

Target reliability of civil engineering structures

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Abstract. The structural reliability recommended in Eurocodes and other international documents vary within a broad range, while the reference to the failure consequences and design working life is mentioned only very vaguely. In some cases the target reliability indexes are indicated for one or two reference periods (in Eurocodes for 1 year and 50 years), however no explicit link to the design working life is usually provided. This article attempts to clarify the relationship between the target reliability levels, failure consequences, the design working life and the discount rate. The theoretical study based on probabilistic optimization is supplemented by recommendations useful for code makers and required by practicing engineers. It appears that the optimum reliability indices depend primarily on the ratio of the cost of structural failure to the cost per unit of structural parameter, and less significantly on the design working life and on the discount rate.

1. Introduction

The target reliability levels recommended in various national and international documents for new structures are inconsistent in terms of the values and the criteria according to which the appropriate values are to be selected. Almost no recommendations are available for temporary structures. In general, optimum reliability levels can be obtained by considering both the cost of the structure and the expected cost of failure over the design working life.

The design working life is understood as an assumed period of time for which a structure is to be used for its intended purpose without any major repair work being necessary. Indicative values of design working life (10 to 100 years for different types of new structures) are given in EN 1990 (2002) [2]. Recommended values of reliability indexes are given for two reference periods, 1 year and 50 years (see Table 1), without any explicit link to the design working life that generally differs from the reference period, while no specific indicative values are available for temporary structures.

It should be emphasized that the reference period is understood as a chosen period of time used as a basis for statistically assessing the time variant basic random variables, and the corresponding probability of failure. The concept of reference period is therefore fundamentally different from the concept of design working life. Confusion is often caused when the difference between these two concepts is not recognized.

It should be recognized that the couple of β values (for 1 year and 50 years) given in Table 1 for each reliability class correspond to the same reliability level. Practical application of these values, however, depends on the time period considered in the verification, which may be linked to available probabilistic information concerning time variant basic variables (imposed load, wind, earthquake,



etc.). It should be noted that the reference period of 50 years is also accepted as the design working life for common structures (see the discussion by Diamantidis (2009) [1]).

Table 1. Reliability classification according to EN 1990 (2002) [2].

| Reliability classes | Consequences of structural failure | Reliability index β for reference period | | Examples of buildings and civil engineering works |
|---------------------|------------------------------------|--|----------|---|
| | | 1 year | 50 years | |
| RC3 – high | High | 5.2 | 4.3 | Bridges, public buildings |
| RC2 – normal | Medium | 4.7 | 3.8 | Residences and offices |
| RC1 – low | Low | 4.2 | 3.3 | Agricultural buildings |

For example, considering a structure of reliability class 2 having a design working life of 50 years, the reliability index $\beta = 3.8$ should be used, provided that probabilistic models of basic variables are available for this period. The same reliability level is achieved when a reference period of 1 year, and a target of $\beta = 4.7$ are applied using the theoretical models for a reference period of one year. Thus, when designing a structural member, similar dimensions (reinforcement area) would be obtained considering $\beta = 4.7$ and basic variables related to 1 year or $\beta = 3.8$ and basic variables related to 50 years.

A more detailed recommendation concerning the target reliability is provided by ISO 2394 (1998) [7], where the target reliability indexes are indicated for the whole design working life without any restriction concerning its length, and are related not only to the consequences, but also to the relative costs of safety measures (Table 2).

Table 2. Life-time target reliability indexes β according to ISO 2394 (1998) [7].

| Relative costs of safety measures | Consequences of failure | | | |
|-----------------------------------|-------------------------|------|----------|-------|
| | small | some | moderate | great |
| High | 0 | 1.5 | 2.3 | 3.1 |
| Moderate | 1.3 | 2.3 | 3.1 | 3.8 |
| Low | 2.3 | 3.1 | 3.8 | 4.3 |

Similar recommendations are provided in the JCSS (2001) [8] Probabilistic Model Code (Table 3) based on the previous study of Rackwitz (2000) [9]. The recommended target reliability indexes are also related to both the consequences and to the relative costs of safety measures, though for a reference period of 1 year. The consequence classes in JCSS (2001) [8] (similar to EN 1990, 2002[2]) are linked to the ratio ρ defined as the ratio $(C_{str} + C_f) / C_{str}$ of the total cost induced by a failure (cost of construction C_{str} plus direct failure costs C_f) to the construction cost C_{str} as follows:

