IOP Conf. Series: Materials Science and Engineering **366** (2018) 012056 doi:10.1088/1757-899X/366/1/012056

A Virtual Array Extension Method for Multiple Partial Discharge Sources Localization Based on Fourth-order Cumulant and Signal Sparse Decomposition

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Abstract. The localization of multiple partial discharges based on UHF signal is difficult to accomplish due to the excessive localization error. A virtual array extension method based on four-order cumulant and signal sparse decomposition is proposed in this paper. The virtually extended array has higher directional resolution and it could detect partial discharge sources with higher accuracy. The 2×2 UHF array that commonly used in existed partial discharge localization methods is virtually extended to 3×3 and 5×5 array. The virtual array extension is accomplished by extending the corresponding steering vector matrix of PD signal spatial spectrum to a overcomplete dictionary, which is then used to realize the sparse decomposition of the PD signal. Eventually the DOA estimation of multiple partial discharge sources is achieved. Simulation and field test results show that the virtually extended UHF array can detect and locate two PD sources simultaneously. The directional angel estimation error is about 6° and the pitch angle error is lower than 4°, which has proved the feasibility of the proposed method.

1. Introduction

The Partial Discharge (PD) localization based on Ultra-High Frequency (UHF) signals has been widely studied in recent years, due to its high detection accuracy and strong ability of anti-interference. Many innovative PD detection methods have been proposed by researchers [1-4] and several successful applications have been reported. However, most studies focus on the detection or accurate localization of single PD source, which has limited their practical application.

The detection and localization of multiple PD sources is an important research area while few studies are addressed. In [5], each PD source is represented as a feature vector and a clustering algorithm is applied to separate the PD sources. Reference [6] proposed a self-adaptive PD separation algorithm based on optimized feature extraction of cumulative energy function. A probability-based algorithm is presented in [7], which could locate multiple PD sources in electrical equipment. These researches generally follow a similar roadmap of “signal processing, feature extraction, cluster analysis and multiple sources separation.” Those algorithms based on feature extraction and clustering analysis can successfully separate multiple PD sources while suffer to its algorithm complexity and huge computation burden.

In this paper, an effective multiple UHF PD DOA estimation method based on virtual array extension is proposed. The fourth-order cumulant matrix [8-9] is applied to the PD signals received by the sensor array to accomplish the sparse decomposition. The weighted l1-norm method is used to solve the DOA estimation equations. Eventually, the spatial spectrum of multiple PD sources is established and the DOA estimation is achieved. As most of the multi-channel synchronous GHz sampling rate devices that having an acceptable price are usually four-channel, the UHF sensor array is limited to a 2×2 array. Therefore, the 2×2 actual array is virtually extended by the proposed method, and simulation and fields tests are performed. Test results show that the extended array could estimate the PD source more accurately than the actual array. The directional angel estimation error about 6° and the pitch angel error lower than 4°, which has verified the feasibility of the proposed method in practical application.

2. Signal Sparse Decomposition Based on Fourth-order Cumulant Matrix

2.1. Application of Fourth-order Cumulant Matrix

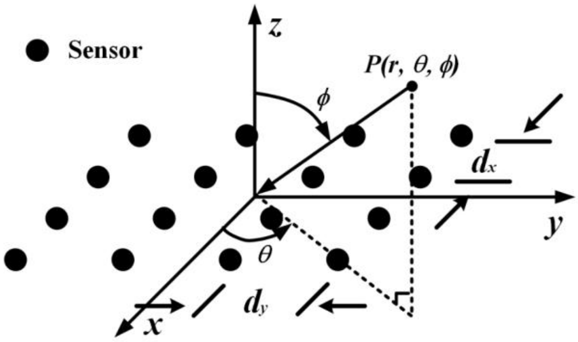

Figure 1. Partial Discharge DOA Estimation by Sensor Array.

Suppose the plane UHF sensor array is composed of M×M UHF sensors, as shown in Fig. 1. The sensors are distributed evenly with the interval as dx along x-axis and the interval as dy along y-axis. Suppose P stands for the PD source, the distance from P to axis origin is r, the directional angel is □ and the pitch angel is □.

