

The estimation of spatial cracks formation in reinforced concrete structures under the action torsion with bending

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Abstract. It is proposed a computational model for the spatial cracks formation of the first, second and third types in reinforced concrete structures under the action torsion with bending, based on working assumptions and solving equations, with allowance for physical nonlinearity, deplaning cross sections, prestressing in the longitudinal and transverse reinforcement and the influence of the local stress field. Solving equations are constructed in a special way, so that their system does not become decaying for functions of many variables with the Lagrange multipliers use for finding the minimum generalized load corresponding to the spatial crack formation of the first, second and third types and the coordinates of their formation points. It is obtained experimental data for checking the proposed design model, including the values of the general crack-forming load, the coordinates of the points of crack formation along the axes (x ; y , z); cracks and angles of their tilt; distance between cracks at different levels of cracking; the width of the opening of the spatial cracks along the entire helical-shaped line; angles of twisting, deflections, deformation of concrete and reinforcement.

1. Introduction

As one of the main directions in the development of technical progress of concrete and reinforced concrete, as noted by a number of international conferences of recent years, is the deepening and improvement of theoretical studies of the work of reinforced concrete structures with various combinations of force effects [1–3]. A very common form of such a combination is the joint action on the construction of bending and torque (beams of monolithic overlays, onboard elements and support contour of ribbed carvings and bridges, substructure and crane beams, contour beams of buildings with a monolithic frame, supports of power lines, reinforced concrete pylons and etc.).

At the same time, since in the case of torsion with bending, the moment of cracks formation and the magnitude of the angle of their inclination significantly affect the stress state [4], first of all, there is a need to study the problems of cracking, which to date have not been investigated sufficiently.

2. The purpose of scientific research

The purpose of the present research is to study the process and develop a calculated model for the formation of spatial cracks in reinforced concrete structures in torsion with bending [5, 6] over a wide range of ratios of active twisting and bending moments, spans of their joint action and other significant parameters of the stress-deformed state.

3. The scientific novelty of the work consists of:



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- the calculated model of spatial cracks formation of the first, second and third types in reinforced concrete structures under the action torsion with bending, based on working conditions and permitting equations, taking into account physical nonlinearity, deplanation of transverse sections, preliminary stress in longitudinal and transverse reinforcement and the influence of the local stresses fields [7];
- resolving equations, constructed in a special way, so that their system does not turn into decomposable for functions of many variables using lagrange multipliers in finding the minimal generalized load corresponding to the formation of the first spatial crack of the first, second and third types and the coordinates of their formation points [5–8].

4. The main part

The basis of the construction of the calculated crack-forming model under the action torsion with bending for spatial cracks of the first type is the following calculation prerequisites [7]:

- formation of the first type spatial crack (crossing only longitudinal reinforcement at $M > M_{crc}$, $M_t > M_{t,crc}$ and $Q \geq Q_{crc}$), second or third type (crossing only transverse reinforcement at $M < M_{crc}$, $M_t > M_{t,crc}$ and $Q > Q_{crc}$ and oriented at further development in the direction of the point of application of the concentrated force or with an arbitrary orientation, respectively) occurs at an arbitrary point A located on the lower or lateral faces or at an arbitrary point of the complex figure of the cross-section (Fig. 1).

- diagrams of tangential stresses in torsion τ_t ; positive and negative zones of deplanation of transverse rectangular sections and the approximation of the figure of a cross section using the squares of the $ABCDEFGH$ and the inscribed circles, are executed in accordance with the diagrams of Fig. 2, *a, b*;

- the diagrams of normal σ_x and tangential stresses τ_{zx} in the cross-section, passing through the point A are approximated between the points 1 and 2 (Fig. 2, *c, d*) by linear dependences.

The following equations are used to evaluate the resistance of rod concrete structures to the formation of the first spatial crack.