- Class 1 Minor Consequences: ρ is less than approximately 2; risk to life, given a failure, is small to negligible and the economic consequences are small or negligible (e.g. agricultural structures, silos, masts);
- Class 2 Moderate Consequences: ρ is between 2 and 5; risk to life, given a failure, is medium and the economic consequences are considerable (e.g. office buildings, industrial buildings, apartment buildings);
- Class 3 Large Consequences: ρ is between 5 and 10; risk to life, given a failure, is high, and the economic consequences are significant (e.g. main bridges, theatres, hospitals, high rise buildings).

However, it is not quite clear what is meant in JCSS (2001) [8] by “the direct failure costs”. This term indicates that there may be some other “indirect costs” that may affect the total expected cost. Here it

is assumed that the failure costs C_f cover all additional direct and indirect costs (except the structural costs C_{str}) induced by the failure. The structural costs are considered separately and related to the costs needed for an improvement of safety (costs per unit of decision parameter C_1).

Both the documents ISO 2394 (1998) [7] and JCSS (2001) [8] seem to recommend reliability indexes that are lower than those given in EN 1990 (2002) [2] even for the “small relative costs” of safety measures. It should be noted that EN 1990 (2002) [2] gives the reliability indexes for two reference periods (1 and 50 years) that may be accepted as the design working life for common structures (see also the discussion provided by Diamantidis (2009) [1]). ISO 2394 (1998) [7] recommends indexes for “life-time, examples”, thus related to the design working life, without any restrictions, while Probabilistic Model Code by JCSS (2001) [8] provides reliability indexes for the reference period of 1 year.

Table 3. Tentative target reliability indexes β (and associated target failure rates) related to one year reference period and ultimate limit states according to JCSS (2001) [8].

| Relative costs of safety measures | Minor consequences of failure | Moderate consequences of failure | Large consequences of failure |
|-----------------------------------|---------------------------------------|--|--|
| Large | $\beta = 3.1$ ($p \approx 10^{-3}$) | $\beta = 3.3$ ($p \approx 5 \times 10^{-4}$) | $\beta = 3.7$ ($p \approx 10^{-4}$) |
| Normal | $\beta = 3.7$ ($p \approx 10^{-4}$) | $\beta = 4.2$ ($p \approx 10^{-5}$) | $\beta = 4.4$ ($p \approx 5 \times 10^{-6}$) |
| Small | $\beta = 4.2$ ($p \approx 10^{-5}$) | $\beta = 4.4$ ($p \approx 5 \times 10^{-6}$) | $\beta = 4.7$ ($p \approx 10^{-6}$) |

However, a clear link between the design working life and the target reliability level is not apparent from any of the above-mentioned documents. Thus, it is not clear which target reliability index should be used for a given design working life different from 50 years (say 10 years).

A new promising approach to specify the target reliability based on the concept of Life Quality Index (Fischer et al., 2012) [3] is considered in an on-going revision of the International Standard ISO 2394 (1998) [7].

The basic aim of this contribution is to clarify the link between the design working life and the reliability index, and to provide guidance for specification of the target reliability level for a given design working life. The submitted theoretical study based on probabilistic optimization is supplemented by practical recommendations. This contribution is an extension of the previous study by Holicky and Retief (2011) [6], and Holicky [10].

2. General principles of probabilistic optimization

Probabilistic optimization may be based on a certain objective function adjusted to given condition of heritage structure. A simplified form (not covering monitoring and maintenance) may be expressed as the present value of the total expected cost $C_{tot}(x, o, q, n)$

$$C_{tot}(x, o, q, n) = C_{str} \sum_1^n P_f(x, i) Q(o, i) + C_f \sum_1^n P_f(x, i) Q(q, i) + C_0 + x C_1 \quad (1)$$

The cost of construction C_{str} including artistic value is discounted as it is paid in the future after number of years i . Here x denotes the decision parameter of the optimization (a parameter of structural resistance), o is the annual obsolescence (oldness) rate of heritage structure enhanced by annual discount rate q .

The cost of failure C_f including relevant artistic values is also discounted as it is paid after number of years i , q is the annual discount rate (without obsolescence rate o), e.g. 0.03, an average long run value of the real annual discount rate in European countries, n is the number of years to the failure, which may differ from the design working life (specified usually as 50 or 100 years).

