IOP Conf. Series: Materials Science and Engineering **365** (2018) 042003 doi:10.1088/1757-899X/365/4/042003

Particle transport in a porous medium with initial deposit

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Abstract. Modelling of the suspended particles transport in a porous medium is used in the analysis of methods for strengthening foundations. To strengthen the porous soil, a low concentration cement-based grout is pumped into it under pressure. The suspension is filtered in a porous medium and fills the cavity of the soil. Grains of the grout are distributed along the network structure of the porous medium and strengthen the soil. The transfer of particles by a flow of a carrier fluid is accompanied by an uneven formation of a deposit on the porous medium frame.
The purpose of the paper is to determine the mobile two-phase boundary between water and the particles during the injection of a suspension into a porous medium and to obtain an analytical solution of the nonlinear filtration problem for a general case of variable porosity and permeability.
The mathematical model of one-dimensional deep bed filtration with size-exclusion particles retention includes the equations of mass balance and kinetic rate of a deposit and unsteady boundary conditions with unknown dimensionless concentrations of suspended and retained particles. Methods of non-linear asymptotic analysis are used to obtain the analytical solution and to construct an asymptotics near the porous medium inlet. The asymptotics is determined on the basis of a local exact solution of the problem.
It is shown that the mobile two-phase boundary moves with variable speed. At the boundary of two phases, the concentration of suspended particles is discontinuous, and the concentration of retained particles is continuous and loses its smoothness. The exact explicit formula for the two-phase boundary and its asymptotics in a form convenient for calculations are obtained. An exact solution is obtained on the boundary of the porous medium and the asymptotics of the filtration problem is constructed near the porous medium inlet. Numerical calculation of the asymptotic solution is performed; graphs of the dependence of concentrations on time and coordinate are presented.
In contrast to the numerical solution, analytical methods make it possible to determine the dependence of the solution of the filtration problem on the controlled external parameters. This allows the construction engineers to choose the best size of injected grout grains and the properties of the carrier fluid, optimize the filtration process and form a grouted porous soil of the required strength and density.

1. Introduction

The construction of buildings and structures on loose ground requires the formation of a solid foundation. Many soil layers and building materials are porous media capable of accumulating


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moisture and particles contained in it. One way to strengthen the soil is to inject into the soil a low-concentration concrete solution [1].

The watery grout spreads over the porous soil and fills the voids of the soil. The liquid grout spreads over the porous soil and fills the voids of the soil. Grains of the grout are distributed along the network structure of the porous medium and strengthen the soil after solidification.

The filtration problems describe the transport of suspended solid fine particles by the fluid flow through a porous medium. Some particles pass through a porous medium, while a part gets stuck in the pores and forms a deposit. Accumulation of the deposit changes the structure of the porous medium. The pumped grout increases the strength of the porous soil.

Theoretical research, field studies and laboratory experiments have allowed the construction of various filtration models [2, 3]. In this paper, a one-dimensional model of deep bed filtration with a mechanical-geometric mechanism of particle capture is considered [4, 5]. Two first-order partial differential equations describing the mass balance of suspended and retained particles and the growth rate of the deposit form a nonlinear hyperbolic system. In some cases, exact solutions have been obtained [6, 7]. In the absence of an exact solution, asymptotics are constructed modeling the problem approximately [8, 9], and numerical calculations are performed [10].

Periodic alternate injection of the suspension and water into a porous medium leads to gradual accumulation of the deposit in the pores. Consider one of the steps of this process. At the initial moment, the porous medium is filled with pure water and the pores already contain the deposit unevenly distributed along the porous frame. A suspension of constant concentration injected at the frame inlet, moves through the pores and gradually displaces the water. The model takes into account the change in the porosity and permeability of the porous medium with increasing deposit concentration.

Section 2 presents a mathematical model of the particle transport in a porous medium with the initial deposit. The asymptotic model near the porous medium inlet and the asymptotics of the two-phase boundary are constructed in Section 3. Section 4 is devoted to the results of numerical simulation. Discussion and Conclusion in sections 5 and 6 finalize the article.

