Model of grout filtration in a porous soil

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Abstract. To strengthen the foundations and protect them from ground water, liquid grout is pumped under pressure into a loose porous soil. Modern construction techniques use colloids or suspensions which form a waterproof layer upon solidification. Calculation of grout distribution in a porous soil is an important part of designing buildings and structures. The distribution of suspended particulates in unconsolidated soil is described by mathematical equations of particles filtration in the porous media. The aim of this work is modelling of suspended particles transport and deposit formation in a porous soil for different filtration modes. The distributions of solid particles of different sizes transported by the carrier fluid and retained on the frame of the porous medium at different kinetic rates of deposit growth are examined. Model of one-dimensional filtration with size-exclusion particle retention mechanism includes a hyperbolic system of the first order differential equations with inconsistent initial and boundary conditions that leads to discontinuous solutions. A modified mathematical model for polydisperse media with the competition between particles of different sizes for small pores is considered. The computational scheme for numerical solutions is constructed by finite difference method. Optimization methods to improve convergence and reduce computation time are used. A multi-particle model of grout filtration in a porous soil is constructed, considering the variety of sizes of suspended particles. A numerical calculation of the problem is performed for various blocking filtration coefficient. Solutions are obtained with a discontinuity on the concentration front. Approbation of the obtained numerical solutions is carried out. Plots of the suspended and retained particles concentrations depending on time and coordinates are constructed. A new model of multi-particle transport in a porous medium widely extends the design capabilities of waterproof underground walls. The solution of the filtration problem for solid particles of different sizes makes it possible to calculate the optimal composition of grout for constructing a waterproof layer in a porous soil. The developed computing finite-difference methods allow to obtain the numerical solution of complex systems of equations with good accuracy and a moderate amount of computation.

1. Introduction

In the construction of buildings and structures on unstable ground it is necessary to strengthen the soil and create a solid foundation. Construction of underground structures, tunnels, storages of hazardous chemical and radioactive wastes requires the creation of a reliable protection against ground water and flood water. To strengthen the foundations and create a waterproof wall in unstable porous soil, a liquid solution of the grout is pumped under the pressure. The solution fills the pores of the soil and upon solidification strengthens the foundation and creates a waterproof layer [1, 2].

The aim of this work is the simulation of the motion of suspended particle suspension and colloids and precipitate formation in a porous soil for different filtration modes. The distribution of solid particles of different sizes carried by the fluid carrier and deposited on the frame of the porous medium at different growth rates of the precipitate is considered.

Filtration of suspensions and colloids in a porous medium is a complex physicochemical process that determines the transport of particles and the formation of a precipitate on the framework of a porous medium. Depending on the type of solution and the porous medium, the filtration is more or less affected by electrical and gravitational forces, diffusion, viscosity, etc. If the particle and pore size distributions overlap, and the grout and the soil do not react chemically, the main reason for particle precipitation is the size-exclusion particle retention mechanism [3, 4]. Suspended particles pass freely through large pores and get stuck at the inlet of pores whose dimensions are smaller than the diameter of the particles.

To determine the distribution of the suspension with identical solid particles in loose ground a one-dimensional mathematical model of the filtration of a monodisperse suspension in a porous medium is considered [5-7]. The model includes a first order hyperbolic system of equations with inconsistent initial and boundary conditions that generate discontinuous solutions. In this paper we study a modified mathematical model for polydisperse suspension with competition of particles of different sizes for the small pores. The particles of each size obey the classical filtration equations for a monodisperse suspension; the interaction of particles of different sizes provides the dependence of the filtration coefficients on the total deposit [8, 9].

A numerical calculation of the filtration problem of a polydisperse suspension in a porous medium for different filtration coefficients is performed. The computational scheme is constructed on the basis of the method of finite differences [10]. Solutions with a discontinuity on the concentrations front of the suspended and retained particles are obtained. Approbation of the computational method and the found numerical solutions is carried out.

In section 2, we construct a mathematical model of filtration of the solution with solid particles of different sizes in a porous medium. Section 3 is devoted to methods of numerical solution of the problem. In section 4 the results of numerical calculations are considered. Discussion and Conclusions in sections 5 and 6 conclude the work.