Further, $P_f(x, i)$ is the failure probability in year i , $Q(o, i)$ is the discount factor dependent on the annual obsolescence rate o , $Q(q, i)$ is the discount factor dependent on the annual discount rate q and the number of years i , C_0 is the initial cost of intervention independent of the decision parameter x and

failure (a quantity not affecting the optimization), and C_1 is the cost per unit of the decision parameter x (a structural parameter quantity affecting the structural resistance and optimization).

Note that the design working life may generally differ from the time to failure denoted by the number of years n and considered here as an independent variable affecting the probability of failure. Maintenance and possible repair of the structure is not included in the objective function (1), and these aspects are to be considered in further studies. Assuming independent failure events in subsequent years, the annual probability of failure $P_f(x,i)$ in year i may be approximated by the geometric sequence

$$P_f(x,i) = p(x) (1 - p(x))^{i-1} \quad (2)$$

The initial annual probability of failure $p(x)$ is dependent on the decision parameter x . Note that annual failure probabilities can be assumed to be independent when failure probabilities are chiefly influenced by time-variant loads (climatic actions, traffic loads, accidental loads). Then the failure probability $P_{fn}(x)$ during n years can be estimated by the sum of the sequence $P_f(x,i)$, that can be expressed as

$$P_{fn}(x,n) = 1 - (1 - p(x))^n \approx n p(x) \quad (3)$$

Note that the approximation indicated in equation (3) is fully acceptable for small annual probabilities $p(x) < 10^{-3}$.

The discount factor of the present value of the expected future costs in year i is considered in the usual form as

$$Q(q,i) = 1 / (1+q)^i \quad (4)$$

Thus, the cost of malfunctioning C_f is discounted by the factor $Q(q,i)$ depending on the discount rate q and the point in time (year number defined as i) when the loss of structural utility occurs.

Considering equations (2) and (4) the total costs $C_{tot}(x,q,n)$ described by equation (1) may be written in a simplified form as

$$C_{tot}(x,o,q,n) = C_{str} PQ(x,o,n) + C_f p(x) PQ(x,q,n) + C_0 + x C_1 \quad (5)$$

Here the total sum of expected structural cost after n years depends on present structural cost C_{str} , the annual probability $p(x)$ and on the sum of the geometric sequence having the quotient $(1 - p(x))/(1 + o)$, denoted as the time factor $PQ(x,o,n)$. Similarly malfunctioning costs after of n years is dependent on the product of the present value of malfunction cost C_f , the annual probability $p(x)$ and a sum of the geometric sequence having the quotient $(1 - p(x))/(1 + q)$, denoted as the time factor $PQ(x,q,n)$:

$$PQ(x,o,n) = \frac{1 - \left[\frac{1 - p(x)}{1 + o} \right]^n}{1 - \left[\frac{1 - p(x)}{1 + o} \right]}, \quad PQ(x,q,n) = \frac{1 - \left[\frac{1 - p(x)}{1 + q} \right]^n}{1 - \left[\frac{1 - p(x)}{1 + q} \right]} \quad (6)$$

In general the total cost $C_{tot}(x,o,q,n)$ depends on the costs C_0 , C_1 , C_{str} , C_f , the annual probability of failure $p(x)$, the oldness rate o , on the discount rate q , and the number of years n . Note that for small probabilities of failure $p(x)$ (for appropriate structural parameter x) and very small (zero) rates o and q , the time factor $PQ(x,o,n) \approx PQ(x,q,n) \approx n$. Variation of the time factor $PQ(x,o,n)$ with n for $o = 0, 0.03, 0.06$ and 0.13 is shown in figure 1. The same variation holds for the time factor $PQ(x,q,n)$.

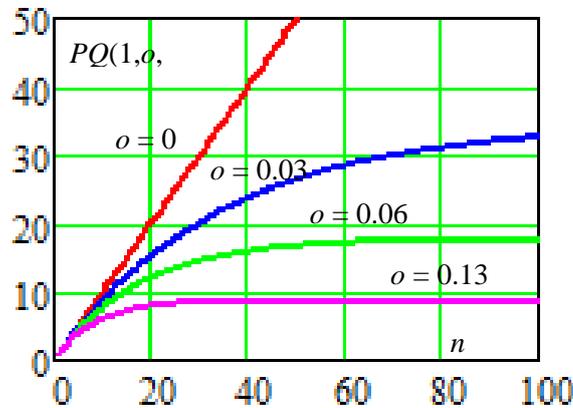


Figure 1. Variation of the time factor $PQ(x,o,n)$ with n for $o = 0, 0.03, 0.06$ and 0.13 .