Suppose K independent narrowband signals are received by the M×M sensor array, the PD signal received by the mth sensor in the array could be expressed as:

$$y_m(t) = \sum_{k=1}^{K} s_k(t) \cdot a_m(\theta_k, \phi_k) + v_m(t) \tag{1}$$

And the vector form could be expressed as:

$$\boldsymbol{y(t) = A(\theta,\phi) \cdot s(t) + v(t)} \tag{2}$$

where $\boldsymbol{y(t)}$ stands for the signal received by the array. $\boldsymbol{A(\theta,\phi)} = (a(\theta_1,\phi_1), a(\theta_2,\phi_2), \ldots, a(\theta_N,\phi_N))$ is the overcomplete steering vector and N is the number of divided angles. $a(\theta_i,\phi_i) = (1, e^{j\pi \sin\theta_i \cos\phi_i}, \ldots, e^{j\pi(M-1)\sin\theta_i \cos\phi_i})^{\mathrm{T}}$. $\boldsymbol{v(t)}$ stands for the environment noise. $\boldsymbol{s(t)}$ stands for the signal source vector. The diagram of DOA estimation with overcomplete steering vector is shown in Fig. 2.

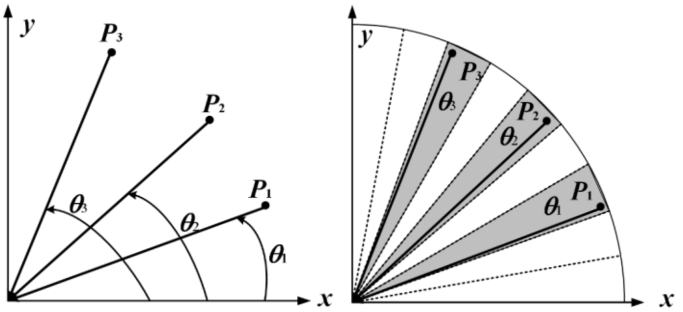

Figure 2. DOA Estimation with Overcomplete Steering vector.

The $M^2 \times M^2$ fourth-order cumulant of the received signal $\boldsymbol{y(t)}$ is calculated as follows [10]:

$$\begin{aligned}\boldsymbol{C}_Y &= cum\{\boldsymbol{Y}(t),\boldsymbol{Y}^{\mathrm{T}}(t),\boldsymbol{Y}^*(t),\boldsymbol{Y}^{\mathrm{H}}(t)\}\\ &=(\boldsymbol{A}(\theta,\phi)\otimes\boldsymbol{A}^*(\theta,\phi))\boldsymbol{C}_s(\boldsymbol{A}(\theta,\phi)\otimes\boldsymbol{A}^*(\theta,\phi))^{\mathrm{H}}\end{aligned} \tag{3}$$

where $\boldsymbol{C}_s$ is expressed as follows:

$$\begin{aligned}\boldsymbol{C}_s &= cum\{\boldsymbol{S}(t),\boldsymbol{S}^{\mathrm{T}}(t),\boldsymbol{S}^*(t),\boldsymbol{S}^{\mathrm{H}}(t)\}\\ &=E\{(\boldsymbol{S}(t)\otimes\boldsymbol{S}^*(t))(\boldsymbol{S}(t)\otimes\boldsymbol{S}^*(t))^{\mathrm{H}}\}-\\ &\quad E\{(\boldsymbol{S}(t)\otimes\boldsymbol{S}^*(t))\}E\{(\boldsymbol{S}(t)\otimes\boldsymbol{S}^*(t))^{\mathrm{H}}\}-\\ &\quad E\{(\boldsymbol{S}(t)\boldsymbol{S}^{\mathrm{H}}(t)\}\otimes E\{(\boldsymbol{S}(t)\boldsymbol{S}^{\mathrm{H}}(t))^*\}\end{aligned} \tag{4}$$

$\boldsymbol{C}_s$ is the fourth-order cumulant matrix of the signal source vector $\boldsymbol{s(t)}$. The value of the matrix element at the $((i_1\text{-}1)\mathrm{M}+i_3)^{\mathrm{th}}$ row and $((i_2\text{-}1)\mathrm{M}+i_4)^{\mathrm{th}}$ column of $\boldsymbol{C}_s$ is $cum(s_{i_1}(t),s_{i_2}(t),s_{i_3}^*(t),s_{i_4}^*(t))$, $i_1,i_2,i_3,i_4\in\{1,2,\ldots,N\}$.

