

# The field of crack directions in reinforced concrete bending elements

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**Abstract.** The evaluation of the design projection is essential in design of bending elements for inclined sections. A new method of determination of the inclination angle of a crack at any point along the span of an element is proposed. The design angle of inclination corresponds to the minimum of load causing its opening, which is determined from the equation of balance of energy. The experimental tests of proposed formula have been carried out and they have approved the theoretical data.

## 1. Introduction

A great number of investigations are devoted to shear behavior of bending elements. One of the most essential factors in the design of bending elements for inclined sections is the direction of dangerous shear crack, through which the ultimate forces in elements are estimated. The field of cracks directions is variable along the span of the element. Different models are proposed to determine trajectories of shear cracks [2], [3], [7], [8]. According to the current Russian code the value of projection of dangerous inclined section is estimated from the terms of minimal strength in shear force [1]. In this case the action of shear forces and bending moments in inclined section are considered separately. It is assumed that a shear force is carried by the concrete of compressed zone and shear reinforcement. Ultimate internal forces in concrete are determined on a basis of empirical formula.

A new method is offered allowing to determine the only possible direction of a crack propagation from a specific point in the span of bending element. The advantage of this approach is that the action of shear force and bending moment are considered simultaneously.

## 2. Main assumptions

It is supposed that:

- The crack can propagate from any point along the span of an element. The design direction of crack corresponds to the minimum of load causing its opening [4]. The value of that load is expressed from the equation of energy balance and varies by the angle of the inclination of a crack to horizontal line  $\alpha$ .
- The crack trajectory is approximated by straight line.
- The « $\sigma$ - $\varepsilon$ » diagram of concrete is parabolic under compression and under tension. The destruction of stretched concrete in a case of uniaxial stress state takes place when the ultimate stress  $R_{bt}$  and the ultimate strain  $\varepsilon_{ubt}$  are achieved.



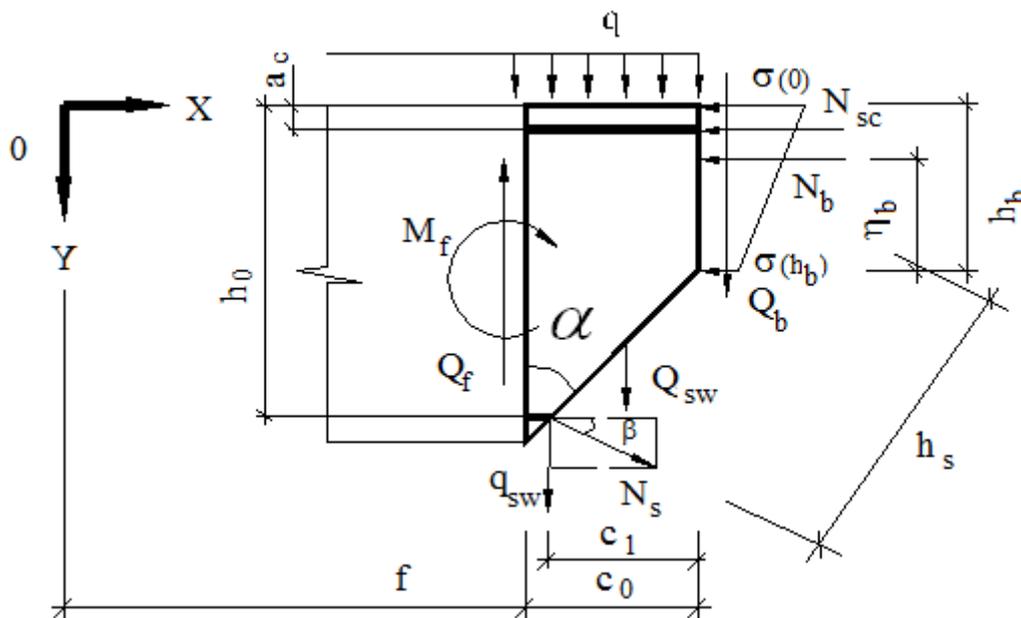
- The process of the crack formation ends before the development of large plastic deformations in the compressed area of concrete.
- The process of the crack formation ends before the achievement of ultimate shear strength in the compressed area of concrete, so the height of the compressed zone  $h_b$  doesn't depend on the angle  $\alpha$ .
- The process of the crack formation ends before the beginning of the plastic deformation of the longitudinal and shear reinforcement, crossing the crack. It means that the influence of plastic behavior of reinforcement on crack direction is not considered.
- The «moment  $M$  – curvature  $\chi$ » in inclined section with regard to the influence of stretched concrete behavior is approximated by the linear function.

