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Analysis of systems with nonlinear auxiliary mass dampers by means of a special selection of linear generating functions

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Abstract. Nonlinear vibration isolation systems have a wider range of effective damping than linear ones. However, their analysis with an arbitrary load is quite laborious. In the article the method of special selection of linear generating systems for analyzing nonlinear systems proposed by professor Chernov Yu T is used. The linear generating system is chosen in such a way that the difference between the solutions of the linear and nonlinear systems in the first approximation for the first harmonic is minimal. Such an approach makes it possible to reduce the complexity of constructing the amplitude-frequency characteristics of nonlinear systems. In this paper, the amplitude-frequency characteristics were constructed to evaluate the efficiency of nonlinear auxiliary mass dampers at different frequencies of the external load. Tuned mass damper (TMD) was chosen as the type of auxiliary mass damper. The article looks at numerical examples of two systems with nonlinear TMD as a three degrees of freedom system. In the first numerical example, a stand with an equipment installed on it was analyzed. The TMD is installed on the stand. In the second numerical example the equipment is installed on a pedestal. The equipment is equipped with a nonlinear TMD. In both examples, there is cubic reaction-displacement relationship in the link that attaches the TMD. The amplitude-frequency characteristics for both systems are constructed for different variants: with a nonlinear, linear TMD and without a damper. The linear TMD was tuned to the first natural frequency of oscillations of the system without a damper. A comparative analysis of the options is given. The efficiency of using nonlinear TMD is determined from the numerical examples. There is a three times reduction in displacements in the nonlinear system as compared to the linear system.

1. Introduction

Nonlinear vibration isolation systems have been found to be more effective in vibration isolation than linear ones [1,2,3,4]. Various descriptions of nonlinear isolation systems could be found in [5].

Tuned mass damper (TMD) was chosen as the type of auxiliary mass damper. TMD have been used over many years to protect structures from the negative effects due to vibration. Optimum tuning parameters of TMD were given by Den Hartog [6] and in [7].

In this study the efficiency of a TMD with a nonlinear link between the structure and the TMD (nonlinear TMD) is assessed using the method of special selection of linear generating systems for analyzing nonlinear systems proposed by Prof. Chernov Yu T [8].

The linear generating system is chosen in such a way that the difference between the solutions of the linear and nonlinear systems in the first approximation for the first harmonic is minimal. Such an approach makes it possible to reduce the complexity of constructing the amplitude-frequency characteristics of nonlinear systems. The amplitude-frequency characteristics were constructed to evaluate the efficiency of nonlinear TMD at different frequencies of the external load.

The article looks at numerical examples of two systems with nonlinear TMD as the three degrees of freedom systems.

2. Numerical examples

2.1. First numerical example

In the first numerical example (figure 1), a system with 3 degrees of freedom was selected, representing a stand (mass m_2) with dynamic equipment (mass m_1) installed on it. The mass m_3 in this system is a nonlinear TMD.

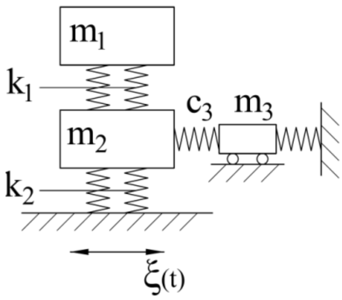

Figure 1. Calculation scheme of 2 degrees of freedom system with a TMD

The equations of motion of such system have the form:

$$\begin{cases} m_1 \dfrac{d^2 x_1}{dt^2} + \left(1 + 2\nu_1 \dfrac{d}{dt}\right) k_1 (x_1 - x_2) = -m_1 \dfrac{d^2 \xi(t)}{dt^2}; \\ m_2 \dfrac{d^2 x_2}{dt^2} - \left(1 + 2\nu_1 \dfrac{d}{dt}\right) k_1 (x_1 - x_2) + \left(1 + 2\nu_2 \dfrac{d}{dt}\right) k_2 x_2 + \left(1 + 2\nu_3 \dfrac{d}{dt}\right) c_3 (x_2 - x_3) = -m_2 \dfrac{d^2 \xi(t)}{dt^2}; \\ m_3 \dfrac{d^2 x_3}{dt^2} - \left(1 + 2\nu_3 \dfrac{d}{dt}\right) c_3 (x_2 - x_3) = 0, \end{cases} \tag{1}$$

where m_1, m_2, m_3 – masses of elements of the system;
k_1, k_2, c_3 – stiffness of elements of the system;
x_1, x_2, x_3 – horizontal displacements of elements of the system;
ν_1, ν_2, ν_3 – damping coefficients of the system;
$\xi(t)$ – function of ground motion.

