

Magnetic attitude control torque generation of a gravity gradient stabilized satellite

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Abstract. Magnetic torquer is used to generate a magnetic dipole moment onboard satellites whereby a control torque for attitude control purposes is generated when it couples with the geomagnetic field. This technique has been considered very attractive for satellites operated in Low Earth Orbit (LEO) as the strength of the geomagnetic field is relatively high below the altitude of 1000 km. This paper presents the algorithm used to generate required magnetic dipole moment by 3 magnetic torquers mounted onboard a gravity gradient stabilized satellite operated at an altitude of 540 km with nadir pointing mission. As the geomagnetic field cannot be altered and its magnitude and direction vary with respect to the orbit altitude and inclination, a comparison study of attitude control torque generation performance with various orbit inclination is performed where the structured control algorithm is simulated for 13°, 33° and 53° orbit inclinations to see how the variation of the satellite orbit affects the satellite's attitude control torque generation. Results from simulation show that the higher orbit inclination generates optimum magnetic attitude control torque for accurate nadir pointing mission.

1. Introduction

One of the most significant disturbance torques for small satellites operated in Low Earth Orbit (LEO) is the gravity gradient torque. It exists from the variation of the Earth's gravitational force over the asymmetric body of the satellite where the Earth's gravitational force pulls the satellite axis with a least moment of inertia pointing towards the Earth with a relative angular velocity of the axis points to the Earth equal to the mean motion of the orbit. Therefore, this property can be used to passively stabilize the satellite with nadir pointing mission by extending the axis that is required to point to the Earth with a flexible mechanical device called a gravity gradient boom to significantly reduce its moment of inertia. Stability analysis of a gravity gradient stabilized satellite made by [1] shows that the stable condition is achieved with the satellite having moment of inertia values as $I_y > I_x > I_z$ after the deployment of the gravity gradient boom. However, this stabilization technique only allows 2-axis control, which is along the roll and pitch axis with an average attitude accuracies thus leaving the satellite in a libration motion while orbiting. Therefore, having an additional actuator onboard the gravity gradient stabilized satellite is necessary to provide 3-axis control and significantly improve its attitude performance. The actuator should be lightweight, low power consumption and inexpensive for low-cost missions. In this regards, a magnetic torquer is the primary option.



Magnetic torquer is a magnetic induction device consists of a coil and a core where the magnetic dipole moment is generated when an electrical current passing through the coil. It is used to generate a controllable value of magnetic dipole moment needed to be generated onboard the satellite in order to generate the required control torque when couples with the geomagnetic field. This technique has been considered very attractive for satellites operated in LEO as the strength of the geomagnetic field is relatively high below the altitude of 1000 km. Through the literature survey, several control strategies and approaches of the magnetic control technique implemented on the gravity gradient stabilized satellites have been reviewed. It can be summarized here that there is only a work found implemented a single magnetic torquer while the rest implemented three magnetic torquers. The work proposed a magnetic libration control scheme to eliminate the libration motion which likely to occur due to the external disturbance torques acting on the satellite [2]. The magnetic torquer was placed along the z axis where the controller was developed based on the energy dissipation principle. Numerical simulations were performed for a circular polar orbit. Results proved the effectiveness of the developed scheme by improving the yaw attitude oscillation between 0° and 2° .

As for the three magnetic torquers, [3] found that the B-dot control law was not effective in providing 3-axis stabilization during nominal attitude pointing phase. It was due to the non-uniformity of the geomagnetic field and the effect of the orbital rates. Therefore, they proposed to use the magnetic torquer cross-product control law for the attitude control where the control error was developed based on the PD controller. It was found from simulations performed for three different inclinations that the control along the yaw axis was achieved and consequently granting 3-axis stabilization. However, as stated by [4], this control algorithm imposes some control constraints which are the unavailability of the control torque during certain orbital regions and the generation of the unwanted cross control torques between the attitude axes. This is due to its tendency to generate the most favorable magnetic torque by interleaving or simultaneously switching any of the three magnetic torquers that are normal to the geomagnetic field direction at any control instant. Therefore, Steyn proposed a controller based on the fuzzy control theory where the developed controller chooses the magnetic torquer to generate control torques at any control instant. This controller was implemented for libration damping purposes onboard SUNSAT, a gravity gradient stabilized microsatellite developed by graduate students at Stellenbosch University with its primary mission objectives were for imaging, communication and data interchange [5]. Three reaction wheels were also mounted along this satellite's body axis to perform maneuvers for the image pointing and stabilization during imaging. The magnetic torquers were also used for momentum dumping of the reaction wheels.

