

Modelling of particles retention in a porous soil

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Abstract. Construction of foundations and underground structures requires grouting the soil for groundwater protection. To select the technology of soil grouting and the type of grout, it is necessary to calculate the suspended particles retention for the filtration of bentonite or cement-based suspension in a porous soil in dependence of sizes of the soil pores and the grains of the injected mortar. The study of transport and retention of solid particles in the complex structure of a porous medium makes it possible to calculate the strength and impermeability of a grouted foundation at the stage of pre-construction preparation.

The solid particles retention in the nonlinear process of deep bed filtration of a suspension in a porous medium is considered. It is assumed that the retained particles can not be detached from the frame of porous medium by a fluid flow or suspended particles. The purpose of the study is to calculate the dynamics of deposit formation for different types of variable filtration coefficients with the predominance of the size-exclusion mechanism of particles capture.

The mathematical model of filtration is the system of mass transfer equation and the kinetic equation of deposit rate with the conditions of injection of a constant concentration suspension into an empty porous medium. In this paper an exact solution of the problem at the inlet of a porous medium is constructed. The dynamics of retained particles growth is studied by numerical modeling method, depending on the type of blocking filtration coefficient.

The dependence of retained particles concentration on time at the inlet of a porous medium for various blocking filtration coefficients was obtained. Plots of the dependence of the deposit concentration on time for smooth and non-smooth filtration coefficients are constructed.

When reducing the number of free pores of small sizes, the filtration rate slows down. It is shown that slowing down the filtration process and the growth rate of the deposit are determined by the multiplicity of the root of the blocking filtration coefficient. Depending on the type of filtration coefficient, the process of retention of solid particles in a porous medium can last indefinitely, or it may cease after some time with complete blocking of small pores by retained particles. The limiting time for the end of the filtration is given by a compact analytical formula.

1. Introduction

Soil grouting is the main method of protecting tunnels and underground structures for groundwater protection. Grout is a fluid with suspended particles or a colloidal solution. It is injected under pressure into a loose ground. The grout locks the pores of the soil and after drying the soil becomes waterproof. The study of the interaction of grout and soil, as well as the problems of filtration of drinking water, sewage and liquid industrial wastes treatment, require the study of the transport and retention of particles in porous media [1-4].

The porous medium is a solid body, dotted with hollow channels of various lengths and shapes, called pores. When the suspension - a liquid with suspended solids - is injected into a porous medium



the fluid flow moves along the pores, gradually filling the entire porous medium. If the particle sizes are comparable with the pore sizes, the solid particles penetrate into a porous medium. Some particles moving along wide pores pass unhindered through the porous medium. Other particles that fall into the pores of a small cross-section get stuck in them and form a deposit.

Consider the process of deep bed filtration, when retained particles gradually accumulate throughout the entire porous medium, and not only in its surface layer. It is assumed that one retained particle completely blocks one pore and stops movement through this channel. A stuck particle in the pore can not be knocked out of it by other particles or fluid flow, and remains immovable forever [5, 6]. The main reason for the retention of solid particles of the suspension in the pores is the size-exclusion mechanism of particle capture: the particles freely pass through large-diameter pores and get stuck at the inlet of pores which are smaller than the particle diameter [7-9].

The mathematical model of filtration consists of two first-order partial differential equations characterizing the transport of solid particles in a porous medium. The concentrations of suspended and retained particles are the unknowns in the system. The first equation is related to the conservation law of mass balance, the second determines the growth rate of the deposit. The proportionality coefficient between the growth rate of the deposit and the suspended particles concentration is called the filtration coefficient. The dependence of this coefficient on the retained particles concentration is determined experimentally. If all small pores are blocked by retained particles, the process of deposit formation ceases and all particles pass unobstructed through large pores. The concentration of retained particles reaches a maximum value at which the filtration coefficient is equal to zero. Such a filtration coefficient is called blocking filtration coefficient [10]. Depending on the multiplicity of the root, a maximum of the deposit can be attained at a finite time or at $t \rightarrow \infty$.