- 1). Equation of communication between normal stresses σ_x in a cross-section located at a distance from x the support and a generalized external load expressed through the support reaction R_{sup} at the time of the formation of the first spatial crack including the bending moment from the external forces $R_{sup} \cdot x$, the bending moment $P_0 \cdot e_{0,p}$ from the prestress force, the longitudinal force N and the force preliminary stresses P_0 , as well as local influences $\frac{R_{sup}}{b \cdot h} \cdot \varphi_x$ (φ_x – the coefficient of calculation of local normal stresses σ_x in the direction of the x axis from the reference reactions), of which follows:

$$R_{sup} = \frac{\sigma_x \cdot A_{red} \cdot 0,85I_{red} - N \cdot 0,85I_{red} - P_0 \cdot e_{0,p} \cdot A_{red} \cdot 0,5h}{x \cdot 0,5h \cdot A_{red} + \varphi_x \cdot 0,85I_{red}}. \quad (1)$$

Note: in the formulas (1)–(21) below, all notations that are not deciphered correspond to the common notions of the mechanics of a solid deformable body and the theory of reinforced concrete.

- 2). The equations for determining the tangential torsion stresses τ_t in the cross-section, located at a distance x from the support, are recorded in accordance with Fig. 2

$$\tau_t = \tau_{t,j} = \frac{M_{t,j}}{I_{t,j}} \sqrt{(\xi z)^2 + y^2} \leq \tau_{t,u}. \quad (2)$$

Here, the moment of torsion inertia in the general case of a complex cross-section consisting of rectangles is equal to the sum of the moments of inertia of the squares on which the rectangles are broken down with their subsequent approximation by the circles inscribed in these squares (in this case, the superimposed part of the intersecting sections in the summation is included with the sign

"minus", And the angular sections in view of their insignificant influence on the values of tangential stresses are not taken into account, Fig. 2, b), –

$$I_t = I_{t,1} + I_{t,2} + \dots + I_{t,j} = \sum I_{t,j}, \quad (3)$$

and each of the torsional moments incident on the inscribed circles are respectively determined, –

$$M_{t,1} = M_t \frac{I_{t,1}}{I_t}; \quad M_{t,2} = M_t \frac{I_{t,2}}{I_t}; \quad \dots M_{t,j} = M_t \frac{I_{t,j}}{I_t}; \quad (4)$$

$I_{t,j}$ – the moment of inertia of the circle of the circle used in formula (2) inscribed in the corresponding square (the lower circle is used, as a rule, for cracks of the first type, the middle circle is used for the second and third types, fig. 2, b); ζ – coefficient of transition to local axes; $\tau_{t,u}$ – the limiting tangential stress caused by torsion.