The equations could be expressed as:

$$\begin{aligned}&\boldsymbol{C}_s((i_1-1)\mathrm{M}+i_3,(i_2-1)\mathrm{M}+i_4)=\\ &cum(s_{i_1}(t),s_{i_2}(t),s_{i_3}^*(t),s_{i_4}^*(t))\end{aligned} \tag{5}$$

2.2. *Signal Sparse Decomposition and Array Extension*

Suppose the signal source vector is non-correlation signal. According to the principle of fourth-order cumulant, (3) could be simplified as:

$$\begin{aligned}\boldsymbol{C}_Y &= \sum_{i=1}^{N}\boldsymbol{C}_s((i-1)\mathrm{M}+i,(i-1)\mathrm{M}+i)(a(\theta_i,\phi_i)\otimes\\ &a^*(\theta_i,\phi_i))(a(\theta_i,\phi_i)\otimes a^*(\theta_i,\phi_i))^{\mathrm{H}}\end{aligned} \tag{6}$$

In (6), the Kronecker product of the steering vector, $a(\theta_i,\phi_i)\otimes a^*(\theta_i,\phi_i)$, would form the virtual array sensors, extending the sensor array from M×M to (2M-1)×(2M-1). However, the size of vector $a(\theta_i,\phi_i)\otimes a^*(\theta_i,\phi_i)$ is $\mathrm{M}^2\times 1$. It is because that many elements in the vector carries the same signal information received by the same sensor [11]. The location of sensor is expressed as P. The relationship between P and the number of elements related to it, N_{num}, is shown as follows:

$$N_{num}=M-\frac{|P|}{d} \qquad -(M-1)d\le P\le(M-1)d \tag{7}$$

where d is the interval between sensors in the array.

As there are many elements in $a(\theta_i,\phi_i)\otimes a^*(\theta_i,\phi_i)$ that are representing the same received signal, add these elements together. Similarly, add the same elements in $\boldsymbol{C}_Y$ together. Thus the $M^2\times M^2$ is

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turned into a (2M-1)×(2M-1) matrix. Express it in the form of $b(\theta_i,\phi_i)b^{\mathrm{H}}(\theta_i,\phi_i)$, where $b(\theta_i,\phi_i)$ is the newly designed steering vector:

$$\begin{aligned} b(\theta_i,\phi_i) = (&e^{-j\pi(M-1)\sin\theta_i\cos\phi_i},\ldots,e^{-j\pi\sin\theta_i\cos\phi_i}, \\ &1,e^{j\pi\sin\theta_i\cos\phi_i},\ldots,e^{j\pi(M-1)\sin\theta_i\cos\phi_i})^{\mathrm{T}} \end{aligned} \tag{8}$$

Thus (6) can be simplified as:

$$\begin{aligned} \boldsymbol{C}_{\mathrm{Y}}' &= \sum_{i=1}^{N} \boldsymbol{C}_s((i-1)\mathrm{M}+i,(i-1)\mathrm{M}+i)b(\theta_i,\phi_i)b^{\mathrm{H}}(\theta_i,\phi_i) \\ &= \boldsymbol{B(\theta,\phi)} \cdot diag\{\boldsymbol{C}_s(1,1),\ldots, \\ &\boldsymbol{C}_s((N-1)M+N,(N-1)M+N)\} \cdot \boldsymbol{B}^{\mathrm{H}}\boldsymbol{(\theta,\phi)} \end{aligned} \tag{9}$$

Suppose $\boldsymbol{X}$ = diag{$\boldsymbol{C}_s$(1, 1) , … , $\boldsymbol{C}_s$((N-1)M+N, (N-1)M+N)}$\boldsymbol{B}^{\mathrm{H}}(\boldsymbol{\theta},\boldsymbol{\phi})$, the signal sparse decomposition could be written as:

$$\boldsymbol{C}_{\mathrm{Y}}' = \boldsymbol{B(\theta,\phi)} \cdot \boldsymbol{X} \tag{10}$$

Compared to (2), (10) is the signal equations received by the extended to a (2M-1)×(2M-1) sensor array.