On the other hand, [6] concern was on the incapability of the magnetic toquer to generate exactly the required action due to the orthogonality rule. Therefore, in their work, two approaches were proposed for generating the actual torque out of the ideal control torque: i) based on the torque closest to the ideally sought and ii) based on the torques with two components equal to the ideal. The ideal control torque law was developed based on the reference modern control theory. According to the proposed methods of generating the actual torque, three different algorithms were formulated. These three formulated algorithms were then compared with the algorithm developed by [3]. It was found that one of the formulated algorithms, which is the one with constant gains and alternate switching for roll and yaw, gives better performances with respect to the others. [7] also carried out a similar research. In their work, two ideal control torques were developed based on the nonlinear control theory. One was based on the energy method and the other one was based on the variable control structure method. These developed algorithms were compared with the algorithm of [6]. Simulation results show that both the energy method and the variable control structure method reached their steady state value faster than that of [6].

On the contrary, [8] proposed a fully autonomous magnetic attitude control system for a small satellite with a remote sensing application operated on a sun-synchronous orbit with an inclination of

97.2° at an altitude of 433 km. The control laws were derived separately for the attitude acquisition and station keeping phases. The former was developed using the B-dot control law while for the latter, the algorithm developed by Martel et al. was implemented. This mission was numerically tested and results show that it fulfills the satellite mission requirement. In other publication, [9] reported the performance of the ACS system of a multi-mission microsatellite developed by a group of Italian universities called UNISAT which employs the same control scheme. In a different way, [10] developed ACS system for a micro-satellite called LEAP. The controller was developed using the well-known B-dot control law for both detumbling and normal modes which managed to detumble the satellite before the gravity boom deployment as well as maintained the satellite attitude accuracy after the deployment. [11] proposed a velocity controller using a cross product between the angular velocity and the geomagnetic field whereby series of control algorithms were developed based on the PD control for recovery from the tumbling motion, nominal operation and recovery from a gravity gradient boom upside-down condition. Numerical simulations were performed for a satellite on a near polar orbit and results showed that global 3-axis stabilization was achieved with a promising attitude performance for all the three axes. These control algorithms were implemented onboard the Danish Orsted satellite and this mission was the first 3-axis stabilized satellite flying with magnetic torquers as the sole actuators onboard.

Another research implementing three magnetic torquers onboard was carried out based on the extension of the H_∞ control technique for controlling the linear time variant system where the periodic controllers for the periodic system can be designed. In this work, [12] have developed the discrete time state-space description for the linear time variant system where its transfer function has many properties analogous to the transfer functions for the linear time invariant system. Thus, allowing the formulation of control law for this system implementing the traditional H_∞ control.

Review of various control laws used for magnetic torquer mentioned here give us options to implement magnetic attitude control technique using three magnetic torquers. All the algorithms developed performed well and fulfill their respective requirements. However, the numerical treatments performed are only for specific orbit inclination. Therefore, in this paper, a comparison study of attitude control torque generation performance with various orbit inclination is performed where the structured control algorithm of a gravity gradient stabilized satellite equipped with three magnetic torquers is simulated for 13°, 33° and 53° to see how the variation of the satellite orbit along these orbits affects the satellite's attitude control torque generation as the geomagnetic field cannot be altered and its magnitude and direction vary with respect to the orbit altitude and inclination. The algorithm used is based on the Proportional-Derivative (PD) controller.

2. Satellite configuration

The configuration of the satellite is as depicted in Figure 1. It is equipped with 3 magnetic torquers that are placed along the +x, +y and +z axes and a gravity gradient boom along the -z axis. Each of these magnetic torquers controls any two axes that are normal to them. Note that the z axis points toward the Earth, the y axis is normal to the orbital plane and the x axis points toward the satellite's orbital motion and complete the right-hand orthogonal system [13].

