

Computation of Stability Derivatives of an oscillating cone for specific heat ratio = 1.66

Aysha Shabana^{1*}, Renita Sharon Monis², Asha Crasta³, S. A. Khan⁴

¹ Research Scholar, Department of Mathematics, M.I.T.E, Moodabidri and Assistant Professor, Sahyadri College of Engineering and Management, Mangaluru, Karnataka, India.

² Research Scholar, Department of Mathematics, M.I.T.E, Moodabidri and Assistant Professor (Sr), SMVITM, Bantakal, Karnataka, India.

³ Associate Professor, Department of Mathematics, M.I.T.E, Moodabidri, Karnataka, India.

⁴ Professor, Department of Mechanical Engineering, Faculty of Engineering, IIUM, Gombak Campus, Kuala Lumpur, Malaysia.

*E-mail: aysha.nawshad@gmail.com

Abstract. In this paper the expressions for stiffness and Damping derivatives are obtained in a closed form for perfect gas where the flow is quasi-steady and axi-axisymmetric, and the nose semi angle of the cone is such that the Mach number M_2 behind the shock $M_2 \geq 2.5$. Results are presented for an oscillating cone for gas with $\gamma = 1.666$, at different Mach numbers and semi cone angles. The Stiffness derivative decreases with pivot position and also with semi vertex angle, there is substantial change in the stiffness derivative when semi-vertex has been increased from 5 degrees to ten degrees, further increase in the semi-vertex angle results in marginal change in the stiffness derivative. Due the marginal change in the Mach number level there is marginal increase in the magnitude of the stability and with further increase in the inertia level the stability derivative conform to the Mach number independence principle. The present theory for Oscillating cone is restricted to quasi-steady case. Viscous effects have been neglected. The expressions so obtained for stability derivative in pitch are valid for a slender ogive which often approximates to the whole fuselage of an aircraft. **Keywords:** High Speed Flow, Hypersonic Flow, Oscillating cone, Stiffness derivative

1. Introduction

The present work estimates stability derivatives in pitch for non-slender axi-symmetric ogives oscillating in hypersonic flow. At hypersonic speeds the “nose cones” are often non-slender and blunt. The reason for such a configuration, is the problem of aerodynamic heating and hence the heat transfer at such speeds. Although the present work is not for blunt bodies with detached shocks, once a theory is developed for the ogives with sharp nose, it can then be extended to more practical shapes to incorporate the bluntness.

It is fascinating to note that the study of hypersonic flows, which was limited to slender bodies and low angles of attack, should attain a stage of non-slender shapes and at large angle of attack flows; promising an emphasized development of efficient future hypersonic systems.



Ghosh [1] developed a new hypersonic similitude with the assumptions of attached bow- shock and Mach number behind the shock being greater than 2.5. This similitude was valid for the windward surface of an aero-foil with large flow deflection. His work was extended to oscillating non-planar wedges by Crasta and Khan to calculate the aerodynamic pitching moment derivatives in both Supersonic [2] and Hypersonic flows [3, 4 and 5].

The large deflection similitude of Ghosh [1] has been extended by Ghosh, K., [2] to axisymmetric bodies with attached shock. Equivalence with a new piston-motion with axial symmetry has been established. The cone (or quasi-cone) results from the revolution of a wedge (or quasi-wedge) round the stream-wise axis and a similar revolution of the independent fluid slab [1] produces an axially symmetric conico-annular space. It is shown by Ghosh, K., [2] that the flow past a cone or quasi-cone is equivalent to a piston-motion in this conico-annular space, which is called the similitudinal slab. Although Ghosh, K., [2] gives similitude for cones and quasi-cones, he gives a solution based on the similitude for a cone only. This solution gives a constant density shock layer. Hence the constant density form of the unsteady Bernoulli's equation is used to find pressure on the cone surface in terms of the piston Mach number. Results are obtained for hypersonic flow for a perfect gas over oscillating cones and ogives of different Mach numbers and semi- angles.