3. The Solution to the Extended Array Model and the Spatial Spectrum PD Estimation

3.1. Model Solution

The extended signal source vector **X** in (10) is the solution to the partial discharge DOA estimation in this paper. To every row in matrix **X**, calculate the l_2-norm vector $x_{l_2} = (\|\boldsymbol{X}(1,:)\|_2,\ldots,\|\boldsymbol{X}(N,:)\|_2)^{\mathrm{T}}$. The l_1-norm of vector x_{l_2} is the objective function of the minimization problem. Thus, the solution to (10) is transferred to the convex optimization problem as shown below [12]:

$$\begin{aligned} &\min \sum_{i=1}^{N} \omega_i \|\boldsymbol{X}(i,:)\|_2 \\ &\text{s.t. } \boldsymbol{C}_{\mathrm{Y}}' = \boldsymbol{B(\theta,\phi)} \cdot \boldsymbol{X} \end{aligned} \tag{11}$$

where ω_i is the weight, which could be calculated by the signal subspace and the noise subspace [13]. In this paper, it is calculated as below:

$$\begin{aligned} \omega_i = b^H(\theta_i,\phi_i)&\boldsymbol{U}(:,(K+1):(2M-1)) \\ &\boldsymbol{U}^{\mathrm{H}}(:,(K+1):(2M-1))b(\theta_i,\phi_i) \end{aligned} \tag{12}$$

where $\boldsymbol{U}$ is the noise subspace of the $\boldsymbol{C}_{\mathrm{Y}}'$.

3.2. PD DOA Estimation by Spatial Spectrum Estimation

The spatial spectrum $P(\theta,\phi)$ is of PD sources are established to estimate the DOA. It is calculated as below [14]:

$$P(\theta,\phi) = \frac{1}{b^{\mathrm{H}}(\theta,\phi)\cdot \boldsymbol{U} \cdot \boldsymbol{U}^{\mathrm{H}} \cdot b(\theta,\phi)} \tag{13}$$

where $\boldsymbol{U}$ is the noise subspace in (12) and $b(\theta,\phi)$ is the steering vector designed in (8).

The corresponding (θ,ϕ) to the maximum point of $P(\theta,\phi)$ is the DOA estimation results of the partial discharge. According to the principle of spatial spectrum DOA estimation, the number of sources K should be smaller than the number of sensors in the array M [15]. Thus, the extended (2M-1)×(2M-1) array could only detect (2M-2) sources at a time.

3.3. Simulation Tests of PD DOA Estimation

Simulation tests based on MATLAB are conducted to verify the feasibility of the proposed method. The double exponential model, added with Gauss noise, is used to simulate the partial discharge signal. Two PD sources are simulated simultaneously, with their DOA coordinates as $(\theta_1,\phi_1)=(-35°,20°)$ and $(\theta_2,\phi_2)=(-40°,55°)$. The propagation speed of the PD signal is set as the speed of light, c. The frequency range is 300~3000MHz, with the center frequency as 1000MHz. The wavelength is 30mm and the interval between sensors is 15mm. The sampling frequency is 5GHz and the sampling point is 1024. The SNR of the simulated signal is 10dB.

In the simulation tests, the simulated sensor array is set to be a 3×3 UHF array, which could locate up to two PD sources simultaneously. By applying the array extension method proposed in this paper, the array is extended to a 5×5 virtual array, which could locate up to four PD sources. Fig. 3 shows the DOA estimation with 3×3 UHF array. Detailed localization results are shown in Table 1.