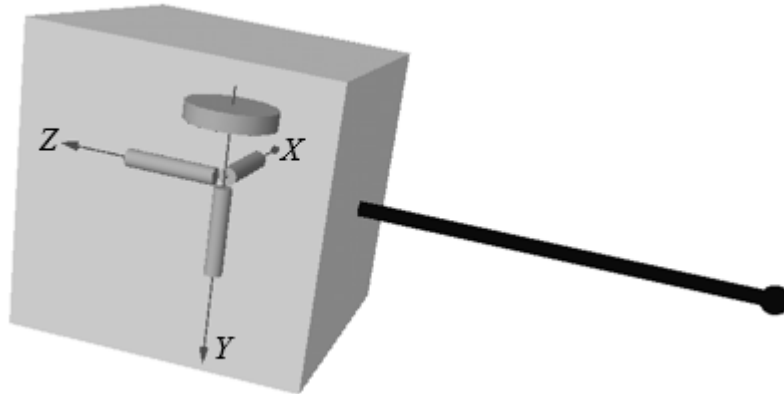


Figure 1. Satellite Configuration.

Based on this configuration, the linearized dynamic equation of motion of the satellite is described in a state-space equation where along the roll/yaw axis the equation becomes

$$\begin{bmatrix} \dot{\phi} \\ \dot{\psi} \\ \ddot{\phi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4\omega_0^2\sigma_x & 0 & 0 & \omega_0(1-\sigma_x) \\ 0 & -\omega_0^2\sigma_z & -\omega_0(1-\sigma_z) & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \psi \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/I_x & 0 \\ 0 & 1/I_z \end{bmatrix} \begin{bmatrix} T_x \\ T_z \end{bmatrix} \quad (1)$$

while along the pitch axis is

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3\omega_0^2\sigma_y & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I_y \end{bmatrix} T_y \quad (2)$$

The external torque T_x , T_y and T_z in equation (1) and equation (2) consists of two essential parts which are the disturbance torque and the control torque. Their relation can be defined as

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} T_{dx} + T_{mx} \\ T_{dy} + T_{my} \\ T_{dz} + T_{mz} \end{bmatrix} \quad (3)$$

where the disturbance torque consists of the gravity gradient torque, aerodynamic torque, magnetic torque and solar radiation torque, while the control torque is generated by the magnetic torquers.

3. Control strategy

The magnetic control torque is produced when the magnetic dipole moment generated by magnetic torquer interacts with the geomagnetic field. This interaction can be mathematically described as follow

$$\mathbf{T}_m = \mathbf{M} \times \mathbf{B} \quad (4)$$

where $\mathbf{T}_m$ is the magnetic torque vector, $\mathbf{M}$ is the satellite's magnetic dipole moment vector and $\mathbf{B}$ is the geomagnetic field vector. This equation clearly explains the generated magnetic control torque direction is normal to both the geomagnetic field vector and the satellite's magnetic dipole moment. Since the control algorithm is for a nominal attitude pointing phase, the magnetic torquer cross-product control law is used and written here as [14]

$$\mathbf{M} = \frac{\mathbf{T}_c \times \mathbf{B}}{|\mathbf{B}|} \quad (5)$$

The PD controller is adopted for controlling this satellite where $\mathbf{T}_c$ in equation (5) is defined as [3]

$$\begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix} = \begin{bmatrix} k_{px}\phi + k_{dx}\dot{\phi} \\ k_{py}\theta + k_{dy}\dot{\theta} \\ k_{pz}\psi + k_{dz}\dot{\psi} \end{bmatrix} \quad (6)$$

Therefore, the total generated magnetic control torque along each axis is defined by inserting equation (5) into equation (4), which yielding the following equation

$$T_{mx} = \frac{-(B_y^2 + B_z^2)}{B^2} \cdot [k_{px}\phi + k_{dx}\dot{\phi}] + \frac{B_x B_y}{B^2} \cdot [k_{py}\theta + k_{dy}\dot{\theta}] + \frac{B_x B_z}{B^2} \cdot [k_{pz}\psi + k_{dz}\dot{\psi}] \quad (7)$$

$$T_{my} = \frac{B_x B_y}{B^2} \cdot [k_{px}\phi + k_{dx}\dot{\phi}] - \frac{(B_x^2 + B_z^2)}{B^2} \cdot [k_{py}\theta + k_{dy}\dot{\theta}] + \frac{B_y B_z}{B^2} \cdot [k_{pz}\psi + k_{dz}\dot{\psi}] \quad (8)$$

$$T_{mz} = \frac{B_x B_z}{B^2} \cdot [k_{px}\phi + k_{dx}\dot{\phi}] + \frac{B_y B_z}{B^2} \cdot [k_{py}\theta + k_{dy}\dot{\theta}] - \frac{(B_x^2 + B_y^2)}{B^2} \cdot [k_{pz}\psi + k_{dz}\dot{\psi}] \quad (9)$$