**3. The method of determination of the design crack direction**

Let's suppose that the height of the compressed zone  $h_b$  above the crack is known and in doesn't depend on the angle  $\alpha$ . Then one can express the internal forces in trapezoidal element (fig. 1) from the equations of the balance and with regard to the stress-strain state of concrete above the inclined crack it is possible to determine its direction. The equation of conservation of energy

$$W_s + W_{sc} + W_{sw} + W_{sh} + W_{bt} + W_{bc} = A_q, \tag{1}$$

$W_s$  is the potential energy of the stretched longitudinal reinforcement deformation;  $W_{sc}$ ,  $W_{sw}$  are the potential energy of the compressed longitudinal reinforcement and of the shear reinforcement deformation.  $W_{bc}$  is the potential energy of deformation of the compressed concrete above the crack,  $W_{bt}$  is the potential energy of destruction of stretched concrete,  $W_{sh}$  is the potential energy of shift of concrete above the crack,  $A_q$  – the work of the external loads.



**Figure 1.** The scheme of forces in trapezoidal element above the crack

The potential energy of the stretched longitudinal reinforcement deformation

$$W_s = \frac{1}{2} \cdot N_s \cdot \epsilon_s = \frac{1}{2} E_s \cdot A_s \cdot \epsilon_s^2$$

$E_s, A_s, \varepsilon_s$  are the modulus of elasticity, the area and the longitudinal strain of the stretched reinforcement correspondingly.

The potential energy of the compressed longitudinal reinforcement deformation

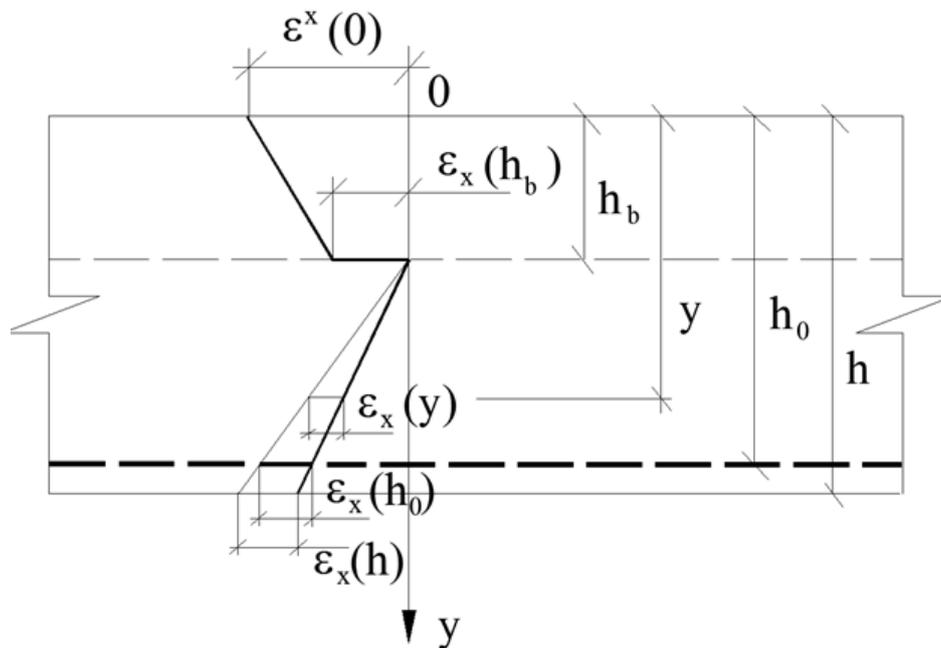
$$W_{sc} = \frac{1}{2} \cdot N_{sc} \cdot \varepsilon_{sc} = \frac{1}{2} E_{sc} \cdot A_{sc} \cdot \varepsilon_{sc}^2, \tag{3}$$

$E_{sc}, A_{sc}, \varepsilon_{sc}$  are the modulus of elasticity, the area and the longitudinal strain of the compressed reinforcement correspondingly.