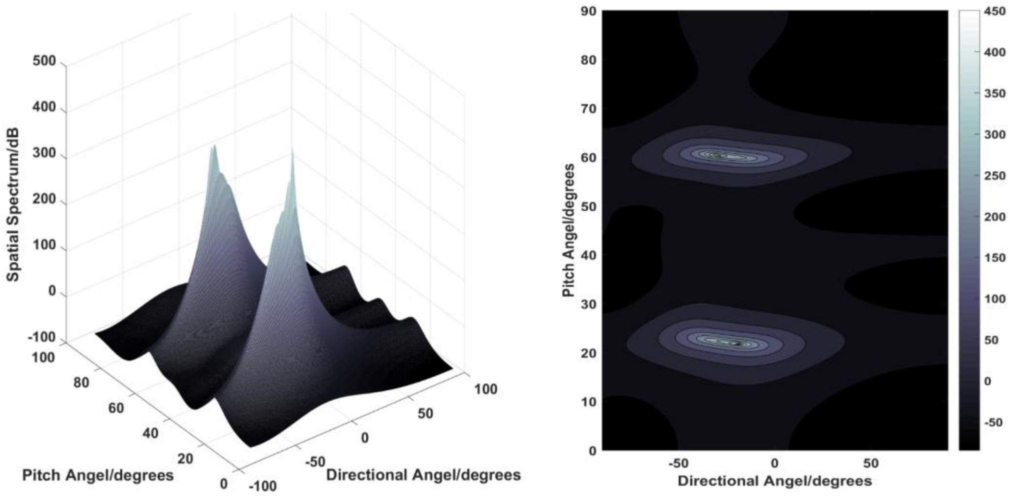

Figure 3. PD DOA Estimation Results with Extended Array.

Table 1. PD DOA Estimation Results with Both Array

Array Model	Simulated PD DOA Estimation		Estimation Error /(°)
	Simulated PD Coordinates (θ, ϕ) /(°)	DOA Estimation Results (θ', ϕ') /(°)	
Simulated Array (3×3)	(-35.0, 20.0) (-40.0, 55.0)	(-25.39, 27.93) (-31.59, 61.20)	9.61, 7.93 8.41, 6.20
Extended Array (5×5)	(-35.0, 20.0) (-40.0, 55.0) (35.0, 70.0)	(-31.57, 22.14) (-36.27, 58.43) (-33.59, 72.93)	3.43, 2.14 3.71, 3.43 -1.41, 2.93

It can be seen from Table. 1 that the DOA estimation error of the 3×3 array is about 8 degrees. Besides, it could only locate up to two PD sources at the same time. By comparison, the DOA estimation error of the extended array is less than 4 degrees. Furthermore, the extended array could locate more PD sources than the actual array. The simulation results have verified the feasibility of the proposed method.

4. Field Tests and Result Analysis

4.1. Experiment Setup

Field tests are conducted to verify the performance of the proposed array extension method in practical application. The PD DOA estimation system is established under actual electromagnetic environment, whose schematic diagram is shown in Fig. 4. The system is composed of the sensor array, the signal pre-processing unit, the UHF signal collector unit and the computer.

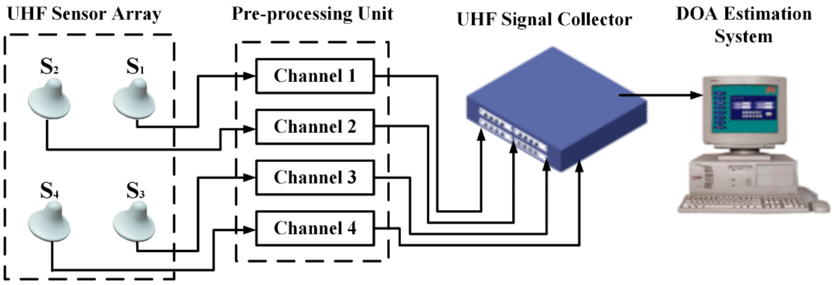

Figure 4. The Multiple UHF Partial Discharge DOA Estimation System.

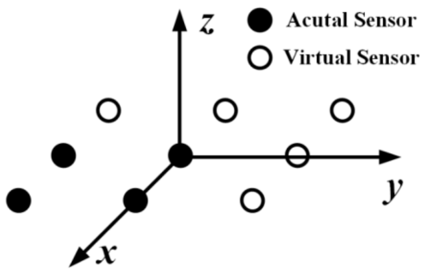

Figure 5. The Schematic Diagram of UHF Array Extension.

As mentioned before, most of the synchronous GHz sampling rate devices have only four channels. Therefore, the UHF sensor array is a 2×2 array and their relative coordinates are shown in Fig. 5. The 2×2 actual UHF array will be extended to 3×3 virtual array by the proposed method.

An EM TEST DITO ESD gun is used to simulate partial discharge. The frequency detection range of the UHF sensor is 300M-1.5GHz. The frequency detection range of the signal amplifier is 300M-1.5GHz. We discharge at different directional angles and pitch angles, while the distance is fixed to 3 meters without changes. The typical two PD sources signal received by the sensor array is shown in Fig. 6 and the localization result is shown in Fig. 7.

