

Mathematical modelling of water exchange in public swimming pools

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Abstract. The article presents information on open-air overflow swimming pools and their water exchange systems. It defines the principle of open-air swimming pool operation and shows the developed mathematical model of its water regime describing the change in concentration time of suspended solids in the pool basin water depending on the qualitative composition of the feed water (drinking or circulating water after cleaning). A number of design parameters (building volume of the pool basin, the water-surface area), technological indicators (initial concentration of suspended solids in water, feed water consumption and concentration of suspended solids), standard ratios (average number of pool users, the amount of pollution coming from one person, the maximum permissible concentration of suspended solids in the pool water), as well as meteorological conditions at the pool location (rain intensity and hours of rainfall, dustiness, and the volume of atmospheric air above the pool basin) have been analyzed as basic elements of pool operation. The article also provides information about a computer program for calculating the water regime of the pool and the results of an automated calculation of the pool performance period (in days) when the technological and regulatory parameters of its operation, designed to provide the required sanitary standards for the concentration of suspended solids in the pool basin water, change. Based on automated calculation, four alternative cases of the pool operation have been reviewed, in which the average number of pool users is used as a variable value of the standard parameter. The change dynamics of the duration (in days) of safe user stay in the pool have been determined and analyzed, indicating the need to reduce the time of its operation in proportion to the number of users, all other things being equal.

1. Introduction

Modern lifestyle in developed countries has a characteristic trend for the wide and intensive use of swimming pools, which can be classified as sports, Wellness (recreation), combined (mixed) and private ones [1, 2].

There are many types of various design structures of swimming pools, among which a separate place is attributed to closed and open ones for swimming and other purposes. They differ in layout design, a variety of equipment and measuring technique for water disinfection in swimming pools, systems of dosing reagents to regulate the indicators of the recycled water quality [3]. All pools, no matter whether they are closed or open ones, must meet the requirements for rational architectural and construction solutions, the water exchange rate to provide sanitary-hygienic requirements, energy



efficiency and other regulations [4, 5]. The swimming pools should not bear any risks to the health of visitors and athletes [6].

The most dangerous phenomena in the operation of swimming pools include the violation of their rules by visitors and the parameters, i.e. the temperature. The work [7] describes the situations, when a greater number of users of swimming pools results in an intensive water pollution due to the increased excess amount of sweat, the skin microparticles, saliva, sebum. The rate of sweat per hour at a water temperature of 29°C is of 0.1-0.2 l/m² and at 35°C it increases to 0.8 l/m² [8]. Other negative factors which affect the efficiency of the pools operation include violation of the technological mode of treatment and disinfection of water [9]. This circumstance causes an increased concentration of pollutants and the appearance of toxic substances [10, 11].

In different countries the questions of operation of swimming pools and provision of the required performance are carried out based on the requirements of normative-technical documents [12, 13]. A crucial role in the operation of swimming pools regardless of their type belongs to optimization of the water cycling pattern (concurrent flow, recirculation or filled) that takes into account many technological and other parameters, which provide sanitary requirements [14, 15]. Hence, the main task of the swimming pool staff is to find the ways of an effective control of the water quality values. It means regulation of various parameters of the swimming pool operation related to the number of visitors within a design period, preliminary bathing of swimmers before going to the pool and the analysis of the surrounding natural environment [16].

This article concerns theoretical studies presented as a model of the water conditions, which shall guarantee the fulfillment of the requirements for maintaining the established pool water norms as to concentration of the weighed substances by regulating some technological and structural indicators.

Such tasks can be categorized as managerial ones, which enable control and correlated quality regulation parameters of water and the hydraulic operation mode of the swimming pool [17, 18]. For these purposes it is expedient to use information technologies [19, 20].

2. Materials and methods

The object of research is a traditional outdoor swimming pool of the appropriate typical structural size, technological performances, regulatory and meteorological conditions. A simplified scheme of a public swimming pool with its main elements, which have been used in the mathematical model, is given in figure 1.