It follows from figure 1 that the obsolescence rate o and discount rate q may affect the total costs and consequently their consideration in optimization procedure should be adjusted to actual situations of the heritage structure. The necessary condition for the minimum of the total cost expressed by objective function (1) is

$$\frac{\partial C_{tot}(x,o,q,n)}{\partial x} = C_{str} \sum_{i=1}^n Q(o,i) \left[\frac{\partial P_f(x,i)}{\partial x} \right]_{x=x_{opt}} + C_f \sum_{i=1}^n Q(q,i) \left[\frac{\partial P_f(x,i)}{\partial x} \right]_{x=x_{opt}} + C_1 = 0 \tag{7}$$

Equation (7) represents a general form of the necessary condition for the minimum of total cost $C_{tot}(x,o,q,n)$, the optimum value x_{opt} of the parameter x , and the optimum annual probability of failure $p_{opt} = p(x_{opt})$. The optimum probability for a given number of years n follows from equation (7) as

$$\frac{C_{str}}{C_1} \sum_{i=1}^n Q(q,i) \left[\frac{\partial P_f(x,i)}{\partial x} \right]_{x=x_{opt}} + \frac{C_f}{C_1} \sum_{i=1}^n Q(q,i) \left[\frac{\partial P_f(x,i)}{\partial x} \right]_{x=x_{opt}} = -1 \tag{8}$$

Equation (8) represents a simplified form of the necessary condition for the minimum of total cost $C_{tot}(x,o,q,n)$, the optimum value x_{opt} of the parameter x , and the optimum annual probability of failure $p_{opt} = p(x_{opt})$. The optimum probability for the total design working life $T_d = n$ years follows from equation (3) as

$$P_{fn,opt} = 1 - (1 - p_{opt})^n \approx n p_{opt} \tag{9}$$

The corresponding optimum reliability index $\beta_{opt} = -\Phi^{-1}(P_{fn,opt})$. These quantities are in general dependent on the cost ratios C_{str}/C_1 and C_f/C_1 , rates o and q , and on the number of years n .

3. Failure probability of a generic structural member

Consider a generic structural member described by the limit state function $Z(x)$ as

$$Z(x) = xf - (G+Q) \quad (10)$$

Here x denotes a deterministic structural parameter (e.g. the cross-section area), f the strength of the material, G the load effect due to permanent load and Q the load effect due to variable load. Theoretical models of the random quantities f , G and Q considered in the following example are given in Table 4 (adopted from the probability model code described in JCSS (2001) [8] and Holicky (2009) [4]).

Table 4. Theoretical models of the random variables f , G and Q (annual extremes).

| Variables | Distribution | Mean | Standard deviation | Coefficient of variation |
|-----------|--------------|------|--------------------|--------------------------|
| f | Lognormal | 100 | 10 | 0.10 |
| G | Normal | 35 | 3,5 | 0.10 |
| Q | Gumbel | 10 | 5 | 0.50 |

Considering the theoretical models given in Table 4, the reliability margin $Z(x)$ may be well approximated by the three parameter lognormal distribution $\Phi_{Z(x)}$ that provides sufficient accuracy. The annual failure probability $p(x)$ is then given as

$$p(x) = \Phi_{Z(x)}(Z(x) = 0) \quad (11)$$

The annual failure probability $p(x)$ in equation (11) is evaluated for the reliability margin $Z(x) = 0$ using three parameter for $Z(x)$; then for $x = 1$ and $n = 50$ the failure probability is $P_{fn}(1) \approx 6.7 \cdot 10^{-5}$ and corresponding reliability index is $\beta \approx 3.8$ (common value indicated in EN 1990 (2002) [2]).