The reaction in the link connecting the TMD is determined in accordance with the equation:

$$c_3 (x_2 - x_3) = k_n \left(1 + \alpha (x_2 - x_3)^2\right)(x_2 - x_3). \tag{2}$$

The characteristics of the system under consideration are as follows: m_1 = 1 t; m_2 = 10 t; m_3 = 1 t; $k_1 = 30 \cdot 10^3$ *kN/m*; $k_2 = 58 \cdot 10^3$ *kN/m*; k_n = 5000 *kN/m*; α = 40.

In accordance with the method of special selection of generating linear system proposed by Prof. Chernov Yu T [8], we denote the stiffness of the TMD in the generating system by k_d.

Then the equations of motion of generating linear system take the form:

$$\begin{cases} m_1 \dfrac{d^2 x_1}{dt^2} + \left(1 + 2\nu_1 \dfrac{d}{dt}\right) k_1 (x_1 - x_2) = -m_1 \dfrac{d^2 \xi(t)}{dt^2}; \\ m_2 \dfrac{d^2 x_2}{dt^2} - \left(1 + 2\nu_1 \dfrac{d}{dt}\right) k_1 (x_1 - x_2) + \left(1 + 2\nu_2 \dfrac{d}{dt}\right) k_2 x_2 + \left(1 + 2\nu_3 \dfrac{d}{dt}\right) k_d (x_2 - x_3) = -m_2 \dfrac{d^2 \xi(t)}{dt^2}; \\ m_3 \dfrac{d^2 x_3}{dt^2} - \left(1 + 2\nu_3 \dfrac{d}{dt}\right) k_d (x_2 - x_3) = 0, \end{cases} \quad (3)$$

In the first approximation, the displacements of masses are determined by the following formulas:

$$x_{i,1}(t) = x_{i,0}(t) - h_i(t), \quad (4)$$

where $h_i(t) = \sum_{j=1}^{3} \int_0^t f_j(t) V_{ij}(t - \tau) d\tau$, (5)

where, after neglecting damping components we have:

$$f_1(t) = 0; \; f_2(t) = c_3(x_2 - x_3) - k_d(x_2 - x_3) = (k_n - k_d)(x_2 - x_3) + k_n \alpha (x_2 - x_3)^3; \quad (6)$$

$$f_3(t) = -f_2(t). \quad (7)$$

With zero initial conditions and with the function of ground motion $\frac{d^2 \xi(t)}{dt^2} = \xi_0 \sin \omega t$ displacement in the linear generating system $x_{i,0}(t)$ takes the form:

$$x_{i,0}(t) = X_{i,0} \sin \omega t. \quad (8)$$

The matrix equation of motion of system (3) has the form:

$$A = \begin{pmatrix} k_1 - m_1\omega^2 & -k_1 & 0 \\ -k & k_1 + k_2 + k_d - m_2\omega^2 & -k_d \\ 0 & -k_d & k_d - m_3\omega^2 \end{pmatrix}; \; B = \begin{pmatrix} -m_1\xi_0 \\ -m_2\xi_0 \\ 0 \end{pmatrix}. \quad (9)$$

Solving the linear system, we obtain the following expressions for the displacements of masses $x_{i,0}(t)$ without taking damping into account:

$$x_{i,0}(t) = -\frac{\xi_0 C_i(\omega)}{D(\omega)} \sin \omega t; \quad (10)$$

where $C_1(\omega) = m_1\left[\left(k_1 + k_2 + k_d - m_2\omega^2\right)\left(k_d - m_3\omega^2\right) - k_d^2\right] + m_2 k_1 \left(k_d - m_3\omega^2\right)$; (11)

$$C_2(\omega) = \left(k_d - m_3\omega^2\right)\left[m_1 k_1 + m_2\left(k_1 - m_3\omega^2\right)\right]; \quad (12)$$

$$C_3(\omega) = k_d\left[m_1 k_1 + m_2\left(k_1 - m_3\omega^2\right)\right]; \quad (13)$$