The main goal of the mathematical modeling was to reveal the possibility of effective management of the pool operation to ensure the established design health and sanitation requirements.

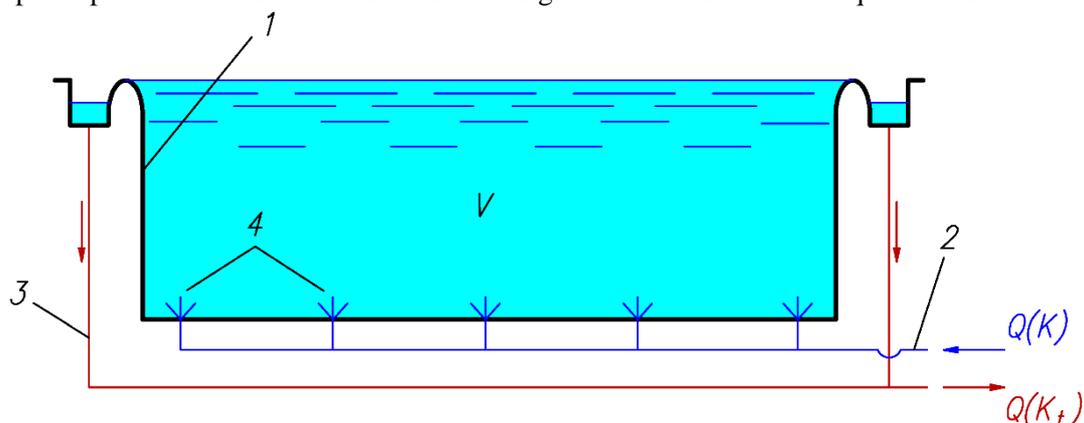


Figure 1. Schematic representation of the pool water bowl with some parameters

1 - pool water bowl; 2, 3 respectively, inflow and outflow water pipelines; 4 – nozzles for supplying recycled water; Q – make-up and drained flow rate of the circulating water, respectively, with the concentrations of suspended solids K and K_t ; V - volume of the pool water bowl

Table 1 presents a list of simulation elements and code numbers.

Table 1. The list of elements for modeling

Code of modeling	Modeling elements
1	The Volume of the pool bowl V, m^3
2	Surface area of water in the pool S, m^2
3	Initial concentration of suspended solids in water $K_0, mg/l$
4	The Average number of the pool users in 1-hour $N, person/h$
5	The Average number of suspended polluting particles n per 1 person in 1 g/h
6	Consumption of make-up (circulation) water $Q, l/h$ per 1 person
7	Concentration of suspended solids in the make-up water $K, mg/l$
8	Maximum allowable concentration of suspended substances in the pool water $K_t^*, mg/l$
9	Rain Intensity $I, l/s$ per 1 <i>hectare</i>
10	Raining hours during 24 h t_1, t_2, \dots, t_n
11	Maximum dust content of atmospheric air over a pool during the rainy periods $Z, g/m^3$
12	Volume of air above the water surface W, m^3

Hereinafter there is the information on individual elements of the model and the methods of compiling balance equations for contaminants penetrating to the pool.

At the beginning of operation, the bowl of the pool is filled with water to a volume V . C contaminations (the value is taken conventionally) rich the pool water from the users every hour. The pool water is drained from the bowl in the quantity Q and is flown back feeding the water amount equal to the deviated one with the suspended solids concentration K . The initial concentration of suspended solids in the pool water makes K_0 .

If a pool is considered as an ideal mixer, i.e. a construction with evenly distributed water pollutants, the concentration of suspended solids will increase by Kdt within the time interval dt . The total amount of suspended solids will make VdK_t . It includes the suspended solids coming from the users of the pool minus some of their number, leaving with the draining water during the time dt , as well as the suspended solids inflowing with the feed water $QKdt$. Ignoring the change in K_t concentration in the pool water for an infinitely small period of time, the quantity of suspended solids leaving with drained water will be expressed with the formula $QKdt$.