The dynamics of retained particles growth at the inlet of a porous medium is considered. Section 2 is devoted to the mathematical model of deep bed filtration. An exact solution of the problem at the inlet of a porous medium is given in Section 3. Results of the numerical modeling of the filtration problem with blocking filter coefficients of various multiplicities are considered in Section 4. Plots of deposit concentration for smooth and non-smooth filtration coefficients are constructed. Discussion and conclusions in Sections 5 and 6 finalize the paper.

2. Mathematical model

In the one-dimensional case for a homogeneous porous medium under the assumption of constant porosity and permeability, the system of dimensionless filtration equations in the domain $\Omega = \{(x, t) : 0 < x < 1, t > 0\}$ has the form

$$\frac{\partial(C+S)}{\partial t} + \frac{\partial C}{\partial x} = 0 \quad (1)$$

$$\frac{\partial S}{\partial t} = \Lambda(S)C. \quad (2)$$

Here $C(x, t)$, $S(x, t)$ are the suspended and retained particles concentrations. The filtration coefficient $\Lambda(S)$ is positive and decreases with increasing S , as the amount of vacant pores of small size decreases with increasing S , and the rate of deposit formation decreases.

The introduction of variable porosity and permeability of a porous medium substantially complicates the equation of mass transfer (1). Relative porosity and permeability increase with increasing retained particles concentration $S(x, t)$. Equation (1) takes the form

$$\frac{\partial}{\partial t}(g(S)C) + \frac{\partial}{\partial x}(f(S)C) + \frac{\partial S}{\partial t} = 0. \quad (3)$$

Here the porosity $g(S)$ and permeability $f(S)$ are smooth increasing functions.

For the uniqueness of the solution of the systems of equations (1), (2) and (2), (3), additional conditions must be specified. Suppose that a suspension with a constant concentration p of suspended

particles is injected into the porous medium, and at the initial moment the porous medium is empty and does not contain any suspended and retained particles. The corresponding conditions have the form

$$C(x, t)|_{x=0} = p, \quad p > 0; \quad (4)$$

$$C(x, t)|_{t=0} = 0; \quad (5)$$

$$S(x, t)|_{t=0} = 0. \quad (6)$$

Mathematical models of filtration are considered in many papers. In some important cases it is possible to obtain an exact solution of the problem [11-13]. In the vicinity of the lines on which the exact solution is constructed, an asymptotics can be constructed [14-16]. If there is no analytical solution, the problem is solved numerically [17-19].

3. Exact solution at the inlet of a porous medium

Substituting condition (4) into equation (2), we obtain the equation for the concentration of retained particles at the inlet of a porous medium $x = 0$

$$\frac{\partial S}{\partial t} = \Lambda(S)p. \quad (7)$$

The equation (7) can be solved by the method of variables separation. Separation of the variables

$$\frac{\partial S / \partial t}{\Lambda(S)} = p, \quad (8)$$

and integration of (8) with respect to the variable t yields

$$\int_0^t \frac{\partial S / \partial t}{\Lambda(S)} dt = pt. \quad (9)$$

Using condition (6), we make the change of the variable in the integral (9):

$$\int_0^S \frac{dS}{\Lambda(S)} = pt. \quad (10)$$

Equation (10) specifies the dependence of the retained particles concentration at the inlet of the porous medium on the time t .

Consider the blocking filter coefficients with a root of multiplicity n , of the form

$$\Lambda(S) = (a - bS)^n. \quad (11)$$

Here a, b, n are positive constants.

In the case $n \geq 1$ the integral on the left-hand side of (10) has a nonintegrable singularity. The solution $S(0, t)$ increases for all t and tends to the limiting value a/b for $t \rightarrow \infty$. For $0 < n < 1$ the singularity in the integral (10) is integrable, and the limiting value of the deposit concentration at the inlet of the porous medium is reached in a finite time t_{\max} . This time is determined by the equation

$$\int_0^{a/b} \frac{dS}{\Lambda(S)} = pt_{\max}. \quad (12)$$

Calculations of retained particles concentrations at the inlet of the porous medium and the corresponding plots for different values of the multiplicity n are presented below.