2. Analysis

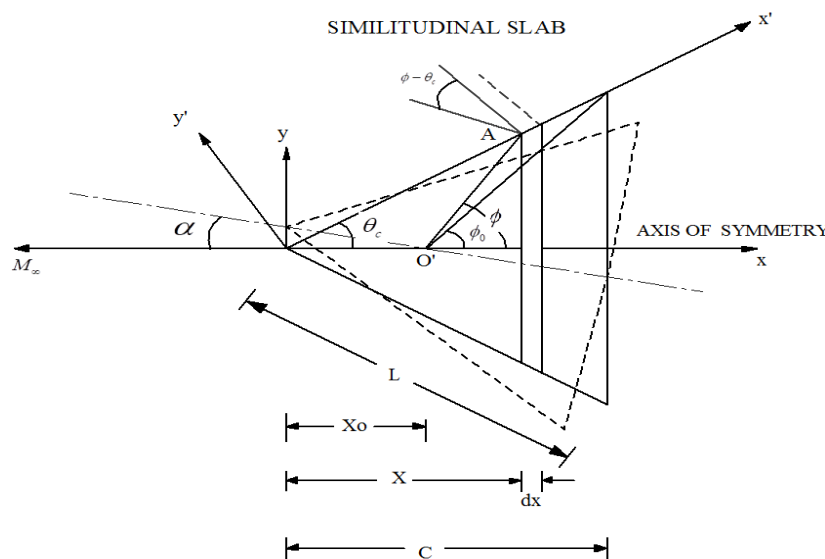


Figure 1. Cone Geometry

From the geometry we have

$$\tan \phi = \frac{x \tan \theta_c}{x - x_0}; \tan \phi_0 = \frac{c \tan \theta}{c - x_0}$$

where ϕ is the angle subtended by A at O' with x-axis, and for different location of A, ϕ varies from π to ϕ_c , θ_c is the cone semi-angle and c is the chord length. The definition of the Stiffness coefficient denoted by $C_{m\alpha}$, is

$$C_{m\alpha} = \left[\frac{\partial M}{\partial \alpha} \right]_{\alpha, q \rightarrow 0} \frac{1}{\frac{1}{2} P_\infty U_\infty^2 S_b c}$$

where S_b = base area of the cone = $\pi(c \tan \theta_c)^2$, c = chord length of the cone. On solving we obtain the following,

The expression for the pressure ratio of a steady cone at zero incidence (Ghosh, K., 1984), provided the bow shock is attached, is

$$\frac{P_{bo}}{P_\infty} = 1 + \gamma M_{po}^2 \left(1 + \frac{1}{4} \varepsilon\right) \quad (1)$$

where the density ratio is

$$\varepsilon = \frac{2 + (\gamma - 1)M_{po}^2}{2 + (\gamma + 1)M_{po}^2} \quad (2)$$

and M_{po} is the piston Mach number of the equivalent piston, operating in a conico-annular space; P_{bo} is the pressure on the body (cone) surface at zero incidences.

$$M_{po} = M_\infty \sin \theta_c$$

where θ_c is the cone semi angle.

Now

$$\frac{dP_{bo}}{dM_{po}} = 2P_\infty M_{po} \left[1 + \frac{1}{4} \left(1 + \frac{1}{2} M_{po} \frac{d\varepsilon}{dM_{po}}\right)\right] \quad (3)$$

where

$$\frac{d\varepsilon}{dM_{po}} = \frac{-8M_{po}}{[2 + (\gamma + 1)M_{po}^2]^2} \quad (4)$$

Thus using the definition of Stiffness and damping derivative and using the above, the expression for Stiffness derivative is obtained as

$$C_{m_\alpha} = D[h^3(1 - 2n^2) - (1 - h)\{H(2 + h) + n^2h(1 + 2h)\}] \quad (5)$$