The basic equation of balance of contaminants entering the pool water and leaving the outlet of the bowl, will be:

$$VdK_t = QKdt + Cdt - QK_tdt \quad (1)$$

Hence, separating the variables, we write the equation (1) in the form (2):

$$\frac{VdK_t}{QK + C - QK_t} = dt \quad (2)$$

Subsequent operations come to identification of the expressions in the numerator and denominator of the left side of the equation through algebraic transformations to bring the expression under the integral sign to a well-known integral. The first step of integrating the left part of equation (2) is the imposition of the integral sign of the constant values of V and Q in the form of (3):

$$-\frac{V}{Q} \int \frac{d(-K_t)}{K + C/Q - K_t} \quad (3)$$

Re-generating algebraic transformations in the form of adding constant values $QK + C$ under the sign of the differential, we obtain the equation (4):

$$-\frac{V}{Q} \int \frac{d(QK + C - K_t)}{QK + C - QK_t} = \int dt + B \quad (4)$$

$$\text{or } \ln(QK + C - QK_t) = -\frac{Q}{V} (t + B)$$

where B – arbitrary constant of integration.

The general solution of the equation has the form (5):

$$QK + C - QK_t = e^{-\frac{Q}{V}t} \cdot e^{-\frac{Q}{V}B} \quad (5)$$

Since is constant, we express it as a constant of integration P and the equation (5) shall be written in the following form (6):

$$QK + C - QK_t = P \cdot e^{-\frac{Q}{V}t} \quad (6)$$

At the initial conditions, i.e. when $t = 0$ and $K_t = K_0$, the P value shall be determined as:

$$P = QK + C - QK_0$$

The found P constant of integration shall be put in the equation (6), from which we obtain three formulas to calculate the concentration of suspended solids in the pool water at any one time for the three modes of the water make-up:

a). for the case of initial concentration K_0 of suspended solids in water and K concentration of suspended solids in make-up water (I make-up mode, equation 7):

$$K_t = \left(K + \frac{C}{Q}\right) \cdot \left(1 - e^{-\frac{Q}{V}t}\right) + K_0 e^{-\frac{Q}{V}t} \quad (7)$$

b). for the case of initial concentration of suspended solids in water K_0 and $K=0$ concentration of suspended solids in make-up water (II make-up mode, equation 8):

$$K_t = \frac{C}{Q} \left(1 - e^{-\frac{Q}{V}t}\right) + K_0 e^{-\frac{Q}{V}t} \quad (8)$$

c). for the case of initial concentration of suspended solids in water $K_0=0$, i.e. $K=0$ when filling the pool bowl and make-up by potable water supply (III make-up mode, equation 9):

$$K_t = \frac{C}{Q} \left(1 - e^{-\frac{Q}{V}t}\right) \quad (9)$$

d). for the effect of rain, which may begin and end at any moment of the working hours of an open swimming pool, the differential equation (1) according to the Euler method can be replaced by finitely differentiable one (10):

$$V \Delta K_t = (QK + C - QK_t) \Delta t \quad (10)$$

Then, the K_t value growth due to the contaminants, which are contained by the air above the water surface and are washed off by the rain will be defined as (11):

$$K_t = K_t + K_t^0, \quad (11)$$

where K_i^0 concentration of contaminants, which penetrate to the pool water bowl from the air within 1 hour at any time of 24 h.

Then ΔK_i shall be determined as (12):

$$K_t = K_i - K_i^0 \quad (12)$$

If we conventionally take the rainfall interval within an arbitrary hour, i.e. $\Delta t=1h$, in case of a rain at this hour of the day a new K_i^n shall be determined by substituting ΔK_i in the equation (10), being expressed as the equation (13):

$$K_i^n = K_i^0 + (1/V)(QK + C - QK_i) t \quad (13)$$

The obtained dependences enable calculation of the duration of the pool operation for the above (a-d) options subject to the required sanitation norms, technological parameters (e.g. the degree of the circulating water purification), meteorological conditions and the number of the pool users. This approach is useful, as it can contribute in practice to improvement of the pool staff management functions.