4. An example

The following example illustrates the general principles, as well as a special case of probabilistic optimization. To simplify the analysis, the total costs $C_{tot}(x,o,q,n)$ given by equation (5) are transformed to the standardized form $\kappa_{tot}(x,o,q,n)$ given as

$$\kappa_{tot}(x,o,q,n) = \frac{C_{tot}(x,o,q,n) - C_0}{C_1} = p(x) \left[\frac{C_{str}}{C_1} PQ(x,o,n) + \frac{C_f}{C_1} PQ(x,q,n) \right] + x \quad (12)$$

The annual probability of failure $p(x)$ considered here for a general structural member is given by equation (11). However, the following procedure may be applied for any relevant dependence of the failure probability $p(x)$ expressed as a function of a suitable structural parameter x .

In the example illustrated in figure 2, it is assumed that the rates $o = 0.13$ is $q = 0.03$, and the year number when the failure occurs is $n = 50$. Under these assumptions, figure 2 shows the variation of the total standardized costs $\kappa_{tot}(x,o,q,n)$ (given by equation (12)), and the optimum reliability index β_{opt} , with structural parameter x .

The following figure 3 indicates variation of the optimum reliability index β_{opt} with the cost ratio C_f/C_1 for the rates $o = 0.13$, $q = 0.03$, number of years $n = 50$ and selected $C_{str}/C_1 = 10, 100, 1000, 10000$.

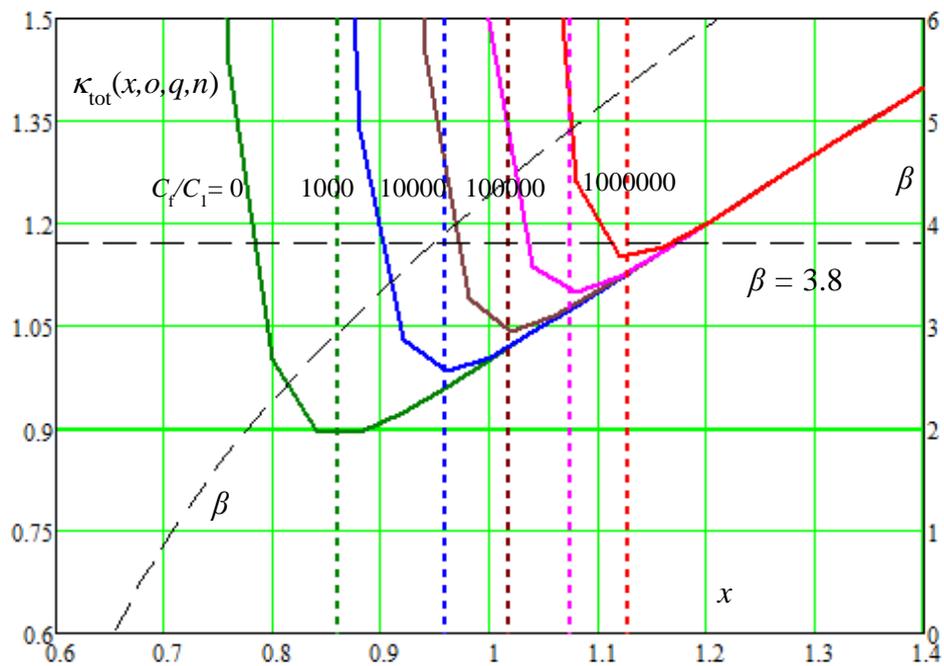


Figure 2. Variation of the total standardized cost $\kappa_{tot}(x, q, n)$ and the optimum reliability index β_{opt} with the decision parameter x for $o = 0.13, q = 0.03, n = 50, C_{str}/C_1 = 100$, and selected $C_f/C_1 = 0, 1000, 10000, 100000$ and 1000000 .