The values of the geomagnetic field magnitude along the orbit can be defined by a series of spherical harmonics as follow [15],

$$(r, \theta, \lambda) = R \sum_{n=1}^k \left(\frac{R}{r}\right)^{n+1} \sum_{m=0}^n (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m(\theta) \quad (10)$$

where R is the equatorial radius of the earth, (r, θ, λ) are the geocentric distance, co-latitude and east longitude from Greenwich, (g_n^m, h_n^m) are the Schmidt normalized Gaussian coefficients and $P_n^m(\theta)$ is the associated Legendre functions. The general solution of this function expressed in spherical coordinate system can be written as follows

$$B_r = \sum_{n=1}^k \left(\frac{R}{r}\right)^{n+2} (n+1) \sum_{m=0}^n (g^{n,m} \cos m\lambda + h^{n,m} \sin m\lambda) P^{n,m}(\theta) \quad (11)$$

$$B_\theta = - \sum_{n=1}^k \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^n (g^{n,m} \cos m\lambda + h^{n,m} \sin m\lambda) \frac{\partial P^{n,m}(\theta)}{\partial \theta} \quad (12)$$

$$B_\lambda = \frac{-1}{\sin \theta} \sum_{n=1}^k \left(\frac{R}{r}\right)^{n+2} \sum_{m=0}^n m (-g^{n,m} \sin m\lambda + h^{n,m} \cos m\lambda) P^{n,m}(\theta) \quad (13)$$

4. Numerical simulation

Series of simulations have been performed using the MATLAB® SIMULINK® software. The components of the geomagnetic field intensity vector expressed in Local Vertical Local Horizontal (LVLH) coordinate system along the 13°, 33° and 53° orbit inclinations with respect to the sampling orbital parameters described in Table 1 have been generated and plotted in **Error! Reference source not found.**

Table 1. Orbital Parameters.

Parameters	Value
Altitude, h	540 km
Eccentricity, e	≈ 0
Right ascension of ascending node, Ω	0°
Argument of perigee, ω	0°
True anomaly, v	0°
Simulation time, t	20 orbits
Orbital frequency, ω_0	0.0011 rads^{-1}