3. Results and discussions

Given the complexity of the manual calculation for the above proposed mathematical dependences, we developed a special automated program, and its dialog box is shown in figure 2.

Indicator	Description	Value
1	The Volume of the pool bowl V, cub m	393
2	Surface area of water in the pool S, sq m	240
3	Initial concentration of suspended solids in water K0, mg/l	0
4	The Average number of the pool users in 1 hour N, person/h	48
5	The Average number of suspended polluting particles n per 1 person in 1 g/h	0.4
6	Consumption of make-up (circulation) water Q, l/h per 1 person	60
7	Concentration of suspended solids in the make-up water K, mg/l	0
8	Maximum allowable concentration of suspended substances in the pool water Kt*, mg/l	2
9	Rain Intensity I, l/s per 1 hectare	150
10	Raining hours during 24 h	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (unchecked); 12, 13 (unchecked); 14, 15, 16, 17, 18, 19, 20, 21 (checked); 22, 23 (unchecked)
11	Maximum dust content of atmospheric air over a pool during the rainy periods Z, g/cub m	0.0008
12	Volume of air above the water surface W, cub m	240

Figure 2. The dialog window of the water exchange simulation program in swimming pools

The program enables simulation of the pool operation modes according to the values listed in the table of indicators. The goal of the computer-aided calculation is to determine the duration (in days) of a safe pool operation from the point of view of ensuring its sanitary and hygienic performances.

A swimming pool with 12 indicators presented in table 2 has been taken as a basic experimental option. For economic reasons the names of the indicators are given as a code number corresponding to the item 2 of table 1.

Table 2 shows the values for four alternative options, where the variable parameter is introduced under the code number 4, i.e. "the Average number of the pool users in 1 hour, N, "person/h" (highlighted in bold). Thus, the number of pool users increases each time by 20 persons, ranging from 40 to 100.

Table 2. Indicators of four alternative calculation options

Code number	Options			
	1	2	3	4
1	393	393	393	393
2	240	240	240	240
3	0	0	0	0
4	40	60	80	100
5	0.4	0.4	0.4	0.4
6	60	60	60	60
7	0	0	0	0
8	2	2	2	2
9	150	150	150	150
10	6, 12, 20	6, 12, 20	6, 12, 20	6, 12, 20
11	0.0008	0.0008	0.0008	0.0008
12	240	240	240	240

The results of the automated calculation show, that the dynamics of changes of a user safe stay duration in the pool (within 24h) have been reducing proportionally to the number of users, when the other parameters remain permanent (table 3). At the given parameters and the number of the pool users, for example, 100 persons per hour, the pool water bowl shall be completely emptied 24 hours after the beginning of its operation.

Table 3. Dynamics of a user stay duration in a swimming pool

Number of the pool users N, person/h	40	60	80	100
Safe duration of the pool operation T, h	60 (2 days and 12 h)	40 (1 day and 16 h)	30 (1 day and 6 h)	24 (1 day)

With the use of the above presented automated program several other tasks, which are aimed at controlling the pool operation, may be solved to ensure the required quality parameters of the water in the pool bowl.

4. Conclusions

1. Herein above is given a brief analysis of potential reasons of deterioration of the water quality parameters in the swimming pools, as well as a range of tasks aimed at improving the efficiency of the pool work. The list of elements has been examined to provide modelling to find the optimal pool operation mode for maintaining the desired concentration of suspended solids.
2. The algorithm and the automated calculation program of the swimming pool water mode, allowing to normalize by modelling the number of operational and other parameters to maintain the desired water quality parameters.