And the expression for damping derivative is obtained as

$$C_{m_q} = \frac{D}{2}[h^4(2n^2 - 3n^4 - 1) - (1 - h)\{H(3H + h(H + 2n^2) + 2h^2n^2) + n^4h^2(1 + 3h)\}] \quad (6)$$

where

$$D = \frac{2}{3(1 + n^2)} \left[1 + \frac{1}{4} \left(\varepsilon + \frac{1}{2} K \frac{d\varepsilon}{dM_{po}}\right)\right] \quad (7)$$

and

$$H = (1 - h + n^2)$$

3. Results and Discussion

The results were obtained for various flow and geometrical parameters & discussed in this section. The variation of Stiffness derivative with pivot position for Mach number $M = 5$ is depicted in Figure 2. It is seen that Stiffness derivative decreases with pivot position and also with semi vertex angle. From the figure it is seen that at $h = 0$ there is substantial change in the stiffness derivative is only when semi-vertex has been increased from 5 degrees to ten degrees, further increase in the semi-vertex angle results in marginal change in the stiffness derivative. However, when we move downstream towards the base of the cone there is substantial increment in the stiffness derivative and this trend could be attributed to the flow field and hence the pressure distribution on the surface of the cone. We also should keep in mind that this is the case of body of revolution and not a simple 2-D case. Here 3-D effect does play an important role.

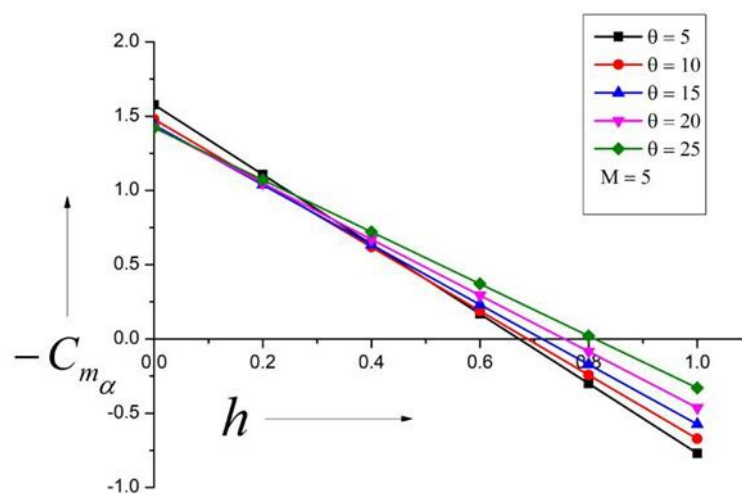


Figure 2. Variation of Stiffness derivative with pivot position for Mach number $M = 5$

Fig. 3 shows the results for stiffness derivatives for Mach number $M = 7$. Since all the parameters are same only the Mach number has been increased from $M = 5$ to 7. Due this marginal change in the inertia level there is marginal increase in the magnitude of the stability derivative and remaining trends are on the similar lines.

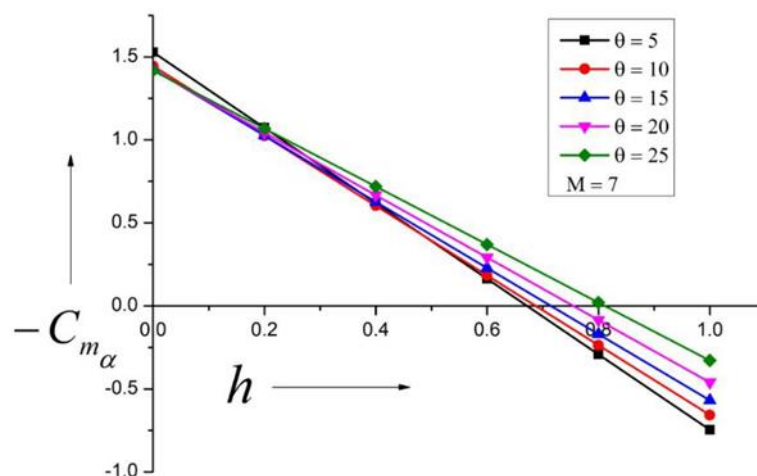


Figure 3. Variation of Stiffness derivative with pivot position for Mach number $M = 7$