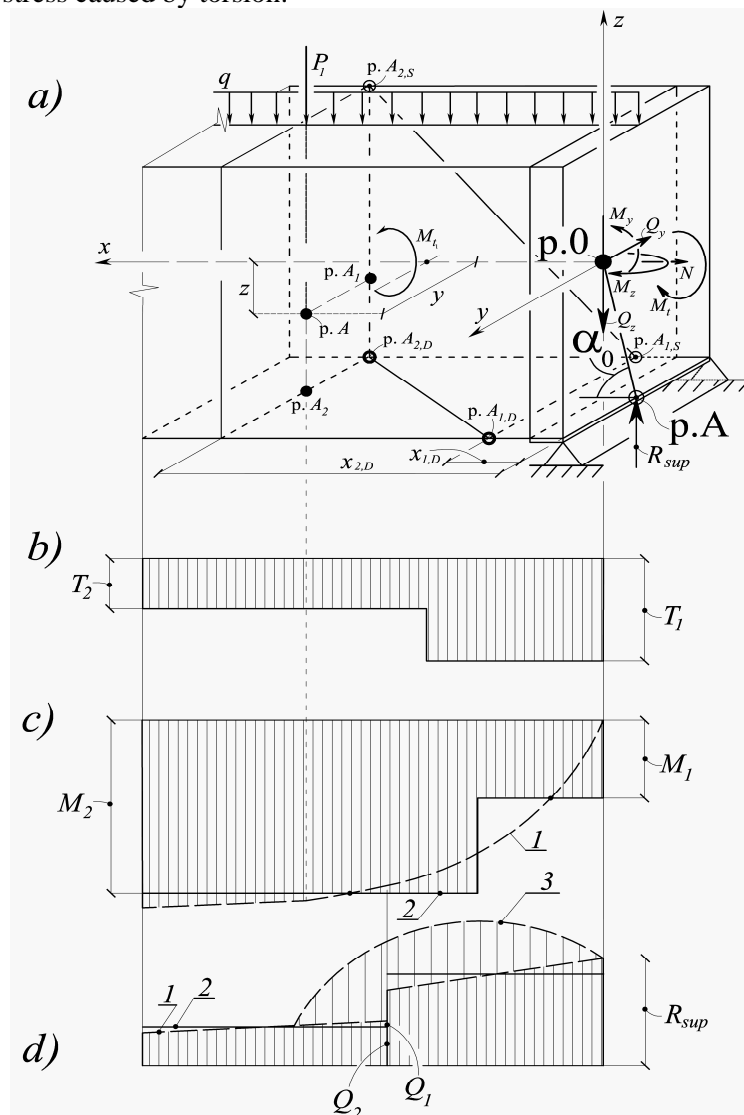


Figure 1. The calculation scheme for the spatial cracks formation of the first, second and third types:

a – the scheme of forces and the choice of the coordinate system to the formation of the first spatial crack; b, c, d – diagrams M_t , M_y and Q_z respectively;

1 – the actual diagram; 2 – accepted for calculation; 3 – diagram of transverse forces from the local field of tangential stresses.

In order to take into account plastic deformations, the moment of inertia $I_{t,j}$ and I_j is recommended to be simplified in the cracking stage:

$$I_{t,j} = 0,85 \cdot I_{t,j,red} ; \quad I_j = 0,85 \cdot I_{j,red} ; \quad (5)$$

for dependencies in which the parameter $I_{t,j}$ does not belong to the same recommendation can be used with respect to the modules E and G , with respect to the stage of crack formation, and in subsequent loading stages, it is necessary to use the secant strain modulus and the variable coefficient of transverse strains for concrete in the calculated dependences.

For cracks of the second and third types, the unknown coordinate y from equation (2) is determined:

$$y = \frac{\sqrt{\tau_t^2 \cdot I_{t,j}^2 - M_{t,j}^2 \cdot (\zeta \cdot z)^2}}{M_{t,j}} \leq 0,5b . \quad (6)$$