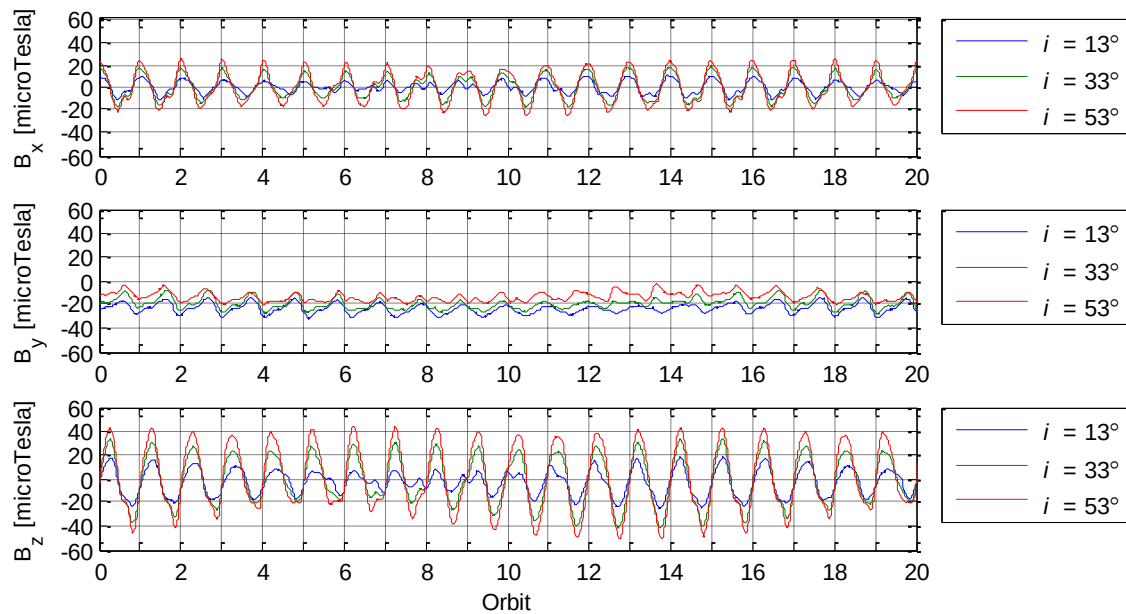


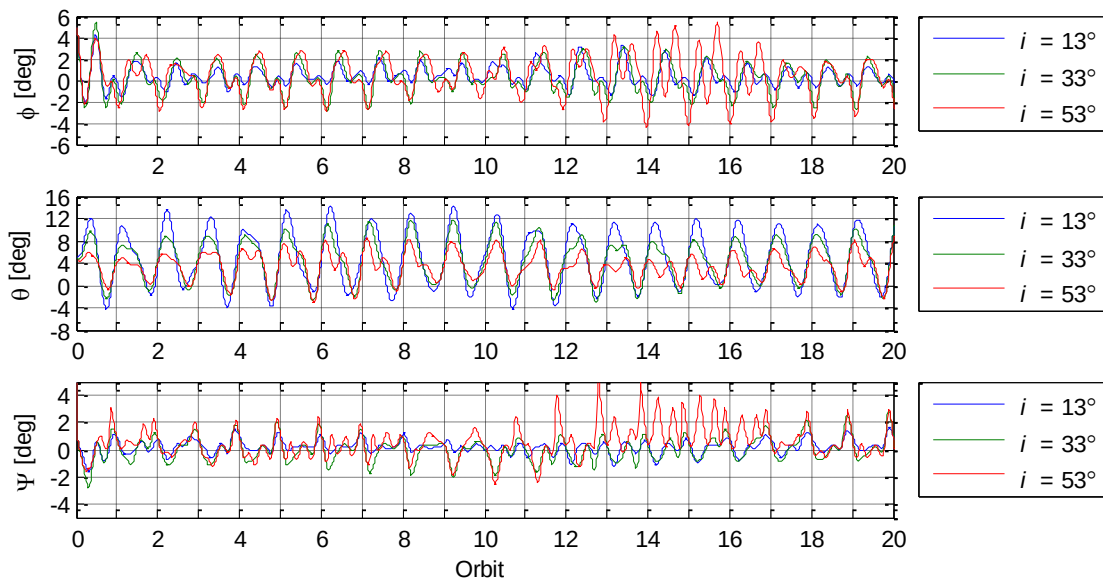
Figure 2. Comparison of the geomagnetic field models along 13°, 33° and 53° orbit inclinations.

The B_x and B_z components vary with respect to the orbital motion as these two components are measured along the $+x$ and $+z$ axis of the LVLH coordinate system which are lying in the orbital plane. Whereas the B_y component is measured along the $+y$ axis which is normal to the orbital plane and only influenced by the rotation of the earth. It is also shown that the magnitude of the geomagnetic field is stronger over the pole and weaker over the equatorial. The tuning of the control gains is based on the geomagnetic field values along the 53° orbit inclination and the same gains are later used to simulate the performance along the 13° and 33° to see the effect of varying the orbit inclination. Details of the satellite parameters and orbital parameters are given in Table 2.

Table 2. Satellite parameters

Parameter	Value
Weight	50 kg
Dimension	$690 \times 366 \times 366$ mm
I_x	178 kgm ²
I_y	181 kgm ²
I_z	4.3 kgm ²
$M_{\text{magnetic torquer}}$	15 Am ²
Input	$T_{dx} = 12.8 \times 10^{-6} + 8.6 \times 10^{-6} \sin(\omega_0 t)$ Nm $T_{dy} = 55 \times 10^{-6} + 55 \times 10^{-6} \sin(\omega_0 t)$ Nm $T_{dz} = 12.8 \times 10^{-6} + 4.3 \times 10^{-6} \sin(\omega_0 t)$ Nm
Initial conditions	$\phi(0) = 5^\circ, \theta(0) = 5^\circ, \psi(0) = 5^\circ$ $\omega_x(0) = 0, \omega_z(0) = 0$
Gains	$k_{px} = 10, k_{py} = 20, k_{pz} = 100,$ $k_{dx} = 1000, k_{dy} = 1000, k_{dz} = 1000$

The attitude performances of the satellite are depicted in figure 3. The plots show that the roll axis oscillates with an accuracy between -1.5° and 3.4° for 13° orbit inclination, between -2.9° and 3.2° for 33° orbit inclination and between -4.6° and 5.5° for 53° orbit inclination. Meanwhile, the pitch axis oscillates with an accuracy between -4° and 14.5° for 13° orbit inclination, between -3.1° and 12° for 33° orbit inclination and between -3.1° and 8° for 53° orbit inclination. As for the yaw axis, it oscillates with the accuracy between -1.2° and 1.5° for 13° orbit inclination, between -2° and 3° for 33° orbit inclination and between -2.6° and 6° for 53° orbit inclination. It can be clearly seen that the attitude performance of the satellite along three axes improves as the orbit increases. This is due to the geomagnetic field value that is stronger over the pole and weaker over the equatorial.