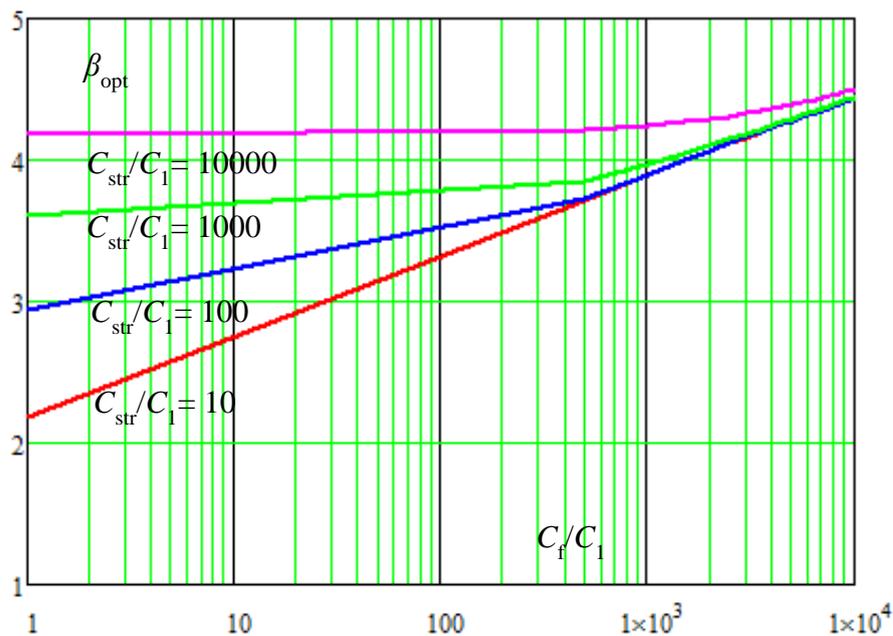


Figure 3. Variation of the optimum reliability index β_{opt} with the cost ratio C_f/C_1 for $o = 0.13, q = 0.03, n = 50$ and selected $C_{str}/C_1 = 10, 100, 1000, 10000$.

5. Transformation of the target reliability for different reference periods

When the main uncertainty comes from actions that have statistically independent maxima in each year, the values of β for a different reference period expressed in years n can be calculated using the following expression [2]:

$$\Phi(\beta_n) = [\Phi(\beta_1)]^n \quad (13)$$

where β_n is the reliability index for a reference period of n years and β_1 is the reliability index for one year. However, the statistical maxima of actions in subsequent years are usually correlated. Then the relationship (13) is not correct and correlation of failure events should be taken into account.

Assume that variation of annual failure probabilities with time is approximated by rectangular wave process with the mean duration of rectangles of k years. When the mean duration $k = 1$, then the failures in subsequent years are assumed to be independent and the relation (13) can be used. If $k = n$, then the annual failures within the whole reference period of n years are fully dependent and the annual target reliability will be valid for the whole reference period of n years. In general the reliability index β_{nk} for the reference period of n years and independence interval of k years can be derived from reliability index β_1 , specified for the reference period of one year, using the following formula:

$$\Phi(\beta_{nk}) = \Phi(\beta_1)^{n/k} \quad (14)$$

Here the independence interval $k \leq n$ corresponds to the mean time period in years for which the failures in subsequent periods of k years are assumed to be mutually independent. An example of determining $\beta_{n,k}$ for $n = 50$ years and $k = 1, 10$ and 50 years is shown in figure 4.

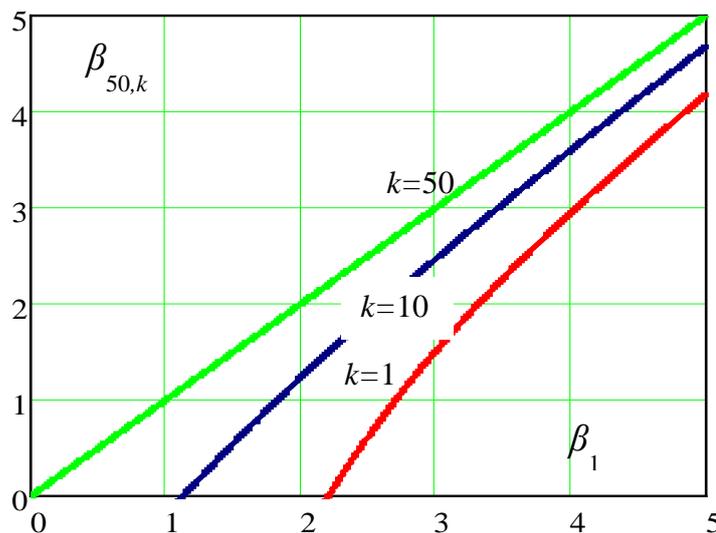


Figure 4. Variation of the target reliability level β_{nk} with annual target index β_1 for the reference period $n = 50$ years and selected independence intervals $k = 1, 10$ and 50 years.

Similar results for the transformation of the target reliability β may be obtained considering dependence of failure probability F_i during all the years $i = 1$ to n . Assume that the maximum failure probability $P\{F_1\}$ refers to a given year 1 and the average failure probabilities during remaining years $i = 2$ to n are reduced by the probability ratio $b = (P\{F_i\}/P\{F_1\})$. Newly derive target reliability index is now denoted β_{nb} is shown in figure 5.