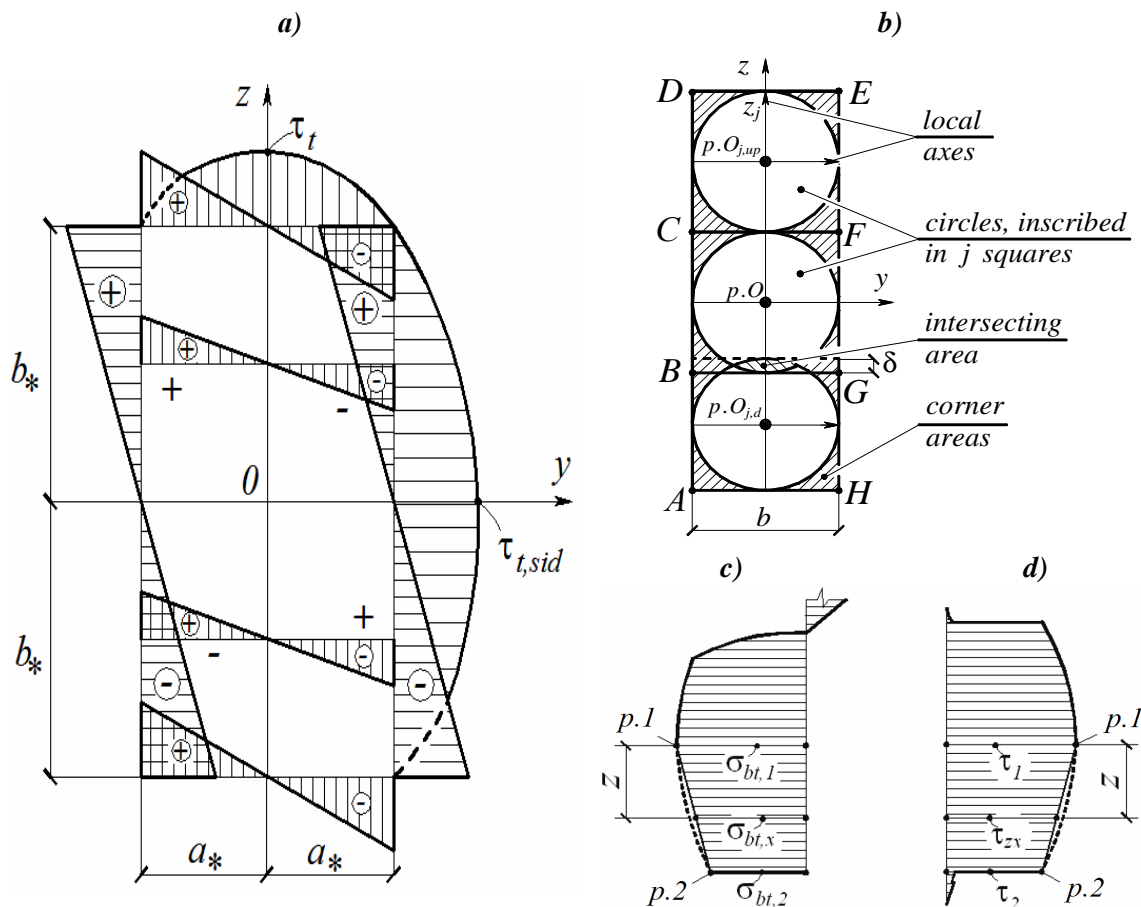


Figure 2. Diagrams of tangential tensions in torsion τ_t , positive and negative zones of deformation of transverse rectangular sections (a); approximation of the $ABCDEFGH$ figure of the cross-section with the help of squares and inscribed circles (b), and the diagrams of the normal and tangential stresses in the cross section passing through the point A (b, c), respectively.

3). Equations for the determination of tangential stresses τ_{xz} in a cross section located at a distance x from the support. In this case, the coupling equations between tangential stresses in the cross-section of the reinforced concrete rod and the generalized load $R_{sup} - \tau_2$ and τ_1 take into account the transverse force from the reference reaction (taking into account local stresses) and the transverse force perceived by the bent rod.

From this equation, the unknown coordinate z of the formation of spatial cracks of the second and third types is determined:

$$z = \frac{(R_{sup} - Q_{inc})B_1 + \frac{R_{sup}}{A_{red}} \cdot \varphi_{xz} - \tau_{xz}}{(R_{sup} - Q_{inc}) \cdot B_2} \leq 0,5h, \quad (7)$$

where φ_{xz} is the coefficient of local shear stress τ_{xz} in the z direction from the reference reactions.

With respect to cracks of the first type, there is no need in finding the coordinate z (the equation degenerates into an equality $z = -0,5h$), in this case it is expedient to use equation (7) for determination. After algebraic transformations, with respect to cracks of the first type, we get:

$$\tau_{zx} = (R_{sup} - Q_{inc})(-0,5h \cdot B_2 - B_1) - \frac{R_{sup}}{A_{red}} \cdot \varphi_{zx} \leq \tau_u. \quad (8)$$

Here the parameter B_1 is determined by the formula:

$$B_1 = \frac{S_{n.ax}}{0,85I_{red} \cdot b}; \quad (9)$$

the parameter B_2 for the T-section (shelf above), and the T-section (the shelf from below), is determined by the formulas, respectively, –

$$B_2 = \frac{1}{0,85I_{red} \cdot b \cdot (h - z_d - h'_f)}; \quad (10)$$

$$B_2 = \frac{1}{0,85I_{red} \cdot b \cdot (z_d - h_f)}, \quad (11)$$

z_d – the distance from the center of gravity of the section to the lower face, the parameter h'_f for rectangular sections is assumed to be equal $(h - z_d)/3$, and the parameter h_f is assumed to be equal to $z_d/3$; τ_u – the limiting tangential stress caused by transverse forces.