**Figure 3.** Comparison of the attitude performance.

As for the orbit around the equator, the geomagnetic field is nearly parallel to the pitch axis of the satellite where one of the magnetic torquer is mounted. Thus, the magnitude of B_y is high compared to B_x and B_z . Since the magnetic control torque can only be generated perpendicular to the geomagnetic field and magnetic dipole moment of the magnetic torque, each magnetic torquer controls any two axes that are normal to it. This situation reflects the magnetic dipole moments generated by each magnetic torquers onboard as shown in figure 4 and the torque generated along the roll, pitch and yaw axes as shown in figure 5.

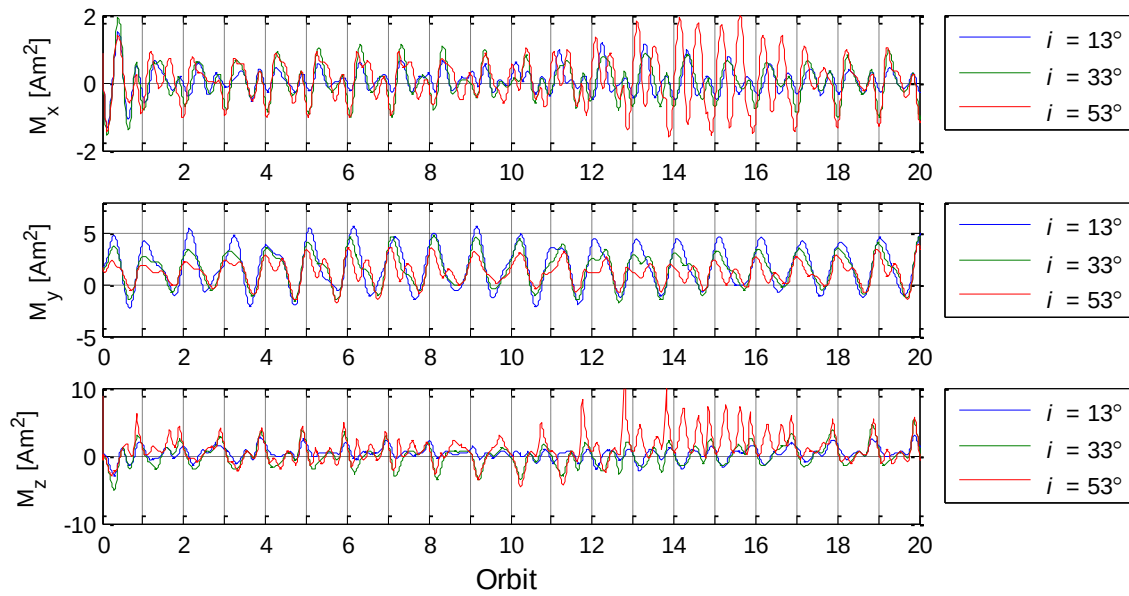


Figure 4. Comparison of the magnetic dipole moment generation.