In order to obtain the strains in inclined section the hypothesis of bilinear sections has been used [3]. In normal section the strains in each fiber can be calculated by the formula:

$$\begin{cases} \varepsilon_x(y) = \varepsilon_x(h_b) + A \cdot (y - h_b) \cdot \chi, & (0 \leq y \leq h_b) \\ \varepsilon_x(y) = (y - h_b) \cdot \chi, & (h_b < y \leq h) \end{cases} \tag{4}$$

$A$  is an empirical coefficient,  $\chi$  is a curvature. If  $A = 1$  the hypothesis of bilinear sections turns to the well-known hypothesis of flat sections.



**Figure 2.** The hypothesis of bilinear sections

As the compressing stresses act above the oblique fissure along its upper side and therefore the strains  $\varepsilon_l$  take place, then in the polyline section at the breaking point  $y = h_b$  the strains change the jump (fig. 2). The stresses  $\sigma_x(y)$  also change the jump in this point. For the polyline section the hypothesis is of the form:

$$\begin{cases} \varepsilon_x(y) = \varepsilon_x(h_b) + A \cdot (y - h_b) \cdot \chi, & (0 \leq y \leq h_b) \\ \varepsilon_x(y) = (y - h_b) \cdot \chi, & (h_b < y \leq h) \end{cases} \tag{5}$$

In this case

$$W_{sc} = \frac{1}{2} E_{sc} \cdot A_{sc} \cdot A^2 \cdot \left( \frac{1}{1-k_\varepsilon} - \frac{a}{h_b} \right)^2 \cdot \left( \frac{h_b}{h_0 - h_b} \right)^2 \cdot \varepsilon_s^2, \quad (6)$$

$a$  is a distance between the compressed face and the center of gravity of the compressed

$$k_\varepsilon = \frac{\varepsilon_x(h_b)}{\varepsilon_x(0)}$$

reinforcement,

The potential energy of the shear reinforcement deformation

$$W_{sw} = \frac{1}{2} \cdot Q_{sw} \cdot \frac{2}{3} \cdot \varepsilon_{sw} = \frac{1}{6} E_{sw} \cdot f_{sw} \cdot (h_0 - h_b) \cdot tg^3 \alpha \cdot \varepsilon_s^2, \quad (7)$$

$Q_{sw}$  is a total shear force in the shear reinforcement,  $\varepsilon_{sw}$  is a maximum strain of shear bars,  $E_{sw}$ ,  $f_{sw}$  are the modulus of elasticity and a distributed area of the shear reinforcement.

The potential energy of deformation of compressed concrete on height  $h_b$ .

$$W_{bc} = \frac{1}{2} \int_0^h \sigma_x(y) \cdot \varepsilon(y) dy, \quad (8)$$

$\sigma_x(y)$  and  $\varepsilon_x(y)$  – stresses and strains of concrete of compressed area.

Let's suppose that the process of the crack formation ends before the achievement of large plastic strains of compressed concrete

e. In this case

$$W_{bc} = \frac{1}{2} b \cdot h_b \cdot E_b \frac{\left( k_\varepsilon + (1-k_\varepsilon) \cdot \frac{1}{3} \right)}{(1-k_\varepsilon)^2} \cdot A^2 \cdot \left( \frac{h_b}{h_0 - h_b} \right)^2 \cdot \varepsilon_s^2 \quad (9)$$

The potential energy of shift of compressed concrete

$$W_{sh} = \frac{1}{2} \cdot Q_b \cdot \phi_{sh} = \frac{1}{2} \frac{Q_b^2}{b \cdot h_b \cdot G_b}, \quad (10)$$

$\phi_{sh}$  is a shift angle,  $G_b$  is the shear modulus of concrete;  $Q_b$  is the shear force in concrete of the compressed zone, which is estimated from the equation of balance of vertical forces.

The potential energy of the stretched concrete destruction

$$W_{bt} = \frac{2}{3} \cdot R_{bt} \cdot \varepsilon_{ubt} \cdot b \cdot l_{crc} = \frac{2}{3} \cdot R_{bt} \cdot \varepsilon_{ubt} \frac{b \cdot (h - h_b)}{\cos \alpha}, \quad (11)$$

$l_{crc}$  is the length of the oblique fissure.