For $b = 0$ probability $P\{F_i\} = 0$, $i = 2$ to n , and the total failure probability is given only by $P\{F_1\}$. This case corresponds to the independence interval $k = n$ year (then $\beta_{nb} = \beta_{nk} = \beta_1$). When the probability ratio $b = 1$ the failure probability during every year i is the same $P\{F_i\} = P\{F_1\}$. This case corresponds to independence interval $k = 1$. Variation of the target reliability level β_{nb} with annual target index β_1 for the reference period $n = 50$ years and selected probability ratios $b = 1; 0,2$ and 0 is illustrated in figure 5.

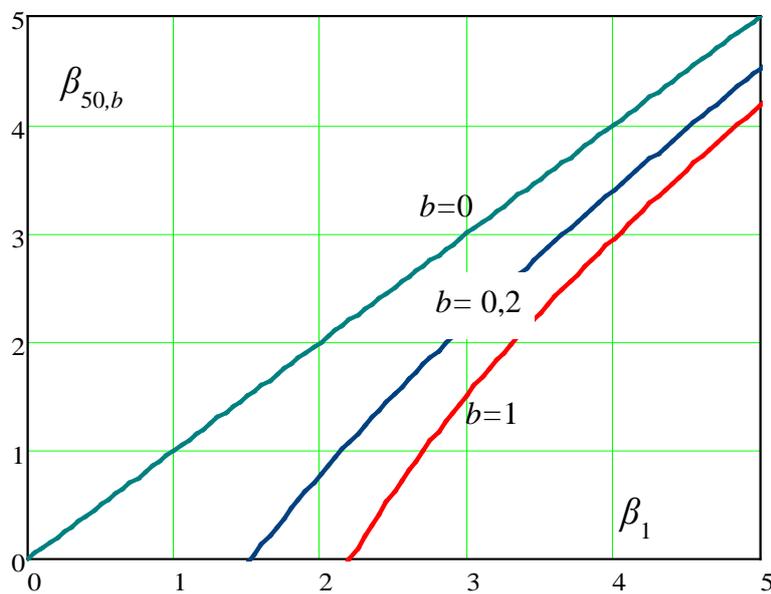


Figure 5. Variation of the target reliability level β_{nb} with annual target index β_1 for the reference period $n = 50$ years and selected probability ratios $b = 1; 0,2$ and 0 .

It follows from figure 4 and 5 that for $n = 50$ (approximately life time) the transformation of the target reliability is equal ($\beta_{nb} = \beta_{nk}$), when $k = b = 1$, or when $k = 50$ and $b = 0,0$. The probability ratio $b = 0,2$, when $P\{F_i\} = 0,2 P\{F_1\}$ for any i not equal to 1, corresponds approximately to the independence interval $k = 5$ years.

6. Conclusion and recommendation

The target reliability levels recommended in various national and international documents are inconsistent in terms of the values and the criteria according to which the appropriate values are to be selected. It is shown that the target reliability of structures can be derived from theoretical principles of probabilistic optimization considering the objective function as the total costs expressed as a sum of the initial costs C_0 , the marginal costs $x C_1$ (where x denotes the decision parameter and C_1 the incremental cost of decision parameter x), and the failure consequences consisting of the construction

costs C_{str} and failure costs C_f (the loss of structural utility at the time of failure), these being taken into account by the relevant cost ratios C_{str}/C_1 and C_f/C_1 .

The construction costs C_{str} is discounted considering an annual obsolescence (oldness) rate q and the time to failure (number of years) n , the failure costs C_f is discounted considering an annual discount rate q and the time to failure (number of years) n . In such a way the total cost is affected (reduced) by the obsolescence rate o and discount rate q , and the number of years n .

An example of the probabilistic optimization of a generic structural member clearly shows (see Figure 1, and 2) that the optimal reliability level, i.e. the reliability index β , depends primarily on:

- the construction costs C_{str} ,
- failure costs (malfunctioning costs) C_f ,
- costs for improving structural safety C_1 .

The obsolescence rate o and discount rate q and the time to failure n seem to be less significant than construction cost, malfunctioning cost and cost for improving structural safety.

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