4). The equation of connection of the external load (expressed through the reference reaction R_{sup}) and normal stresses σ_z , is recorded taking into account local stress fields from the reference reaction and applied to the construction of the concentrated force, and also taking into account the prestress in the clamps and bends. From this equation, with respect to the spatial cracks of the first, second and third types, the unknown σ_z is determined:

$$\sigma_z = \frac{R_{sup}}{A_{red}} \cdot (\varphi_z + k \cdot \varphi_{2,z}) + B_3. \quad (12)$$

Here

$$B_3 = \frac{\sigma_{sw,p} \cdot A_{sw,p}}{s_{sw,p} \cdot b} + \frac{\sigma_{inc,p} \cdot A_{inc,p}}{s_{inc,p} \cdot b} \cdot \sin \theta, \quad (13)$$

φ_z – the coefficient of local normal stresses σ_z in the direction of the z axis from the reference reactions; $k \cdot \varphi_{2,z}$ – the coefficient of local normal stresses σ_z in the direction of the z axis from the concentrated forces.

5). Using the precondition that the main deformations of the elongation of concrete reach their limiting values in the formation of spatial cracks, applied to spatial cracks of the first type, using the following dependencies for stresses and deformations, and bearing in mind that $\sigma_y = 0$, $\tau_{yx} = 0$; $\tau_{zy} = \tau_{yz} = 0$;

$$\gamma_{yz} = 0; \quad \varepsilon_y = -\frac{\mu}{0,85E}(\sigma_x + \sigma_z); \quad \varepsilon_x = \frac{1}{0,85E}[\sigma_x + \sigma_{x,d} - \mu\sigma_z]; \quad \varepsilon_z = \frac{1}{0,85E}[\sigma_z - \mu\sigma_x]; \quad \varepsilon_{x,d} = \frac{w}{x}$$

(deformation deformation of the cross-section of the structure); $w = \frac{M_t}{G \cdot I_t} \cdot f(x, z)$; $f(x, z) = \beta_t \cdot x \cdot z$,

$\beta_t = \frac{a_*^2 - b_*^2}{a_*^2 + b_*^2}$, $\sigma_{x,d} = \varepsilon_{x,d} \cdot 0,85E$, after algebraic transformations, a formula is obtained for

determining the axial deformations of the elongation of concrete in the direction of the axis x :

$$\varepsilon_x = \frac{\gamma_{xz}^2 \left(\varepsilon_y - \varepsilon_{bt,ul} \right) + 4\varepsilon_{bt,ul}^3 - 4\varepsilon_{bt,ul}^2 \left(\varepsilon_y + \varepsilon_z \right) + \varepsilon_{bt,ul} \left(4\varepsilon_z \varepsilon_y - \gamma_{xy}^2 \right) + \varepsilon_z \cdot \gamma_{xy}^2}{4 \left(\varepsilon_z \varepsilon_{bt,ul} - \varepsilon_{bt,ul}^2 + \varepsilon_y \varepsilon_{bt,ul} - \varepsilon_y \varepsilon_z \right)}. \quad (14)$$

Then, using the connection $\sigma_x - \varepsilon_x$, we get:

$$\sigma_x = \varepsilon_x \cdot 0,85E - \sigma_{x,d} + \mu \sigma_z. \quad (15)$$

For fractures of the second and third types, other equations are used. In particular, to determine the normal stresses σ_x , the condition is used to achieve the values of the principal tensile stresses equal to R_{bt} . Then, from the equation for determining the principal tensile stresses, taking into account that $\sigma_y = 0$, $\tau_{zy} = \tau_{yz} = 0$, $\tau_{yx} = \tau_t$, $S_1 = \sigma_x + \sigma_z$, $S_2 = \sigma_x \sigma_z - \tau_{zx}^2$; $S_3 = 0$, after algebraic transformations we obtain:

$$\sigma_x = \frac{\tau_{zx}^2 + R_{bt} \cdot \sigma_z - R_{bt}^2}{\sigma_z - R_{bt}}. \quad (16)$$