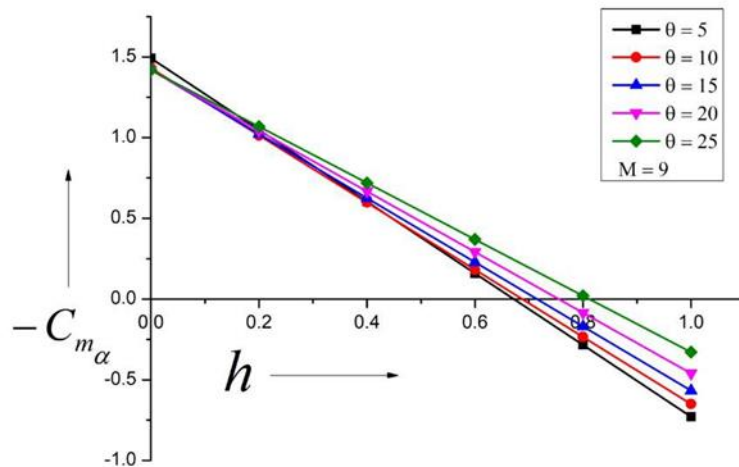


Figure 4. Variation of Stiffness derivative with pivot position for Mach number $M = 7$

Fig. 4 presents the results of stiffness derivative for Mach number $M = 9$. Here it is seen that since the Mach number M has further been increased from $M = 7$ to 9, this has resulted further in the reduction of Stiffness derivatives. However, when we consider the results downstream and more so beyond centre of pressure there is appreciable increase in the magnitude of the stiffness derivatives.

Similar results are shown in figures 5 and 6 for Mach number $M = 10$ and 15. In figure 6 there is marginal change in the stiffness derivative near the vertex of the cone for the pivot position $h = 0$ to 0.1, but this change disappears for $M = 15$. This indicates that Mach number independence principle hold for this Mach number. Further, it is seen that the centre of pressure is independent of Mach number, it remains in the same range, and this may be due to the 3-D effect.

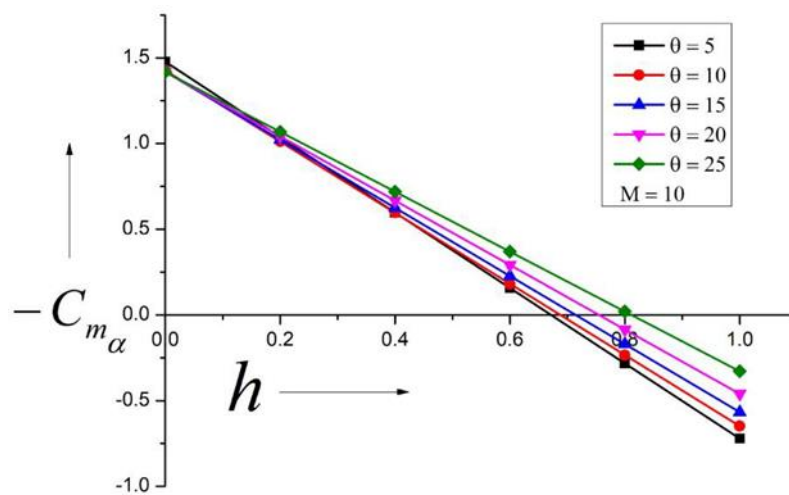


Figure 5. Variation of Stiffness derivative with pivot position for Mach number $M = 10$