2. Mathematical model

Filtration of the suspension in a porous medium is determined by a system of equations with unknown concentrations of suspended $C(x,t)$ and retained particles $S(x,t)$

$$\frac{\partial\big(g(S)C\big)}{\partial t}+\frac{\partial\big(f(S)C\big)}{\partial x}+\frac{\partial S}{\partial t}=0; \tag{1}$$

$$\frac{\partial S}{\partial t}=\Lambda(S)C. \tag{2}$$

Here the filtration coefficient $\Lambda(S)$, porosity $g(S)$ and permeability $f(S)$ are non-negative and smoothly dependent on the deposit $S(x,t)$.

The uniqueness of the solution is provided by the initial and boundary conditions

$$C\big|_{x=0}=1; \tag{3}$$

$$C\big|_{t=0}=0;\ S\big|_{t=0}=p(x). \tag{4}$$

The problem (1)-(4) is considered in the domain $\Omega=\{(x,t):0<x<1,\ t>0\}$. The domain Ω consists of two subdomains Ω_S and Ω_W with water and suspension. The mobile boundary of the two phases - the concentration front of the suspended particles propagates in a porous medium with variable velocity $v(x)=f\big(p(x)\big)/g\big(p(x)\big)$ along the characteristic given by the equation $\mathrm{d}x/\mathrm{d}t=v(x),\ x(0)=0$.

The boundary Γ is determined by formula

$$t_\Gamma(x) = \int_0^x \frac{g\left(p(y)\right)}{f\left(p(y)\right)} \mathrm{d}y \,. \tag{5}$$

Before the concentration front in the domain of pure water Ω_W, the solution does not depend on time: $C = 0,\ S = p(x)$. Behind the concentration front in the suspension domain Ω_S, the solution is positive: $C(x,t) > 0,\ S(x,t) > 0$. On the boundary Γ the solution $C(x,t)$ has a discontinuity, $S(x,t)$ is continuous but loses its smoothness.

Consider a condition on the characteristic boundary Γ

$$S\big|_{t=t_\Gamma(x)} = p(x),\ 0 < x < 1 \,. \tag{6}$$

In the domain Ω_S, the solution of the Goursat problem (1) - (3), (6) coincides with the solution of the original problem (1) - (4) and is smooth. Analytical formulas for the asymptotics of the problem (1) - (3), (6) are obtained below.

3. Asymptotics near the inlet

In the vicinity of the porous medium inlet $x = 0$, the asymptotic solution is constructed in the form

$$C(x,t) = 1 + c_1(x,t)x + O(x^2); \qquad S(x,t) = s_0(x,t) + s_1(x,t)x + O(x^2) \,. \tag{7}$$

Assume that the coefficients of the system (1), (2) can be expanded in the series in powers of the small *x:*

$$\begin{aligned} g(S) &= g(s_0) + g'(s_0)s_1 x + O(x^2) \,; \\ f(S) &= f(s_0) + f'(s_0)s_1 x + O(x^2) \,; \\ \Lambda(S) &= \Lambda(s_0) + \Lambda'(s_0)s_1 x + O(x^2) \,. \end{aligned} \tag{8}$$

Substituting the expansions (7), (8) into the system (1), (2) and equating terms with the same powers of x, we obtain a recurrent system of equations

$$s_0' = \Lambda(s_0) \,; \tag{9}$$

$$g'(s_0)s_0' + f'(s_0)s_1 + f(s_0)c_1 + \Lambda(s_0) = 0 \,; \tag{10}$$

$$\frac{\partial s_1}{\partial t} = \Lambda'(s_0)s_1 + \Lambda(s_0)c_1 \,. \tag{11}$$

Expression of c_1 from equation (10)

$$c_1 = -\frac{g'(s_0)s_0' + f'(s_0)s_1 + \Lambda(s_0)}{f(s_0)} \,, \tag{12}$$

and substitution into (11) gives

$$\frac{\partial s_1}{\partial t} = P(s_0)s_1 - Q(s_0) \,, \tag{13}$$

where

$$P(s_0) = \frac{\Lambda'(s_0)f(s_0) - \Lambda(s_0)f'(s_0)}{f(s_0)}; \quad Q(s_0) = \left(g'(s_0) + 1\right)\frac{\Lambda^2(s_0)}{f(s_0)}.$$