$D(\omega)$ – the determinant of the system (3), equals

$$D(\omega) = D_1 k_d + D_0, \quad (14)$$

where $D_1 = \left(k_1 - m_1\omega^2\right)\left(k_1 + k_2 - m_2\omega^2 - m_3\omega^2\right) - k_1^2$; (15)

$$D_0 = -[\left(k_1 - m_1\omega^2\right)\left(k_1 + k_2 - m_2\omega^2\right) - k_1^2] m_3\omega^2. \quad (16)$$

The function $f_2(t)$ takes the form:

$$f_2(t) = -(k_n - k_d)\frac{\xi_0 \Delta C(\omega)}{D(\omega)} \sin \omega t - k_n \alpha \frac{\xi_0^3 \Delta C^3(\omega)}{D^3(\omega)} \cdot \frac{3 \sin \omega t - \sin 3\omega t}{4}, \quad (17)$$

where $\Delta C(\omega) = C_2(\omega) - C_3(\omega) = -m_3\omega^2[m_1 k_1 + m_2\left(k_1 - m_1\omega^2\right)]$. (18)

In accordance with the method based on the special selection of generating systems, we equate the coefficients of the fundamental harmonic $\sin \omega t$ in $f_2(t)$ to zero. After simplification, we obtain the following equation:

$$4D_1^2 k_d^3 + 4D_1\left(2D_0 - D_1 k_n\right)k_d^2 + 4D_0\left(D_0 - 2D_1 k_n\right)k_d - 4D_0^2 k_n - 3k_n \alpha \xi_0^2 \Delta C^2(\omega) = 0 \qquad (19)$$

The algorithm for constructing the amplitude-frequency characteristic is as follows. For any values of frequency ω and amplitude ξ_0 of the excitation force, the corresponding optimum stiffness of the generating system k_d (from equation (19)) is determined, the solution of which gives the minimum error with respect to the initial nonlinear system. Then, the amplitudes of mass oscillations (taking damping into consideration) of the system are determined by the formula ([8], [9]):

$$X_{i,0} = -\frac{\xi_0}{B}\sum_{s=1}^{3}\frac{C_i(p_s)\cdot R(s)}{p_s^2 A_s} \quad (i = 3), \qquad (20)$$

where $B = m_1 m_2 m_3\left(p_3^2 - p_2^2\right)\left(p_3^2 - p_1^2\right)\left(p_2^2 - p_1^2\right)$;

$C_i(p_s)$ – using formulas (11)–(13);

p_s – natural frequencies of the system (9);

$R(s) = p_{1+\mathrm{Rem}(s,3)}^2 - p_{1+\mathrm{Rem}(s+1,3)}^2$;

$\mathrm{Rem}(s,3)$ – the remainder of dividing the number of the normalized eigenform s by 3;

$A_s = \left(\left(1 - \omega^2 / p_s^2\right)^2 + \gamma_s^2\right)^{1/2}$;

γ_s – damping coefficients corresponding to normalized eigenforms.

Amplitude of ground acceleration $\xi_0 = 1\ m/s^2$.

The amplitudes of displacement of mass m_3 are shown in figure 2. The amplitude-frequency characteristic for the system without TMD is also constructed .

