone angles, however, for pivot position h = 1.0, shows almost constant values up to cone angle of 20 degrees. For cone angle 30 degrees value of damping derivative coincides irrespective of pivot position being at h = 0.8 or 1.0 and this trend is attributed to the 3-D effect of the flow field.

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