4. Numerical modelling

The retained particles concentrations at different values of the multiplicity n are calculated for the parameters

$$a = b = 1, \quad p = 1. \quad (13)$$

1. $n = 1$. The case of the linear filtration coefficient is most often encountered in applications. The integral on the left-hand side of (10) is equal to

$$\int_0^S \frac{dS}{a-bS} = -\frac{1}{b} \ln \left(1 - \frac{b}{a} S \right),$$

and the solution of equation (10) has the form

$$S(t) = \frac{a}{b} \left(1 - e^{-bpt} \right). \quad (14)$$

In Fig. 1 a plot of the deposit concentration is constructed for $n = 1$.

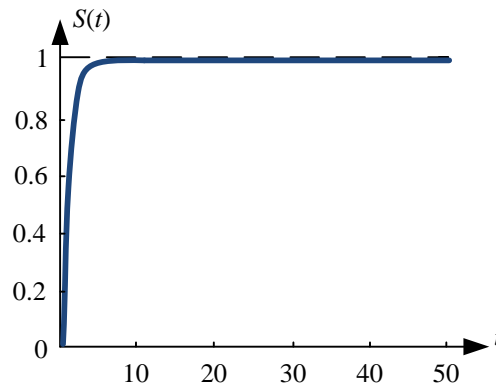


Figure 1. The deposit concentration for the linear filtration coefficient.

2. $n > 1$. In the case of a multiple root (11), the solution of the equation (10) is given by formula

$$S(t) = \frac{1}{b} \left(a - \frac{1}{\sqrt[n-1]{a^{1-n} + (n-1)bpt}} \right). \quad (15)$$

In Fig. 2, 3 the plots of the deposit concentration are constructed for $n = 2$ and $n = 3$

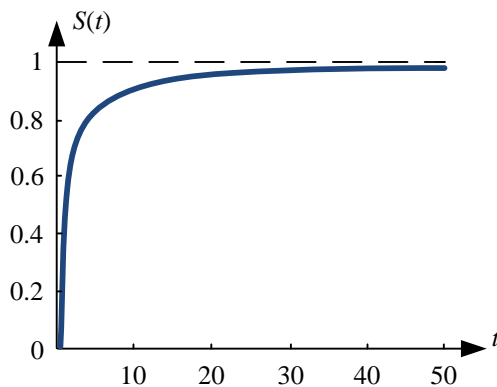


Figure 2. The deposit concentration for $n = 2$.

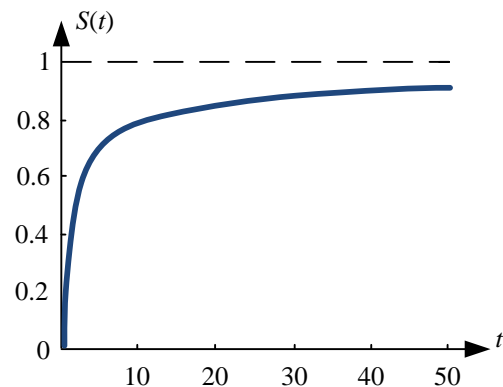


Figure 3. The deposit concentration for $n = 3$.

When $n \geq 1$ the filtration rate so drastically slows down that the filtration process lasts indefinitely. The less vacant small pores remain in the porous medium, the more difficult for the particle to block the free pore, since the main flow of the suspension moves through large pores.

3. $0 < n < 1$. In the case of a fractional power of n , the solution of equation (10) has the form

$$S(t) = \frac{1}{b} \left(a - \left(a^{1-n} + (n-1)bpt \right)^{\frac{1}{1-n}} \right), \quad t \leq t_{\max}; \quad S = \frac{a}{b}, \quad t > t_{\max}. \quad (16)$$

For $n < 1$ the maximum deposit is reached at the time $t = t_{\max}$, when the deposit concentration becomes constant

$$t_{\max} = \frac{a^{1-n}}{bp(1-n)}. \quad (17)$$

In particular, for the values of the parameters (13) we have $t_{\max} = 2$ for $n = 0.5$ and $t_{\max} = 4/3$ for $n = 0.25$.