3. The use of an automated calculation program can be considered as the most important element in the water exchange control procedure in the swimming pools and provision of its sanitary and environmental parameters.

References

- [1] Enrico U 2007 Swimming pool chemistry *Can. Chem. News.* **59** **6**, pp 11-12
- [2] Liguoria G Capellib G and Carraro E et all 2014 A new checklist for swimming pools evaluation: A pilot study *Microchemical Journal* **112** pp 181-185
- [3] Andreas D Wasser K 2003 *Wasserwirt.-Wassertechn* **1-2**, pp 43-44
- [4] Cloteaux A Gérardin F and Midoux N 2011 Numerical simulation and modelling of a typical swimming pool for disinfection by-products assessment *4th International Pool and Spa Conference*, Porto, Portugal
- [5] Cloteaux A Gérardin F and Midoux N 2013 Influence of swimming pool design on hydraulic behavior: A numerical and experimental study, *Engineering* **5** (**5**) pp 511-24
- [6] Chowdhury S Alhooshani K and Karanfil T 2014 Disinfection byproducts in swimming pool: Occurrences, implications and future needs *Water Research* **53** pp 68-109
- [7] Keuten M G A Peters M C F M Daanen H A M de Kreuk M K Rietveld L C and van Dijk J C 2014 Quantification of continual anthropogenic pollutants released in swimming pools *Water Research* **53** pp 259-70
- [8] Lewis L J Woodley I Colbourne J and Pond K 2011 The effects of user groups and rest periods on swimming pool water quality, *Proceedings of the 4th International Swimming Pool and Spa Conference* Porto
- [9] Mogucheva D Yu Orlov V A and Korsakov N A 2016 Disinfection of water in swimming pools *J Water Treatment* **97** pp 42-47
- [10] De Laat J Feng W Adams D Florence and Dossier-Berne 2011 Concentration levels of urea in swimming pool water and reactivity of chlorine with urea *Water Research* **45** pp 1139-46
- [11] Zare M Ernest A and Blatchley R 2015 Seasonal dynamics of water and air chemistry in an indoor chlorinated swimming pool *Water Research* **68** pp 771-83
- [12] SanPiN 2.1.2.1188-03 *Design, construction and operation of residential buildings, businesses, public utilities, educational institutions, culture, recreation, sport. Swimming pools. Hygiene requirements for the design, operation and water quality. Quality control* Moscow
- [13] *American National Standard for Water Quality in Public Pools and Spas.* APSP-11 2009 pp 1-62
- [14] PWTAG *Swimming Pool Water - Treatment and quality standards for pools and spas*, London 2nd Edition 2009
- [15] Ryan U Lawler S and Reid S 2017 Limiting swimming pool outbreaks of cryptosporidiosis – the roles of regulations, staff, patrons and research *Journal of Water and Health* **15**
- [16] Keuten M G A Schets F M Schijven J F Verberk J Q J C and van Dijk J C 2012 Definition and quantification of initial anthropogenic pollutant release in swimming pools *Water Research* **11** pp 3682-92
- [17] *Digitale Schwimmbadsteuerung TAB* Techn. Bau. 2003 **6** p 68
- [18] Yuce B Li H Rezgui Y Petri I Jayan B and Yang C 2013 Utilizing artificial neural network to predict energy consumption and thermal comfort level: An indoor swimming pool case study, Application of real-time PCR to *Pseudomonas aeruginosa* monitoring in a public swimming pool *Microchemical Journal* **110** pp 656-59
- [19] Barenboim G M 2013 By-products of disinfection of swimming pool water: substances, evaluation of their hazard and the ways of effective removal *J Water: chemistry and ecology* **12** pp 3-14
- [20] Yuce B Li H Rezgui Y Petri I Jayan B and Yang C 2014 Utilizing artificial neural network to predict energy consumption and thermal comfort level: An indoor swimming pool case study, *Energy and Buildings* **80**, pp 45-56