From the equation for determining the main deformations of elongation of concrete (taking them equal $\varepsilon_{bt,ul}$), angular deformations γ_{xz} are sought. As a result, using the dependence $\tau_{xz} - \gamma_{xz}$, for tangential stresses τ_{xz} in the cross section, the following equation is obtained:

$$\tau_{xz} = \pm \frac{0,85E}{(1+\mu)} \left(\frac{4\varepsilon_{bt,ul} \varepsilon_x \varepsilon_z - 4\varepsilon_{bt,ul}^2 (\varepsilon_x + \varepsilon_z) + 4\varepsilon_{bt,ul} \varepsilon_x \varepsilon_y \varepsilon_z \cdot (\varepsilon_z + \varepsilon_x) - \varepsilon_y (4\varepsilon_{bt,ul}^2 + 4\varepsilon_x \varepsilon_z)}{(\varepsilon_{bt,ul} - \varepsilon_y)} \right)^{1/2} \leq \tau_u. \quad (17)$$

6). Note that tangential torsional stresses τ_t applied to cracks of the first type are determined in accordance with formula (2), and applied to cracks of the second and third type τ_t , are determined from the relationship between the bending and torsion moments $M_{bend} / M_t = \eta$:

$$\tau_t = \tau_{t,j} = \frac{(R_{sup} \cdot x)_j \cdot \sqrt{(\zeta \cdot z)^2 + y^2}}{I_{t,j} \cdot \eta} \leq \tau_{t,u}. \quad (18)$$

7). Using the resulting equations and the relationship between the bending (M_{bend}) and torque (M_t) moment $M_{bend} / M_t = \eta$, with respect to the cracks of the second and third types, we form the function of several variables $F(R_{sup}, y, z, \sigma_z, \sigma_x, \tau_{xz}, \tau_t, x, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7)$, which has the following form:

$$F(R_{sup}, y, z, \sigma_z, \sigma_x, \tau_{xz}, \tau_t, x, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7) = \frac{\sigma_x \cdot A_{red} \cdot 0,85I_{red} - N \cdot 0,85I_{red} - P_0 \cdot e_{0,p} \cdot A_{red} \cdot z}{x \cdot z \cdot A_{red} + \varphi_x \cdot 0,85I_{red}} +$$

$$+ \left[y - \frac{\sqrt{\tau_t^2 \cdot I_{t,j}^2 - M_{t,j}^2 (\zeta \cdot z)^2}}{M_{t,j}} \right] \lambda_1 + \left[z - \frac{\left((R_{sup} - Q_{inc}) B_1 + \frac{R_{sup}}{A_{red}} \cdot \varphi_{xz} - \tau_{xz} \right)}{(R_{sup} - Q_{inc}) \cdot B_2} \right] \lambda_2 +$$

$$\begin{aligned}
& + \left[\sigma_z - \left(\frac{R_{sup}}{A_{red}} \cdot (\varphi_z + k \cdot \varphi_{2,z}) + B_3 \right) \right] \lambda_3 + \left[\sigma_x - \frac{\tau_{zx}^2 + R_{bt} \cdot \sigma_z - R_{bt}^2}{\sigma_z - R_{bt}} \right] \lambda_4 + \\
& + \left[\tau_{xz} - \frac{0,85E}{(1+\mu)} \left(\frac{4\varepsilon_{bt,ul} \varepsilon_x \varepsilon_z - 4\varepsilon_{bt,ul}^2 (\varepsilon_x + \varepsilon_z) + 4\varepsilon_{bt,ul} \varepsilon_x \varepsilon_y \varepsilon_z \cdot (\varepsilon_z + \varepsilon_x) - \varepsilon_y (4\varepsilon_{bt,ul}^2 + 4\varepsilon_x \varepsilon_z)}{(\varepsilon_{bt,ul} - \varepsilon_y)} \right)^{1/2} \right] \lambda_5 + \\
& + \left[\tau_t - \frac{(R_{sup} \cdot x)_j \cdot \sqrt{(\zeta \cdot z)^2 + y^2}}{I_{t,j} \cdot \eta} \right] \lambda_6 + \left[R_{sup} - \frac{\sigma_x \cdot A_{red} \cdot 0,85I_{red} - N \cdot 0,85I_{red} - P_0 \cdot e_{0,p} \cdot A_{red} \cdot z}{x \cdot z \cdot A_{red} + \varphi_x \cdot 0,85I_{red}} \right] \lambda_7. \quad (19)
\end{aligned}$$