2. Mathematical model

The dimensionless equations of the filtration problem are considered in an infinite half-strip $\Omega = \{0 < x < 1, t > 0\}$, the unknowns are the volumetric concentrations of suspended and precipitated particles $S_i(x,t)$. The equations of mass transfer describe the mass balance of suspended and retained particles; the equations of kinetic rate determine the growth of the precipitate

$$\frac{\partial C_i}{\partial t} + \frac{\partial C_i}{\partial x} + \frac{\partial S_i}{\partial t} = 0 \; ; \tag{1}$$

$$\frac{\partial S_i}{\partial t} = \Lambda_i(S) C_i, \quad i = 1,....,n \tag{2}$$

with boundary and initial conditions

$$x = 0: \quad C_i(x,t) = p_i, \quad p_i > 0 \tag{3}$$

$$t = 0: \quad C_i(x,t) = 0, \quad S_i(x,t) = 0 \tag{4}$$

Here the total deposit $S = S_1 + S_2 + ... + S_n$; functions $\Lambda_i(S)$; $i = 1,...,n$ are continuous and positive.

The functions $\Lambda_i(S)$ are called filtration coefficients. They are determined experimentally. If $\Lambda_i(S)$ has a positive root S_M, then it is called blocking filtration coefficient. S_M is the ultimate maximum of the total deposit corresponding to the locking of all the small pores of the porous medium.

The inconsistency of conditions (3) and (4) at the origin of coordinates generates a solution discontinuity. The concentrations front of the suspended and retained particles moves at a constant velocity $v = 1$ and divides the domain Ω on two subdomains $\Omega_0 = \{0 < x < 1, 0 < t < x\}$ and $\Omega_S = \{0 < x < 1, t > x\}$. In the domain Ω_0, system (1) - (4) has a zero solution; in the domain Ω_S the solution is positive. Since the boundary and initial conditions for the suspended particles concentration are not agreed at the origin, the solution $C_i(x,t)$ has a strong discontinuity (jump) at the concentration front – the characteristic straight line $t = x$. The solution $S_i(x,t)$ is continuous in Ω and has a weak discontinuity at the concentration front (a jump in the derivatives).

Equations (1), (2) form a quasilinear hyperbolic system of equations of the first order.
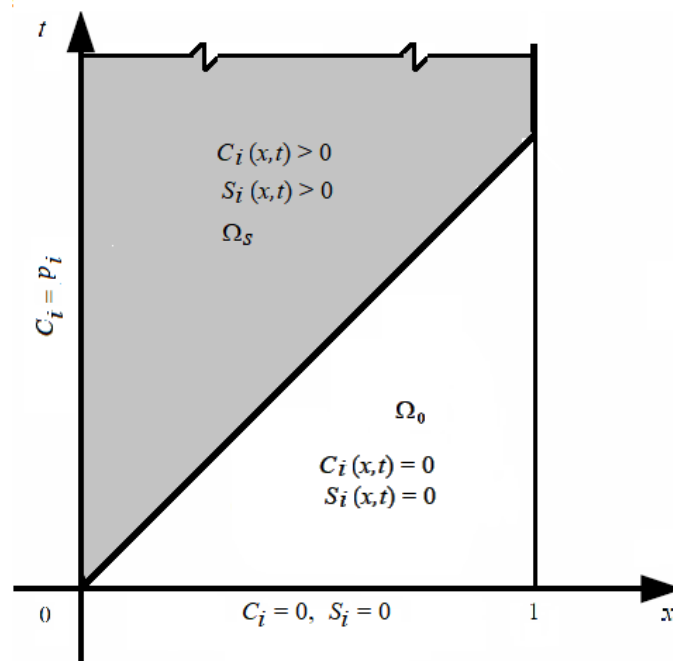A schematic representation of the solution of the considered model is shown in Fig. 1.

Figure 1. The scheme of the solution of the problem (1) – (4).

3. Methods
Non-linear filtration models admit exact analytical solutions only in individual cases [11-13]. In the absence of exact solutions, one can construct an asymptotic formula [14-16]. Most problems do not have analytical solutions, in these cases it is necessary to use numerical methods [17-19].