In Fig. 4, 5 the plots of the deposit concentration are constructed for $n = 0.5$ and $n = 0.25$.

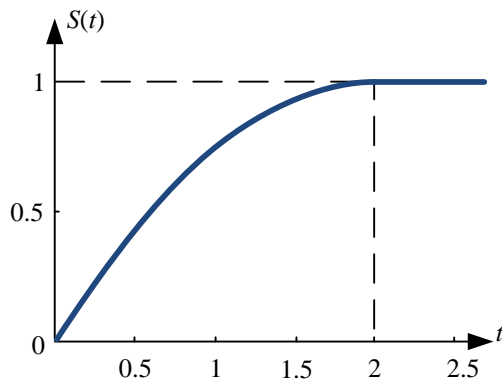


Figure 4. The deposit concentration for $n = 0.5$.

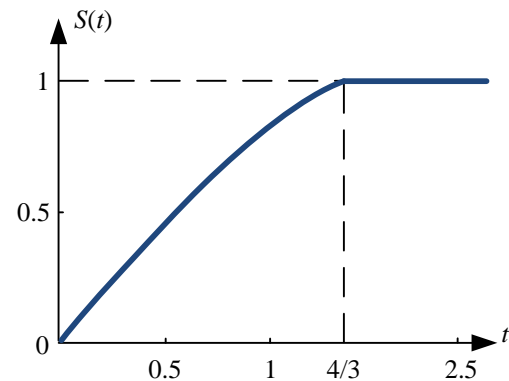


Figure 5. The deposit concentration for $n = 0.25$.

For $0 < n < 1$ the filtration rate also decreases with increasing deposit, but to a lesser extent. In this case, the deposit reaches a maximum value in a finite time. At time $t = t_{\max}$ all small pores are blocked by retained particles. The filtration process stops, and all suspended particles of the suspension pass freely through the large pores from the inlet to the outlet of the porous medium.

5. Discussion

The filtration processes proceed in different ways, depending on the properties of the porous medium and the injected mortar. In this paper the model of size-exclusion particles retention is used. This model is applicable if distributions of sizes of suspended particles and pores overlap.

The study of the filtration process at the inlet of a porous medium is of particular interest, since deep bed filtration occurs most intensively at the injection point of the suspension. The characteristic features of filtration depend on the filtration coefficient. To describe the long-term filtration process, a blocking filtration coefficient is used, taking into account the gradual slowing down of the particles capture with increasing number of blocked pores.

The calculation of various models shows that the concentration of retained particles at the inlet of the porous medium tends to the maximum limit value, regardless of the multiplicity of the root of the blocking filtration coefficient. Modeling of filtration inside a porous medium, taking into account the change in the concentration of suspended particles and the possible heterogeneity of the porous medium, requires a special study.

6. Conclusions

When the suspension is injected into a porous medium, the filtration process begins at the inlet. At any time the concentration of retained particles is maximal at the inlet of the porous medium. A detailed study of the deposit dynamics at the inlet makes it possible to evaluate the process of long-term filtration inside a porous medium.

It is shown that the concentration of the deposit at the inlet of the porous medium continuously increases with time and tends to the limiting value. The maximum limit value is determined by the filtration coefficient.

When decreasing a number of free pores of small sizes, the filtration rate slows down. It is shown that the rate of deceleration is determined by the multiplicity of the root of the blocking filtration

coefficient (9). If the multiplicity is less than 1, then the filtration lasts a finite time; when a multiplicity is greater than 1 or equal to 1 there are vacant small pores at any time and the filtration does not cease.

Mathematical modeling of filtration processes in porous ground allows to calculate the composition and volume of the grout which is necessary for waterproofing underground structures [20].

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