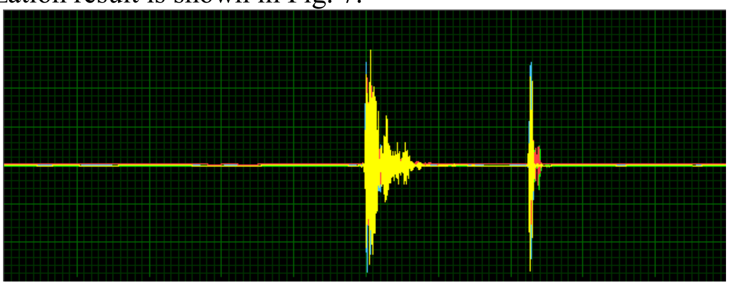

Figure 6. Two UHF Signals Received by the Sensor Array.

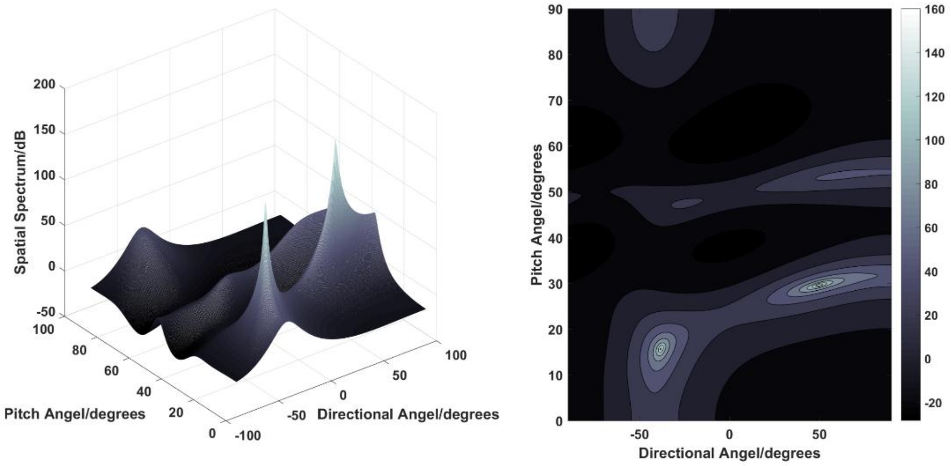

Figure 7. DOA Estimation of Actual PD Sources.

It can be seen from Fig. 7 that the extended UHF array could locate the two PD sources, and the localization results are (-43.19°, 13.30°) and (54.63°, 17.48°) respectively.

To further evaluate the performance of proposed array extension method, several field tests are done and the averaged estimation results are shown in Table 2. For comparison, the DOA estimation of a single PD source with the actual 2×2 array is also shown in Table 2.

Table 2. PD DOA Estimation Results with Both Array

Array Model	No.	Simulated PD DOA Estimation		Estimation Error /(°)
		Simulated PD Coordinates (θ, ϕ) /(°)	DOA Estimation Results (θ', ϕ') /(°)	
Actual Array (2×2)	1	(30.0, 20.0)	(-15.91, 11.38)	14.09, 8.62
	2	(40.0, 25.0)	(-24.46, 13.23)	15.54, 11.77
Extended Array (3×3)	1	(-30.0, 20.0) (30.0, 20.0)	(-23.16, 15.78) (-24.66, 17.49)	-6.84, 4.22 5.34, 2.51
	2	(-40.0, 20.0) (40.0, 20.0)	(-33.28, 23.41) (-45.28, 17.24)	-6.72, -3.41 -5.28, 2.76

As shown in Table. 2, to estimate the DOA of a single PD source with the actual 2×2 array, the estimation error is about 15 degrees. Besides, the 2×2 array could only locate one PD source at a time, which has limited it utilization in practical applications.

By applying the proposed array extension method, the actual 2×2 array is extended to a 3×3 array, which could locate up to two PD sources at a time. More importantly, the extended array could estimate the PD source more accurately, with the directional angel estimation error about 6 degrees and the pitch angel error lower than 4 degrees. The extended array has a better performance in PD localization than the actual array.