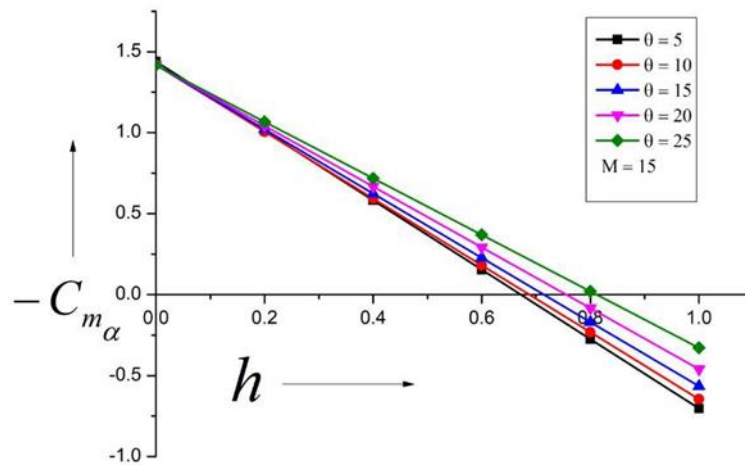


Figure 6. Variation of Stiffness derivative with pivot position for Mach number $M = 15$

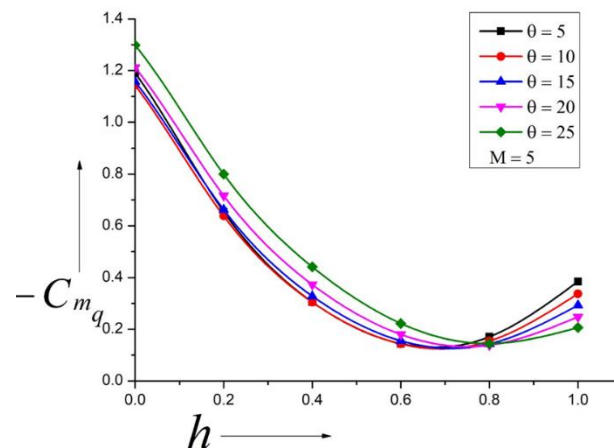


Figure 7. Damping derivative variation with pivot position for Mach number $M = 5$.

Figs. 7 to 11 present the results of damping derivatives for Mach $M = 5, 7, 9, 10$ and 15 . From figure 7 it is found that when angle θ was increased from 5 to 10 degrees there is 33% increase in the stability derivative and the same trend continues till it attains minimum value and then there is reversal in the trend. This is due to the location of pivot position. As long as the values of the pivot positions are ahead of centre of pressure it will have specific behaviour and later just opposite behaviour will start. It is also seen that there is forward shift in the minima point as compared to the oscillating wedge, this may due to the 3-D effect and due to the oscillating cone. Further the position of centre of pressure remains in the range of $h = 0.7$ to 0.8 . With further increase in the value of angle θ keeping Mach number same the increase in the damping derivatives is marginal. This may be due to the combined effect of Mach number, angle θ , and the pivot position.

Similar results are seen in Figs. 8 to 11. In all these figures it is observed that with further increase in the Mach number there is marginal decrease in the magnitude of the damping derivative as expected that with the increase in the Mach number the damping derivatives will decrease. Another important observation is that the band of centre of pressure which was between ($h = 0.7$ to 0.8) is narrowed down with the increase in Mach number. Once the Mach number is $M = 10$ and above there is change in the values of the damping derivatives due to the attainment of Mach number independence principle.

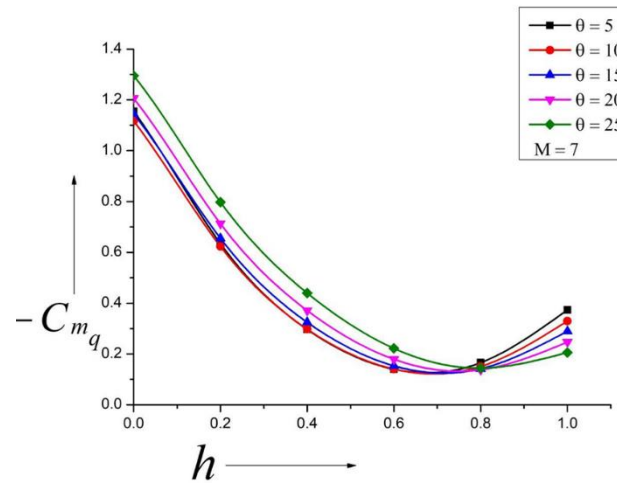


Figure 8. Damping derivative variation with pivot position for Mach number $M = 7$