Expansion of the initial function (6) in a series in powers of x

$$p(x) = p_0 + p_1 x + O(x^2); \quad p_0 = p(0), \quad p_1 = p'(0), \tag{14}$$

and substitution of (7), (14) into condition (6) gives the initial conditions for the equations (9), (13)

$$s_0\big|_{t=t_\Gamma(x)} = p(x); \tag{15}$$

$$s_1\big|_{t=t_\Gamma(x)} = 0. \tag{16}$$

Integration of equation (9) with the initial condition (15) gives the solution $s_0(x,t)$ in an implicit form:

$$\int_{p(x)}^{s_0} \frac{\mathrm{d}S}{\Lambda(S)} = t - t_\Gamma(x). \tag{17}$$

In particular, the retained particles concentration at the inlet $x = 0$ is determined in implicit form by the formula [11]

$$\int_{p(0)}^{S(0,t)} \frac{\mathrm{d}S}{\Lambda(S)} = t. \tag{18}$$

The solution of (13), (16)

$$s_1 = -\int_{t_\Gamma(x)}^{t} \exp\left(\int_{\varsigma}^{t} P\big(s_0(x,\tau)\big)\mathrm{d}\tau\right) Q\big(s_0(x,\varsigma)\big)\mathrm{d}\varsigma. \tag{19}$$

The function c_1 is determined by the formula (12) with known functions s_0, s_1.

Formulas (12), (17), (19) determine the main terms of the asymptotics (7) in the domain Ω_S. These expansions satisfy the equations (1), (2) and conditions (3), (6) with accuracy $O(x^2)$.

Substitution of the expansions (8), (14) into the integrand in (5) gives

$$\frac{g\big(p(y)\big)}{f\big(p(y)\big)} = \frac{g(p_0) + g'(p_0)p_1 y + O(y^2)}{f(p_0) + f'(p_0)p_1 y + O(y^2)} = \frac{g(p_0)}{f(p_0)} + \frac{\delta p_1 y}{f^2(p_0)} + O(y^2).$$

Here $\delta = g'(p_0)f(p_0) - g(p_0)f'(p_0)$.

The asymptotics of the mobile two-phase boundary Γ is obtained by integrating (5)

$$t_\Gamma(x) = \frac{g(p_0)}{f(p_0)}x + \frac{\delta p_1 x^2}{2f^2(p_0)} + O(x^3). \tag{20}$$

The inverse formula is obtained by inverting the asymptotics (20)

$$x_\Gamma(t) = \frac{f(p_0)}{g(p_0)}t\left(1 - \frac{\delta p_1 t}{2g^2(p_0)}\right) + O(t^3). \tag{21}$$

Consider a simple system (1), (2) with constant coefficients $g(S) = f(S) = 1$ and a linear filtration coefficient $\Lambda(S) = a - bS, \ a > 0, \ b > 0$.

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial S}{\partial t} = 0; \tag{22}$$

$$\frac{\partial S}{\partial t} = (a - bS)C \,. \tag{23}$$

The mobile boundary Γ is the straight line $t = x$.

In the domain Ω_S the asymptotic terms of the equations (22), (23) with conditions (15), (16) have the form

$$s_0(t) = \frac{a - \left(a - bp(x)\right)e^{-b(t-x)}}{b}; \quad s_1(t) = -\frac{(a - bp_0)^2}{b}\left(1 - e^{-b(t-x)}\right)e^{-b(t-x)}; \quad c_1(t) = -(a - bp_0)e^{-b(t-x)} \,. \tag{24}$$

The asymptotics of the similar problem without the initial deposit is constructed in [12].

4. Numerical results

Many works are devoted to numerical modeling of filtration problems [13-16]. In this paper the asymptotic solution is calculated.

Calculation of the asymptotics of (22), (23) is performed for $\Lambda(S) = 1 - 0.5S;\ p(x) = 1 - 0.5x$. Fig. 1, 2 present the dependence of the suspended and retained particles concentrations on time in a fixed point *x*: $x = 0$ (solid line), $x = 0.25$ (broken line), $x = 0.5$ (dotted line). In Fig. 1 horizontal line corresponds to the maximum value of the deposit $S_M = 2$.

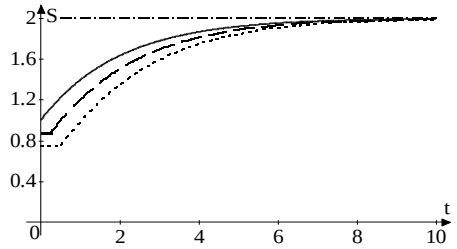


Figure 1. Retained concentrations $S(x_0, t)$.