5. Conclusion

(1) Aiming at the seldom addressed multiple PD sources localization problem, a multiple UHF PD DOA estimation method is proposed in this paper. The actual UHF sensor array is extended to a virtual array based on the fourth-order cumulant matrix and signal sparse decomposition. Results of

simulation tests and field tests have shown that the proposed method could increase the PD estimation number and simultaneously increase the DOA estimation accuracy.
(2) The steering vector of the PD signal is extended to the overcomplete steering vector based on the fourth-order cumulant matrix. The signal sparse decomposition method is applied to simplify and extend the signal DOA equation. Lastly, the spatial spectrum of the PD signal is established and the PD sources are located.
(3) The proposed method could be applied in other devices such as ultrasound sensor array, which should also increase the accuracy of PD localization.

References

[1] P. J. Moore, I. E. Portugues, and I.A. Glover, "Radiometric location of partial discharge sources on energized high-Voltage plant", *IEEE Trans. Power Del*, Vol. 20, No.3, pp. 2264-2272, 2005.
[2] S. Biswas, D. Dey, B. Chatterjee and S. Chakravorti, "Cross-spectrum analysis based methodology for discrimination and localization of partial discharge sources using acoustic sensors", *IEEE Trans. Dielectr. Electr. Insul*, Vol. 23, No.6, pp. 3556-3565, 2016.
[3] M. Mondal and G.B. Kumbhar, "A technique based on an archimedean copula for the localization of partial discharge in a transformer winding", *IEEE Trans. Dielectr. Electr. Insul*, Vol. 23, No.5, pp. 2908-2916, 2016.
[4] H. Hou, G. Sheng and X. Jiang, "Robust time delay estimation method for locating uhf signals of partial discharge in substation", *IEEE Trans. Power Del*, Vol. 28, No.3, pp. 1960-1968, 2013.
[5] R. D. Nimmo, G. Callender, and P. L. Lewin, "Methods for Wavelet-Based Autonomous Discrimination of Multiple Partial Discharge Sources", *IEEE Trans. Dielectr. Electr. Insul*, Vol. 24, No.2, pp. 1131-1140, 2017.
[6] M. X. Zhu, Q. Liu, J. Y. Xue, J. B. Deng, and G. J. Zhang, "Self-adaptive separation of multiple partial discharge sources based on optimized feature extraction of cumulative energy function" *IEEE Trans. Dielectr. Electr. Insul*, Vol. 24, No.1, pp. 246-258, 2017.
[7] M. X. Zhu, Y. B. Wang, Q. Liu, J. N. Zhang, and J. B. Deng, "Localization of multiple partial discharge sources in air-insulated substation using probability-based algorithm" *IEEE Trans. Dielectr. Electr. Insul*, Vol. 24, No.1, pp. 157-166, 2017.
[8] E. Masry, "The Estimation of the Fourth-Order Cumulant for Dependent Data: Consistency and Asymptotic Normality" *IEEE Press*, Vol. 58, No.4, pp. 1991-1998, 2010.
[9] A. Ahmed, Y. D. Zhang, and B. Himed, "Effective nested array design for fourth-order cumulant-based DOA estimation" *2017 IEEE Radar Conference (RadarConf)*, pp.0998-1002.
[10] P. Boaz, and F. Benjamin, "Direction findings algorithms based on high-order statistics", *IEEE Trans. on Signal Process.* Vol. 39, No.9, pp. 2016-2024, 1991.
[11] B. Jiang, and L. J. Zheng, "Fourth-order music algorithm based on minimum redundant virtual extended array", *IEEE Trans. on Signal Process.*, Vol. 42, No.6, pp. 517-600, 2006.
[12] B. Stephen and V. Lieven, *Convex Optimization*, Translated by S. N. Wang and X. L. Huang, vol. I. Beijing: *Tsinghua University Press*, 2013, p. 149-153.
[13] C. D. Zheng, G. Li, and Y. M. Liu, "Subspace weighted $l_{2,1}$ minimization for sparse signal recovery", *EURASIP Journal on Advances in Signal Processing,* Vol. 98, No.1, pp. 1-11, 2012.
[14] S. Cai, "A normalized spatial spectrum for DOA estimation with uniform linear arrays in the presence of unknown mutual coupling", *2016 IEEE International Conference on Acoustics*, pp. 3086-3090.
[15] O Cholnam, D Kang, and J Cholyong, "Spatial spectrum for direction of arrival estimation of multiple non-stationary wideband sources in the time–frequency plane", *IET Signal Processing*, Vol. 10, No.9, pp. 1105-1111, 2017