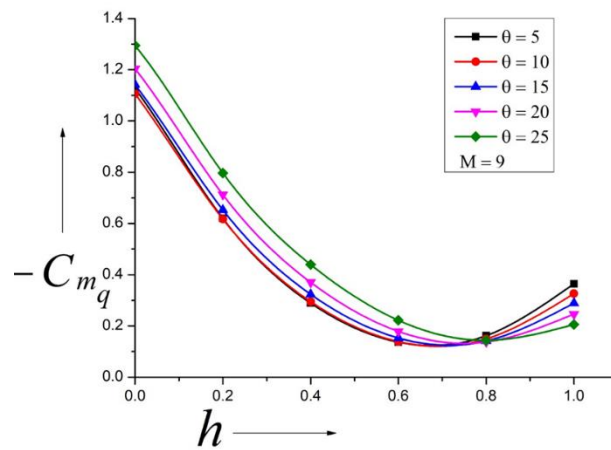


Figure 9. Damping derivative variation with pivot position for Mach number $M = 9$.

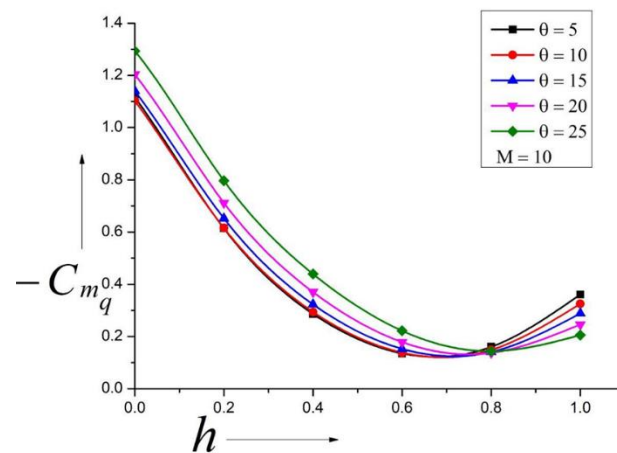


Figure 10. Damping derivative variation with pivot position for Mach number $M = 10$.

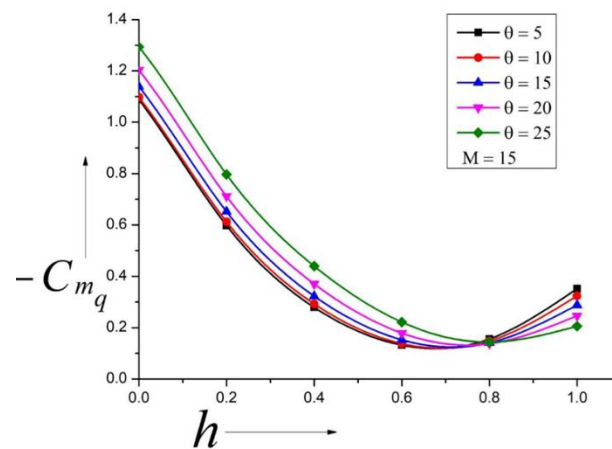


Figure 11. Damping derivative variation with pivot position for Mach number $M = 15$.

4. Conclusions

Based on the above discussion we can draw the following conclusions:

- The Stiffness derivative decreases with pivot position and also with semi vertex angle, there is substantial change in the stiffness derivative when semi-vertex has been increased from 5 degrees to ten degrees, further increase in the semi-vertex angle results in marginal change in the stiffness derivative, this is due to the case of body of revolution and not a simple 2-D case. Here 3-D effect does play an important role.
- Due the marginal change in the inertia level there is marginal increase in the magnitude of the stability.
- It is found that when angle theta was increased from 5 to 10 degrees there is 33 % increase in the stability derivative and the same trend continues till it attains minimum value and then there is reversal in the trend. This is due to the location of pivot position. It is also seen that there is forward shift in the minima point as compared to the oscillating wedge.
- The present theory for Oscillating cone is restricted to quasi-steady case. The main difficulty encountered in treating the unsteady case would be due to the flow being unsteady as well as non-uniform in the conical- annular space even for a steady piston.
- These results are likely to find wide applications in high speed flow problems. Viscous effects have been neglected. The expressions so obtained for stability derivative in pitch are valid for a slender ogive which often approximates to the fuselage of an aircraft.