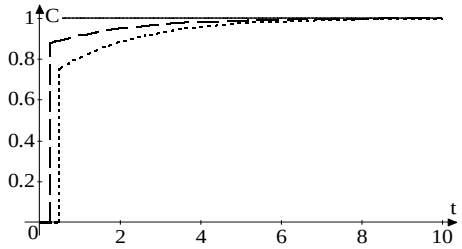


Figure 2. Suspended concentrations $C(x_0, t)$.

Fig. 1 and 2 show that for a blocking filtration coefficient $\Lambda(S) = 1 - 0.5S$ the suspended particles concentration $C(x,t) \to C(0,t) = 1$, and the retained particles concentration $S(x,t) \to 2$ with an unlimited increase in time *t*.

Fig. 3 and 4 characterize the dependence of the suspended and retained particles concentrations on the coordinate *x* at the fixed moment of time $t = 0$ (solid line), $t = 0.5$ (dashed line), $t = 1$ (dotted line), $t = 2$ (dot-dash).

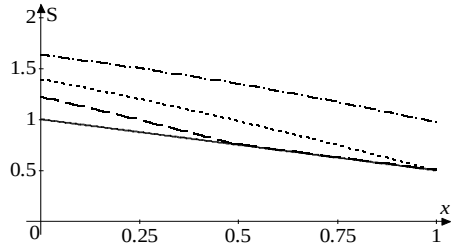


Figure 3. Retained concentrations $S(x, t_0)$.

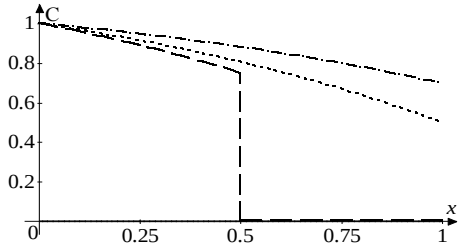


Figure 4. Suspended concentrations $C(x, t_0)$.

For a constant time $t_0 = const$, the suspended and retained particles concentrations decrease in coordinate *x*, since the filtration process expands from the inlet $x = 0$. As the time t_0 increases, the graphs approach the limiting values $C_M = 1$ and $S_M = 2$.

5. Discussion

A filtration model for a porous medium with partially blocked pores is proposed, which generalizes the standard model for the filtration of suspension [17]. The proposed model determines the step of the batch process, in which the injection of the suspension into a porous medium is replaced by a flow of pure water, ejecting the suspension from the porous medium.

The presence of an initial deposit means that the porous medium is inhomogeneous. The exact solution of this mathematical problem cannot be given by a finite analytical formula. To obtain an approximate solution, the asymptotic methods are used [18, 19].

The asymptotics determine a solution near the inlet of the porous medium. Asymptotic solution determines an asymptotic expansion of the curvilinear mobile two-phase boundary with high accuracy.

The local asymptotics is obtained from a system of recurrent differential equations with conditions on the mobile boundary, which is far from the inlet of the porous medium. To prove the applicability of the asymptotics, it is required to show that it is an expansion of the exact solution of the original problem. The proof of the correctness of the asymptotics requires a separate study.

6. Conclusions

It is shown that the mobile two-phase boundary, separating the suspension from pure water, moves with variable speed. The concentration of suspended particles is discontinuous on the boundary, and the concentration of retained particles is continuous and loses its smoothness. The exact explicit formula for the two-phase boundary and its asymptotics in a form convenient for calculations are obtained. An exact solution is obtained at the entrance of the porous medium and the asymptotics of the filtration problem is constructed near the porous medium inlet. Numerical calculation of the asymptotic solution is performed; graphs of the suspended and retained particles concentrations depending on time and coordinate are presented.

It is proved that in the case of the blocking filtration coefficient, the suspended and retained particles concentrations of the suspension match natural physical conditions. With an unlimited increase in time, the suspended particles concentration tends to a constant concentration at the entrance of the porous medium, and the retained particles concentration approaches the maximum value of the deposit.

In contrast to the numerical solution, analytical methods make it possible to determine the dependence of the solution of the filtration problem on the controlled external parameters. This allows the construction engineers to choose the best size of injected grout grains and the properties of the carrier fluid, optimize the filtration process and form a grouted porous soil of the required strength and density [20].

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