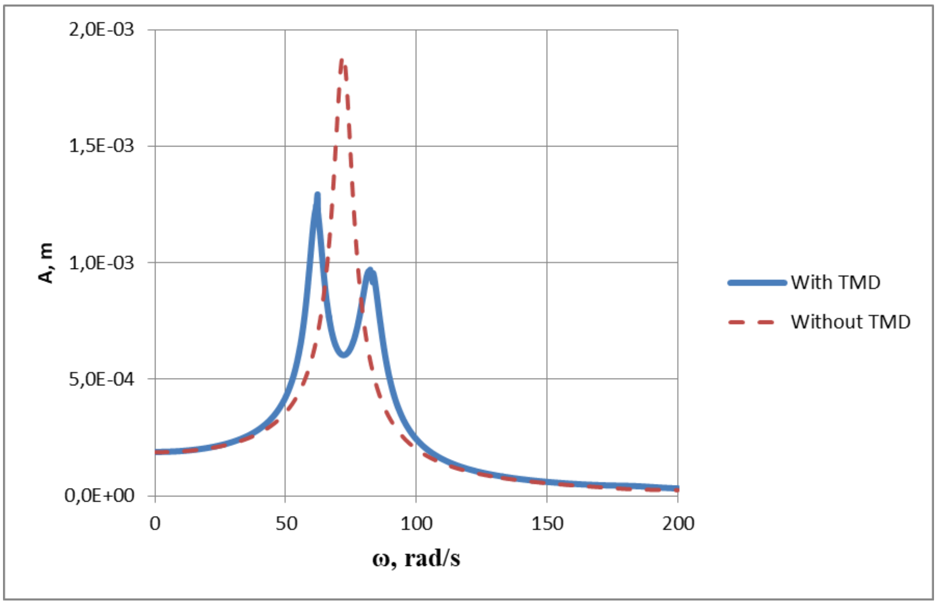

Figure 2. Amplitude-frequency characteristics for the lower masses of systems: with TMD (blue solid line); without damper (red dotted line).

Since according to the existing practice, TMD is usually tuned to the first frequency of the system without a damper, then we will evaluate their efficiency by comparing the amplitudes of the lower mass of the system with and without the damper. For the system under consideration, this frequency is

71.9 rad/s. When using a nonlinear TMD, the amplitude of displacements of the lower mass at this frequency decreases by 3.1 times.

2.2. Second numerical example

In the second numerical example, we considered a system with 3 degrees of freedom, the design scheme of which is shown in figure 3. The system is a stand (mass m_3) on which an equipment (mass m_2) with a nonlinear TMD m_1 is installed. The characteristics of the system are as follows: m_1 = 1.5 t; m_2 = 8 t; m_3 = 15 t; $k_2 = 20\cdot 10^3$ *kN/m*; $k_3 = 90\cdot 10^4$ *kN/m*; $k_\text{n} = 10\cdot 10^3$ *kN/m*; $\alpha = 40$.

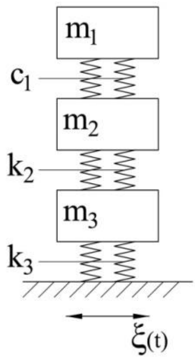

Figure 3. Calculation scheme of 3 degrees of freedom system

The reaction in the link connecting the TMD is determined by the expression:

$$c_1(x_1-x_2)=k_\text{n}\left(1+\alpha(x_1-x_2)^2\right)(x_1-x_2). \tag{21}$$

The steps of solving is similar to the solution in the first numerical example.

We denote the stiffness of the TMD in the generating system by k_d. The equations of motion of the generating linear system will have the form:

$$\begin{cases} m_1\dfrac{d^2x_1}{dt^2}+\left(1+2\nu_1\dfrac{d}{dt}\right)k_\text{d}(x_1-x_2)=-m_1\dfrac{d^2\xi(t)}{dt^2}; \\ m_2\dfrac{d^2x_2}{dt^2}-\left(1+2\nu_1\dfrac{d}{dt}\right)k_1(x_1-x_2)+\left(1+2\nu_2\dfrac{d}{dt}\right)k_2(x_2-x_3)=-m_2\dfrac{d^2\xi(t)}{dt^2}. \\ m_3\dfrac{d^2x_3}{dt^2}-\left(1+2\nu_2\dfrac{d}{dt}\right)k_2(x_2-x_3)+\left(1+2\nu_3\dfrac{d}{dt}\right)k_3x_3=-m_3\dfrac{d^2\xi(t)}{dt^2}, \end{cases} \tag{22}$$

In the first approximation, the masses are determined by the formulas (4).

Functions $f_i(t)$ have the form:

$$f_1(t)=c_1(x_1-x_2)-k_\text{d}(x_1-x_2)=(x_1-x_2)(k_\text{n}-k_\text{d})+k_\text{n}\alpha(x_1-x_2)^3;\ f_2(t)=-f_1(t); \tag{23}$$

$$f_3(t)=0. \tag{24}$$

The displacements in the system with harmonic external action are determined by formula (10), where in accordance with the formulas obtained by M.V. Volkova in [9], the formulas for $C_i(\omega)$ have the following forms:

$$C_1(\omega)=m_1\left[\left(k_\text{d}+k_2-m_2\omega^2\right)\left(k_2+k_3-m_3\omega^2\right)-k_2^2\right]+m_2k_\text{d}\left(k_2+k_3-m_3\omega^2\right)+m_3k_\text{d}k_2; \tag{25}$$