References

- [1] Ghosh, K. (1977), "A New Similitude for Aerofoils in Hypersonic Flow", *Proceedings of the 6th Canadian Congress of Applied Mechanics, Vancouver, Canada, May 29-June 3*, pp. 685-686.
- [2] Ghosh K., (1984), *Hypersonic large deflection similitude for oscillating delta wings*, the Aeronautical Journal, pp. 357-361.
- [3] Asha Crasta and Khan S. A., (2012), *High Incidence Supersonic similitude for Planar wedge*, International Journal of Engineering research and Applications, 2(5), pp. 468-471.
- [4] Asha Crasta and S. A. Khan, (2012), *Estimation of Stability derivatives of an Oscillating Hypersonic delta wings with curved leading edges*, IJMET, 3(3), pp. 483-492.

- [5] Asha Crasta, M. A. Baig, S. A. Khan, (2012), *Estimation of Stability derivatives of a Delta wing in Hypersonic flow*, International Journal of Emerging trends in Engineering and Developments 6(2), pp. 505-516.
- [6] Asha Crasta and S. A. Khan, (2012), *Oscillating Supersonic delta wing with Straight Leading Edges*, International Journal of Computational Engineering Research, 2(5), pp. 1226-1233.
- [7] Asha Crasta and S. A. Khan, (2014), *Effect of Angle of Incidence on Stability derivatives of a wing*, International Journal for Research in Applied Science and Engineering Technology, 2(V), pp. 411-422.
- [8] Asha Crasta and S. A. Khan, (2014), *Hypersonic Similitude for Planar Wedges*, International Journal of Advanced Research in Engineering and Technology, 5(2), pp. 16-31.
- [9] Asha Crasta and S. A. Khan, (2014), *Estimation of Stability Derivatives for a Planar Wedge in the Newtonian Limit*, IOSR Journal of Mathematics, 10(2), pp. 01-06.
- [10] Asha Crasta and S. A. Khan, (2014), *Effect of Mach number on Stiffness and Damping Derivatives for Oscillating Hypersonic Non-Planar Wedge*, IOSR Journal of Mechanical and Civil Engineering, 11(2), pp. 04-11.
- [11] Asha Crasta and S.A. Khan, (2014), *Effect of Angle of Incidence on Stiffness and Damping derivatives for Oscillating Hypersonic Non-planar Wedge*, International Journal for Research in Applied Science and Engineering Technology, 2(IV), pp. 229-242.
- [12] Asha Crasta and S. A. Khan, (2014), *Supersonic Similitude for Oscillating Non-Planar wedge*, IOSR Journal of Mathematics, 10(2), pp. 15-24.
- [13] Khan S. A. and Asha Crasta, (2010), *Oscillating Supersonic delta wings with curved leading edges*, Advanced Studies in Contemporary Mathematics, 20(3), pp. 359-372.
- [14] Asha Crasta and S. A. Khan (2015), "Effect of Angle of attack on Stiffness derivative of an oscillating supersonic delta wing with curved leading edges" IOSR-JMCE issue1, Volume12, December, pp12-25.
- [15] Asha Crasta and S. A. Khan, "Effect of Angle of attack on Damping derivative of a delta wing with full sine curved leading edges" IJETED Journal issue5, Volume1, December- January 2015.
- [16] Asha Crasta and S. A. Khan, (2015) "Estimation of Damping derivative of a delta wing with half sine curved leading edges" IOSR Journal of Mechanical and civil engineering, Vol.12, issue1, February, pp40-44.
- [17] Asha Crasta and S. A. Khan (2015), "Estimation of Damping derivative in pitch of a Supersonic Delta wing with curved leading edges" IOSR Journal of Journal of Mathematics, Vol. 11, issue 1, Jan-feb, pp.07-15.
- [18] Asha Crasta and S.A.Khan (2015), "Effect of Aspect ratio with angle of attack of an oscillating Hypersonic delta wing with Straight Leading edges", Mathematical Sciences International research Journal", volume 4, issue2, pp.28-33., ISSN2278-8697.
- [19] Aysha Shabana, Renita Monis, Asha Crasta and S.A.Khan, "Pressure and its derivative with respect to piston Mach number for an oscillating cone" IOSR Journal of Mechanical and Civil Engineering, volume 2, pp. 74-78.
- [20] Asha Crasta, Aysha Shabana, Renita Monis, and S.A.Khan, "Analysis of Variation of Stiffness Derivative with Mach number and angle of attack for a supersonic flow", IOSR Journal of Mechanical and Civil Engineering, volume 2, pp. 97- 104.
- [21] Renita Monis, Aysha Shabana, Asha Crasta and S.A.Khan, "Supersonic Flow Analysis and Evaluation of Damping Derivative", IOSR Journal of Mechanical and Civil Engineering, volume 3, pp. 29- 36.