The peculiarity of this problem is the presence of discontinuous initial-boundary conditions, which leads to difficulties in obtaining acceptable solutions close to the break line – the concentrations front. As a rule, many computational methods give reliable results for smooth solutions and are not applicable in the vicinity of the discontinuities and zones of fast oscillations.

For the numerical solution of the problem (1)–(4), computational schemes based on the finite difference method were used. A rectangular grid with constant step h along the coordinate x and the τ

– grid step at time t is applied to the domain Ω_S. The relationship between the steps τ and h is chosen from the Courant convergence condition $\tau \leq h / v$.

To solve equation (2), the modified Euler method was used, where the predictor is the "forward" difference for a half-step in time, the corrector is also the "forward" difference, considering the predictor obtained and throughout the time step [20].

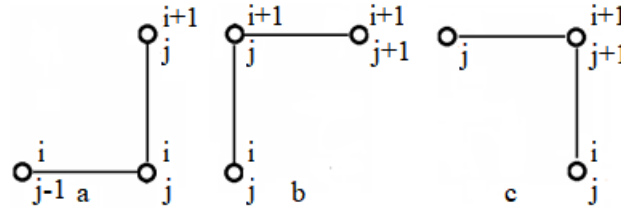Several point schemes, shown in Fig. 2, were studied.

Figure 2. Templates of finite difference calculation schemes.

The study of these schemes and the comparison of numerical calculations with the exact solution for known cases made it possible to establish that the scheme b is optimal for this model, because its convergence rate is equal to the order of the grid step.

4. Numerical calculation

The time and coordinate steps are set to 0.001. Calculations were carried out for the time $t = 0$ and $t = 5$. The quadratic decreasing blocking filtration coefficients with a common root $S_M = 1$ are chosen. Two cases of 2-size particles are considered.

1. The filtration coefficients

$$\Lambda_1(S) = -1.1S^2 + 1.1; \quad \Lambda_2(S) = -1.01S^2 + 1.01$$

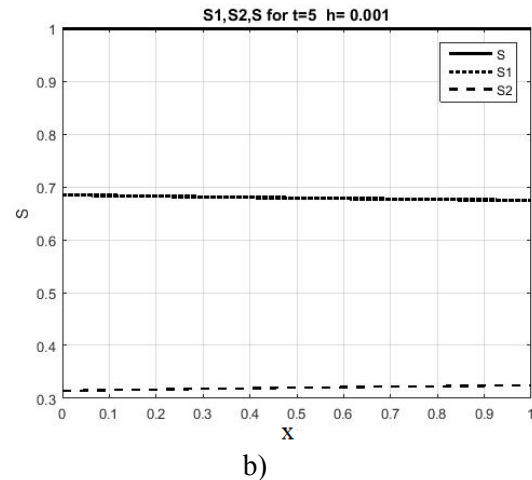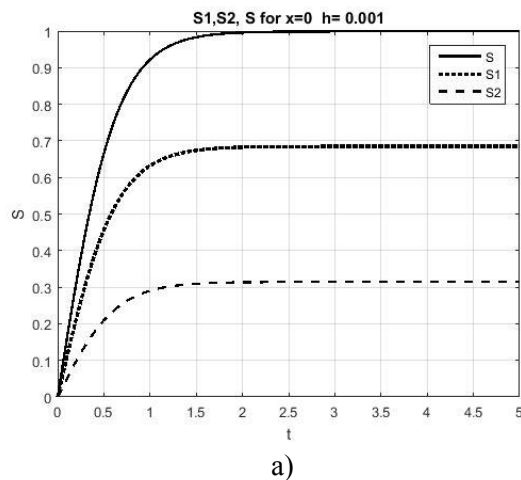a)   b)

Figure 3. Concentrations of partial precipitation and total deposit
a) at the inlet of a porous medium $x = 0$; b) distribution of precipitation in a porous medium at $t = 5$.

a)   b)

Figure 4. a) Concentrations at the outlet of the porous medium $x = 1$
a) partial precipitation and total deposit; b) suspended particles of two sizes.