$$C_2(\omega)=\left(k_2+k_3-m_3\omega^2\right)\left(m_1k_\text{d}+m_2\left(k_\text{d}-m_1\omega^2\right)\right)+m_3k_2\left(k_\text{d}-m_1\omega^2\right); \tag{26}$$

$$C_3(\omega)=k_2\left(m_1k_\text{d}+m_2\left(k_\text{d}-m_1\omega^2\right)\right)+m_3\left[\left(k_1-m_1\omega^2\right)\left(k_\text{d}+k_2-m_2\omega^2\right)-k_\text{d}^2\right]; \tag{27}$$

$D(\omega)$ – is determined by formula (14), in which

$$D_1 = \left(k_2 + k_3 - m_3\omega^2\right)\left(k_2 - \left(m_1 + m_2\right)\omega^2\right) - k_2^2; \quad (28)$$

$$D_0 = -m_1\omega^2\left[\left(k_2 + k_3 - m_3\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2\right]. \quad (29)$$

the function $f_1(t)$ takes the form of (17), where

$$\Delta C(\omega) = C_1(\omega) - C_2(\omega) = m_1 k_2 k_3. \quad (30)$$

The optimum stiffness of the damper in the linear generating system is determined for each angular frequency ω by equation (19), where $\Delta C(\omega)$ is determined by formula (30).

The amplitudes of mass oscillations of the system are determined by formula (20), where $C_i(p_s)$ is calculated from formulas (25) – (27); formulas for the remaining variables are as defined in formula (20).

The amplitude of ground acceleration is taken as $\xi_0 = 1\ m/s^2$.

The obtained amplitudes of displacements of mass m_3 are shown in figure 4.

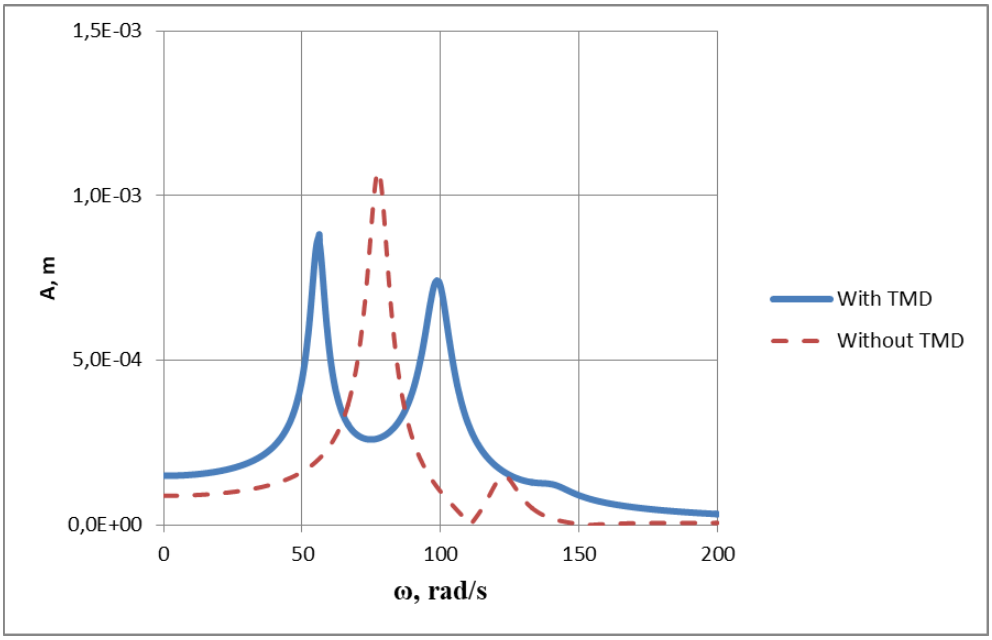

Figure 4. Amplitude-frequency characteristics of systems: with TMD (solid line); without damper (dotted line)

For the system under consideration, the first natural frequency of oscillations of the system without an absorber is 77.5 rad/s. When using a nonlinear TMD, the amplitude of displacements of the lower mass at this frequency decreases by 4.1 times.

3. Conclusion

In analyzed numerical examples there is a three times reduction in displacements in the nonlinear system using tuned mass dampers as compared to the linear systems.

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