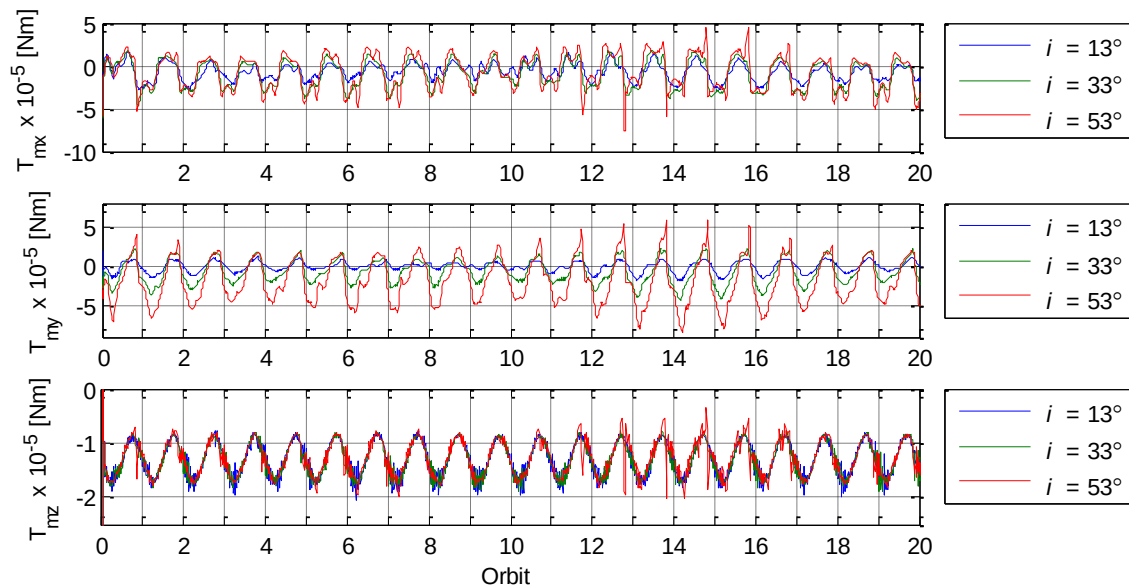


Figure 5. Comparison of the magnetic control torque generation.

5. Conclusion

In this paper, the control torque generation of a gravity gradient stabilized satellite employing 3 magnetic torquers onboard has been simulated for 13°, 33° and 53° orbit inclinations to see how the orbit inclination variation affects the magnetic control torque generation. The control algorithm was structured based on the PD type controller. Simulations were performed using the MATLAB® SIMULINK® codes. Results from the simulations show that the higher orbit inclination generates optimum magnetic attitude control torque for this satellite configuration.

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References

- [1] Kaplan M H 1976 *Modern Spacecraft Dynamics and Control* (New York: John Wiley & Sons)
- [2] Maeda K, Hidaka T and Uo M 1997 A magnetic libration control scheme for gravity gradient stabilized small satellite *Adv. in the Astro. Sci.*, 96 (6) pp 887 – 896
- [3] Martel, F, Pal P K and Psiaki M 1988 Active magnetic control system for gravity gradient stabilized spacecraft *Proc. of the 2nd AIAA/USU Conf. on Small Satellites* (Utah State University)
- [4] Steyn W H 1994 Comparison of low-earth-orbit satellite attitude controllers submitted to controllability constraints *J. Guidance, Control and Dynamics* 17 (4) pp 795–804
- [5] Mostert S and Koekemoer J 1997 The science and engineering payloads and experiments on SUNSAT *Acta Astro.* 41 (4-10) pp 401–411
- [6] Arduini C and Baiocco P 1997 Active magnetic damping attitude control for gravity gradient stabilized spacecraft *J. Guidance, and Control* 20 (10) pp 117–122
- [7] Zhaowei S, Xu Yang and Di Y 2003 Active magnetic control methods for small satellite *J. Aero. Eng.* 16 (1) pp 38–44
- [8] Grassi M 1997a Attitude determination and control for a small remote sensing satellite *Acta Astro.* 40 (9) pp 675–681
- [9] Grassi M 1997b Performance evaluation of the UNISAT attitude control system *J. Astro. Sci.* 45 p 146
- [10] Wang K N, Juang J C and Tsai Y F 2009 Design of the LEAP attitude control system *Proc. of the 7th Asian Control Conf.* (Hong Kong, China)
- [11] Wisnieski R and Blanke M 1999 Fully magnetic attitude control for spacecraft subject to gravity gradient *Automatica* 35 pp 1201–1214
- [12] Kulkarni J and Campbell M 2004 An approach to magnetic torque attitude control of satellites via ' H_∞ ' control for LTV systems 43rd IEEE Conf. on Decision and Control 1 pp 273–277
- [13] Suhadis N M and Varatharajoo R 2012 passive attitude control torque generation performances of a gravity gradient stabilized satellite *App. Mech. and Materials*, 225 pp 458–463
- [14] Stickler A C and Alfriend K T 1974 An elementary magnetic attitude control system *AIAA Mech. and Control of Flights Conf.* (Anaheim California)
- [15] McLean S, Mcmillan S, Maus S, Lesur V Thompson A and Dater D 2004 The US/UK World Magnetic Model for 2005-2010 NOAA Technical Report NESDIS/NGDC-1