2. The filtration coefficients

$$\Lambda_1(S) = 0.5S^2 - 1.5S + 1; \quad \Lambda_2(S) = -1.01S^2 + 1.01$$

a)   b)

Figure 5. Concentrations of partial precipitation and total deposit
a) at the inlet of a porous medium $x = 0$; b) distribution of precipitation in a porous medium at $t = 5$.
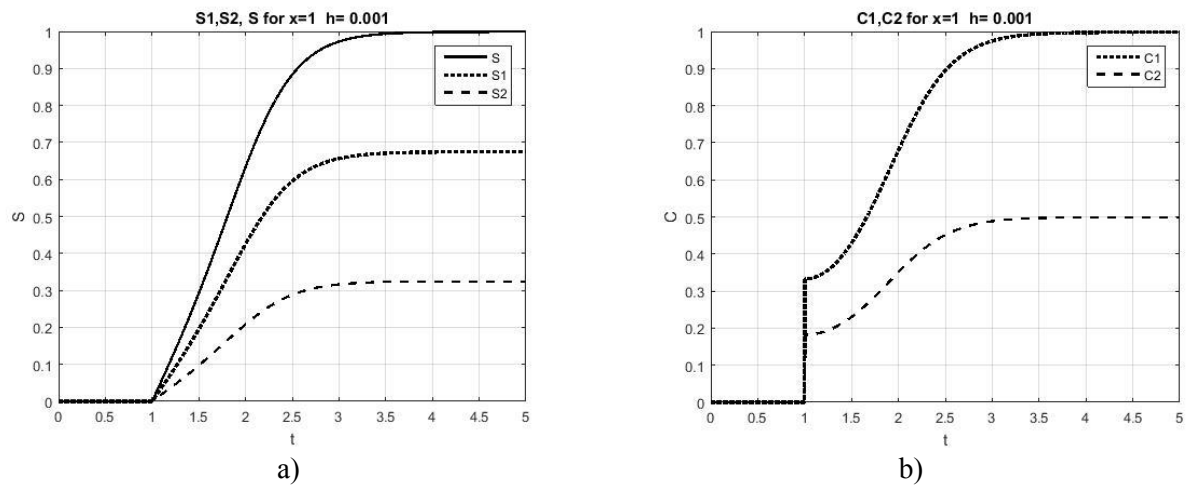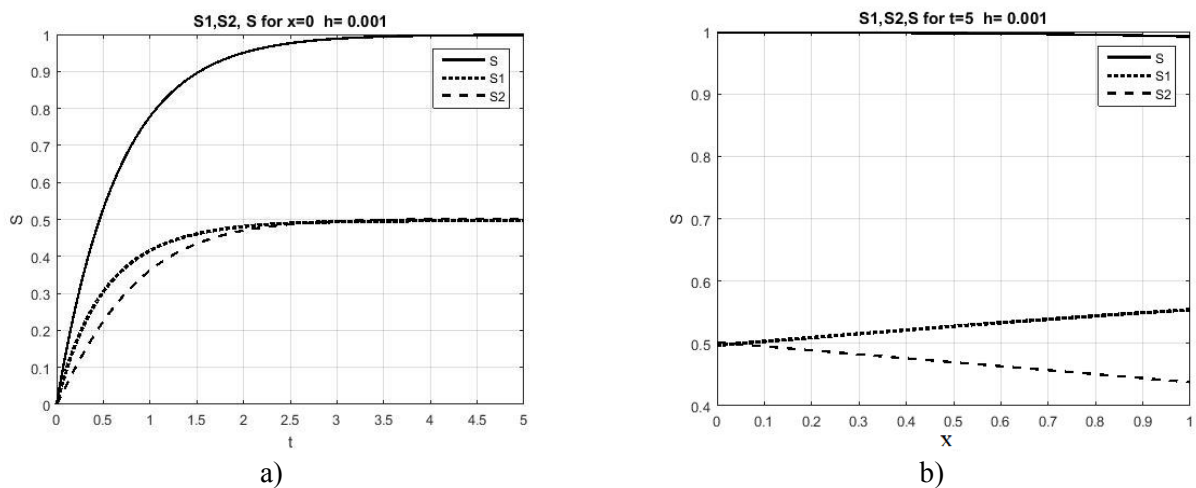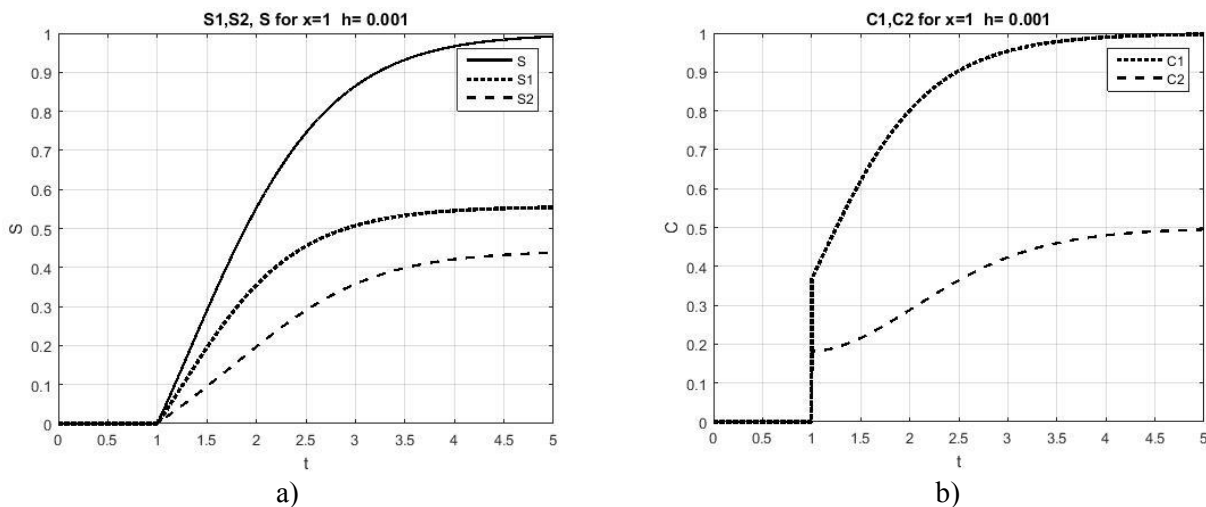
Figure 6. a) Concentrations at the outlet of the porous medium $x = 1$
a) partial precipitation and total deposit; b) suspended particles of two sizes.