Performing the differentiation of the function (19) with respect to the corresponding variables and equating their derivatives with zero, an additional system of equations is obtained using the Lagrange multipliers λ_i . From the solution of the additional system of equations, a formula is obtained for determining the coordinate x of the point of the spatial crack formation:

$$x = \frac{M_{t,j} \cdot I_{t,j} \cdot \eta \sqrt{(\zeta \cdot z)^2 + y^2} \cdot \sqrt{\tau_t^2 \cdot I_{t,j}^2 - (\zeta \cdot z)^2}}{\tau_t \cdot R_{sup} \cdot y}. \quad (20)$$

Similarly, a function of several variables F_1 is compiled, differentiation with respect to the corresponding variables is performed with the equating of the derivatives to zero, and the coordinate x of the point of the first type spatial crack formation is determined.

$$x = \frac{0,85I_{red} [\varphi_x + \varphi_z + k \cdot \varphi_{2,z} - \mu \cdot \varphi_z - \mu \cdot k \cdot \varphi_{2,z}]}{0,5h \cdot A_{red}}. \quad (21)$$

As a result, all the resolving equations and the parameters determined from them turn out to be "closed" in a single solution to the problem associated with determining the minimum generalized load and the coordinates of the formation of spatial cracks of various types in reinforced concrete structures under the action torsion with bending.

After finding the abscissa x of the point A (Fig. 1), in which a spatial crack of the first, second or third type is formed and the search for the general cracking load expressed as a function through the support reaction R_{sup} , the spatial arrangement of the main sites is determined, in the vicinity of this point and the direction of development of the spatial crack.

In this case, the direction cosines l , m , n are found from the stress state equations on the principal and axial areas and the condition of equality to the unit squared direction of the cosines, taking into account that for the problem under consideration $\sigma_y = 0$, $\tau_{zy} = \tau_{yz} = 0$, $\tau_{yx} = \tau_t$, $\sigma = \sigma_1 = \beta \cdot R_{bt}$ (β is the coefficient taking into account the reduction of the limiting principal (minimum) tensile stresses compared with normal tensile stresses $\sigma_x = R_{bt}$).

The physical interpretation of the solution obtained is that it allows us to determine the minimum generalized load, which corresponds to the formation of the first spatial crack at an arbitrary point in the structure and the point of the structure and the corresponding coordinates of its formation.

5. Conclusions

Based on the analysis of the existing scientific research and regulatory documents, scientific developments of domestic and foreign scientists [1–3, 9, 10] devoted to the investigation of reinforced concrete beams in conditions of complex resistance – torsion with bending and the experimental [4] and theoretical studies performed in this paper, it is necessary to do the following conclusions.

1). At the present time in Russia and abroad there are no sufficiently stringent recommendations and corresponding normative documents for determining the limiting states of the first and second group for reinforced concrete structures operating under conditions of a complex stress-strain bending state with torsion. The current normative documents rely either on too simplified models and do not reflect

the actual resistance, or they do not give a clear algorithm for their calculation, and first of all in the investigation of fracture problems, since when torsion with bending the moment of crack formation and the magnitude of their inclination significantly influence the further tensile-deformed state.

2). In the conducted research the classification of spatial cracks in reinforced concrete constructions under the action torsion with bending is generalized; a design model of the spatial cracks formation of the first, second and third types under the action torsion with bending is constructed, based on the criterion for the formation of a spatial crack in the form of the condition for achieving the ultimate values of the elongation of concrete ε_{bt} by its main deformations $\varepsilon_{bt,ul}$. Equations are constructed in such a way that the resolving system does not turn into a decaying one. In this case, the physical dependences are taken into account in the physical nonlinearity, cross-section deplaning, preliminary stress in the longitudinal and transverse reinforcement and the influence of local stress fields. The physical interpretation of the solution obtained is that it allows us to find the minimum generalized load that corresponds to the first spatial crack formation of the first, second, third types and the coordinates of their formation point.

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