The moment from the external loads  $M_f$  at a point with a coordinate  $f$ , which is carried by the inclined section, can be determined in terms of the value of bending moment  $M^*$  in any section and of a shape function of a diagram of bending moments  $\Phi(x)$ .

In this case the work of external loads

$$A_q = \frac{1}{2} \cdot M^* \cdot \Phi_{f+c} \cdot (1 + m_m) \cdot \frac{\varepsilon_{crc}}{l_{crc}}, \quad (12)$$

$\varepsilon_{crc}$  is a strain in direction transverse to the crack trajectory.

The trajectory of a dangerous oblique fissure is determined from the condition:

$$\frac{dM^*}{d\alpha} = 0 \quad (13)$$

#### 4. The influence of different factors on the inclination angle of cracks

In order to evaluate the influence of different factors on the angle of the inclination of a crack the computer program has been written and the design of hinged and clamped beams under uniform loads and concentrated forces has been carried out. The height to span ratio of the beam, the span of the slice value  $l_{sh}$ , the amount of longitudinal and shear reinforcement, the value of the coefficient  $A$  (of the hypothesis of bilinear sections) varied.

On a basis of the obtained result the following conclusions can be made:

1. The influence of the coefficient  $A$  of the hypothesis of bilinear sections is insignificant under uniform load. Under concentrated forces the influence of the value of the coefficient  $A$  on the angle  $\alpha$  is considerable in the area of  $0.4-0.8 l_{sh}$  if  $l_{sh}/h > 2$ .
2. The amount of longitudinal reinforcement has little effect on the inclination angle of expected crack.
3. Under uniform load the amount of shear reinforcement  $\mu_{sw}$  virtually has no effect on  $\alpha$  value of sections beginning within  $0.3-0.7 \cdot l$  area. Within  $0.2 \cdot l - 0.3 \cdot l$  area values of angle  $\alpha$  increase with increasing of  $\mu_{sw}$ . Near the supports (within  $0 - 0.15 \cdot l$  area) values of angle  $\alpha$  decrease with increasing of  $\mu_{sw}$ , and the oblique fissures projections on the horizontal axis decrease. With increasing of the  $l/h$  ratio, values of angle  $\alpha$  decrease in the middle of a span and increase near supports. In this case the area of cracks close to normal expands from the middle to supports. If  $l/h > 10$  big values of  $\alpha$  remain only in sections beginning from supports.
4. Under concentrated forces if  $l_{sh}/h \leq 2.0$  cracks are directed to the point of application of force and intersect at one point. If  $l_{sh}/h \geq 2.5$  angles of inclination of oblique fissures close to the point of application of force sharply decrease, and fissures become close to normal. With increasing of the slice value  $l_{sh}$  this effect expands towards supports. If  $\alpha \leq 50^\circ$  the amount of shear reinforcement virtually has no effect on its value. In sections beginning from supports the angle  $\alpha$  and the oblique fissures projection with increasing of  $\mu_{sw}$  decrease.

### 5. The experimental test

For the experimental verification of proposed formula the tests of 20 specimens on the action of static loads and of 14 specimens on the action of impact concentrated forces have been carried out [4], [5]. The results of the tests are presented in the table 1.