- [22] Asha Crasta, S. A. Khan , “ *Determination of Surface Pressure of an axisymmetric ogive in Hypersonic Flow*” in Mathematical sciences International Research Journal, Vol. 2, Issue2, August 2013, pp.333-335, ISSN:2278-8697.
- [23] Asha Crasta and Khan S. A., 2014, ‘*Stability derivatives of a Delta Wing with Straight leading edge in the Newtonian Limit*’, International Journal of Engineering research and Applications, Vol. 4, Issue 3, Version 2, March, pp. 32-38.
- [24] Asha Crasta and S. A. Khan, 2013, ‘*Stability Derivatives in the Newtonian Limit*’, International Journal of Advanced Research in Engineering and Technology, Volume 4, Issue 7, Nov-Dec pp. 276-289.
- [25] Asha Crasta, S. A. Khan , 2014, ‘*Effect of angle of incidence on Roll damping derivative of a delta Wing*’, International Journal of Emerging trends in Engineering and Developments Vol. 2, Issue4, March 2014, pp. 343-356.
- [26] Asha Crasta and S. A. Khan, 2014, ‘*Effect of Angle of attack on Stability derivatives of a Delta wing with Straight leading edge in Supersonic Flow*’, International Journal of Mathematics, Vol.10, issue5, Sep-oct 2014. Pp. 01-08.
- [27] Asha Crasta, S. A. Khan , 2014, ‘*Effect of sweep angle on Roll damping derivative of a delta Wing*’, International Journal of Emerging trends in Engineering and Developments Vol15, Issue4, Aug-Sep, pp. 45-55.
- [28] Asha Crasta and S. A. Khan, 2014, ‘*Effect of Angle of attack on Stability derivatives of a Delta wing in Hypersonic flow with Straight leading edge*’, International Journal of Mechanical and civil engineering, Vol.11, issue5, Sep-oct (ver.2). Pp. 21-29.
- [29] Asha Crasta and S.A.Khan 2014, “*Effect of sweep angle on rolling moment derivative of an oscillating supersonic/hypersonic delta wing*”, International journal of modern Engg. Research, Vol.4, issue 10, ver3, pp.15-22.
- [30] Asha Crasta and S.A. Khan, 2016, “*Effect of Aspect ratio with roll moment derivative of a delta wing in supersonic flow*”, International Journal of Advances in Engineering Research, (IJAER) 2016, Vol. No. 12, Issue No. I, July, pp.10-15, e-ISSN: 2231-5152, p-ISSN: 2454-1796.