In practice, it is of interest to calculate the solution at the outlet of a porous medium $x = 1$, where it can be compared with experimental data. Fig. 4, 6 show the kink of the concentration S of the retained particles and the break in the concentration C of suspended particles on the concentrations front.

5. Discussion

In this paper, we compute a complex nonlinear filtration model that does not have an analytical solution. In such cases, to select a numerical method, as a rule, a model that has an analytical solution is considered. Numerical solutions obtained by different methods are compared with the exact solution, and the best method is chosen.

However, not always a method suitable for a simple example is applicable for a complex model. In filtration problems, to approbate the chosen numerical method, one can use the analytical properties of solutions, which follow from the physical meaning of deep bed filtration. Since the filtration process starts from the entrance of the porous medium $x = 0$, the solutions $C_i(x,t)$; $S(x,t)$ decrease monotonically with respect to x for a fixed t. At a given point in the porous medium, the precipitate grows, while the filtration process slows down and the concentration of suspended particles increases. Thus, the solutions $C_i(x,t)$; $S(x,t)$ increase monotonically with respect to t for a fixed x.

Fig. 3-6 show that the numerical solution has the necessary monotonicity properties and, with increasing time, tends to the maximum limit values $C_{Mi} = p_i$; $S_M = 1$. This allows us to conclude that the chosen numerical method is acceptable, and the solution is adequate.

6. Conclusions

A mathematical model is proposed for the filtration of polydisperse suspensions and colloids in a porous medium with a size-exclusion particle retention mechanism that describes the competition of particles of different sizes for small pores.

The numerical solution of the problem was found by the computational method on the basis of the finite differences method. The presented explicit-implicit three-point scheme preserves monotonicity, moving a monotonically increasing solution from the previous time layer to the next one with the same direction of change, and shows an acceptable order of convergence. Solutions are obtained with a discontinuity at the concentrations front. Testing of the developed numerical method is carried out.

A new filtration model for particles of different sizes in porous media allows to study the dynamics of the grout penetration in the loose soil. The solution of the problem of the many-particle filtration

enables to calculate the optimal composition of the grout to create a water-resistant layer in the porous soil.

The constructed computing finite difference schemes allow one to obtain the solution of complex systems of equations with good accuracy using a moderate amount of computation.

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