**Table 1.** The comparison of the experimental and the theoretical angles

| Chipper of a beam | $f/l$ | A    | hb/h0  | $\alpha$     |             |
|-------------------|-------|------|--------|--------------|-------------|
|                   |       |      |        | experimental | theoretical |
| 1                 | 2     | 3    | 4      | 5            | 6           |
| I-1-1             | 0.43  | 2.0  | 0.257  | 35           | 35          |
| I-1-2             | 0.69  | 0.4  | 0.483  | 30           | 30          |
| 1                 | 2     | 3    | 4      | 5            | 6           |
| I-2-1             | 0.38  | 1.9  | 0.259  | 30           | 33          |
| I-2-2             | 0.28  | 1.9  | 0.258  | 45           | 45          |
| I-3-1             | 0.    | 2.0  | 0.261  | 44           | 48          |
| I-3-2             | 0.4   | 2.2  | 0.251  | 33           | 34          |
| I-4-1             | 0.31  | 2.1  | 0.254  | 46           | 49          |
| I-4-2             | 1     | 2.0  | 0.252  | 1            | 1           |
| I-5-1             | 0.71  | 2.0  | 0.257  | 23           | 25          |
| I-5-2             | 0.64  | 0.6  | 0.428  | 37           | 37          |
| I-6-1             | 0.364 | 2.0  | 0.249  | 41           | 44          |
| I-6-2             | 1     | 2.0  | 0.273  | 1            | 1           |
| II-1-1            | 1     | 2.0  | 0.281  | 1            | 1           |
| II-1-2            | 1     | 2.0  | 0.281  | 1            | 1           |
| II-2-1            | 0.55  | 2.2  | 0.278  | 33           | 37          |
| II-2-2            | 0.9   | 2.0  | 0.289  | 10           | 13          |
| II-3-1            | 0.44  | 2.0  | 0.2825 | 40           | 42          |
| II-3-2            | 0.27  | 2.0  | 0.283  | 48           | 49          |
| II-4-1            | 0.68  | 0.8  | 0.400  | 40           | 40          |
| II-4-2            | 0.75  | 2.0  | 0.277  | 28           | 28          |
| II-6-1            | 0.54  | 2.1  | 0.275  | 46           | 47          |
| II-6-2            | 0.32  | 2.0  | 0.273  | 50           | 52          |
| III-1-1           | 0.23  | 2.0  | 0.23   | 39           | 43          |
| III-1-2           | 0.48  | 2.0  | 0.239  | 30           | 33          |
| III-2-1           | 0.43  | 1.3  | 0.286  | 38           | 37          |
| III-2-2           | 1     | 2.0  | 0.240  | 1            | 1           |
| III-3-1           | 0.91  | 1.75 | 0.251  | 11           | 10          |
| III-3-2           | 0.67  | 2.0  | 0.246  | 25           | 25          |
| III-4-1           | 1     | 2.0  | 0.238  | 1            | 1           |
| III-4-2           | 0.93  | 1.0  | 0.312  | 11           | 8           |
| III-5-1           | 1     | 2.0  | 0.242  | 1            | 1           |
| III-5-2           | 1     | 2.0  | 0.235  | 1            | 1           |
| III-6-1           | 1     | 2.0  | 0.238  | 1            | 1           |

On a basis of obtained results the following conclusions can be made:

1. The experimental values of angles are close to theoretical values.

2. The values of coefficient  $A$  of the hypothesis of bilinear sections for the cantilever beams with  $l/h$  ratio within from 0.8 to 2 (the conditions of the experiment) can be taken equal to 2, including normal forces.

### References

- [1] SP 63.13330.2012 *Concrete and won concrete construction. Design requirements.* Moscow: GUP CPP
- [2] Zalesov A S and Klimov O A 1989 *Shear strength of reinforced concrete constructions* «Budivel'nik»
- [3] Garnytsky V J 1988 *The development of theory of elasto-plastic reinforced concrete constructions design on dynamic loads*, doctoral thesis, Moscow Civil Engineering Insitute
- [4] Garnytsky V J 2014 *Shear strength of reinforced concrete structures along the sections, coincident with a field of inclined cracks* - Moscow: III All Russian (II International) Conference on concrete and reinforced concrete «Concrete and reinforced concrete – glance at future» Vol 1 pp.27-38
- [5] Garnytsky V J, Belikov A A and Kurnavina S O 2011 *Experimental investigations of reinforced concrete beams under shear forces* - Moscow: «Industrial and Civil engineering» journal., Vol. 3
- [6] Garnytsky V J, Belikov A A and Kurnavina S O 2017 *Failure mechanism of reinforced concrete beams on the inclined sections under the emergency loads-* Ivanono, Textile industry technology journal, Vol. 2
- [7] Kim J H and Mander J B 2000 *Theoretical crack angle in reinforced concrete elements subjected to strong earthquakes* - materials of 12th World Conference on Earthquake Engineering, Aucland, New Zealand
- [8] Caprinteri A, Carmona J R and Ventura G 2007 *Propagation of flexural and shear cracks through reinforced concrete beams by the bridge crack model* - Magazine of concrete research, ISSN 00249831 Vol. 59 No 10 December