To the construction of optimal motions of controlled mechanical systems

R Z Khayrullin

Department of Applied Mathematics, Moscow State University of Civil Engineering, Doctor of Physics and Mathematics, 26 Yaroslavskoye shosse, Moscow,129337, Russia

Abstract. The solution of optimal control problems for mechanical systems is an important practical problem. For the solution of the optimization problem can be used the necessary conditions of the extremum in the form of the maximum principle of L.S. Pontryagin. However, the direct solution of the boundary value problem of maximum principle associated to large computational difficulties. This is due to the nonlinearity of the dynamic system of equations, the need for chose a reasonable first approximation for the conjugate variables at initial time moment, the need for a joint integrate of both the primary and the conjugate system with simultaneous selection of control function from the condition of maximum of the Hamiltonian. The latter circumstance often degrades (or breaks) the properties of continuous dependence of the residuals of the boundary value problem (usually, the values of the conjugate variables in a finite time moment) of variable parameters (typically, the values of the conjugate variables at initial time moment). The effective technology for the study of mechanical systems is developed in the article. The core technology is the integrated use of Direct Optimization Methods for dynamic systems (the method of successive linearization and its modifications); Methods of Solution of Boundary Value Problems (standard methods, based on the many times numerical solution of the system of algebraic equations that provide the required boundary conditions of the maximum principle); Qualitative Methods of study the structure of optimal control functions; Methods for constructing "exact" optimal control function, taking into account the features previously identified properties of the optimal control functions (methods of parametrization of the set of control function); Construction of Simple Techniques to calculate optimal motions of mechanical systems. The results of solution of the following tasks are presented: the problem of optimal control the maximum of angle of rotation of the excavator - dragline on a fixed time interval with finite damping of the oscillations occurring bifilar suspended from the boom of the bucket; the problem of optimal control for movement of foot of the walking machine when it step over through the obstacle.

1. Introduction

It is known that the solution of the boundary value problem (BVP) of Maximum principle of L.S. Pontryagin often causes a rather large computational challenges [1-4]. This is due to the nonlinearity of the original system of equations; to a reasonable chose the first approximation for adjacent variables in the initial time moment; the need for a joint integration of both the main and related systems with synchronous control function selection of the conditions of the maximum of the Hamiltonian. The latter circumstance often degrades (or breaks) the properties of continuous dependence of the residues of the boundary value problem (usually the values of the conjugate variables at the end of the time interval) from variable parameter (usually values of the conjugate variables at initial time moment).

Therefore, a successful parametrization of the family of control functions, in the class of which the optimal control function is sought, which provides certain properties of smoothness, and aimed at improving the reliability of the solution of the BVP seems to be an important practical task. Successful parametrization makes it possible to construct continuous dependence residual dependences of the discrepancies on the variable parameters. Sometimes it is possible so to choose the variable parameters and the corresponding discrepancies so that the need for a joint integration of the original

and the conjugate system is eliminated altogether. As a result, the volume of computations decreases and the convergence of the solution of the boundary value problem is accelerated.

In accordance with the technology proposed in this paper, the choice of a specific set of variable parameters is performed on the basis of an analysis of the form of the approximate solution of the initial problem by means of direct methods. After choosing a set of variable parameters and their corresponding residuals and obtaining the solution of the Auxiliary Boundary Value Problem (ABVP), the conjugate variables for the original boundary value problem are reconstructed. And solution of BVP is checked for the fulfillment of the necessary conditions for the extremum. The fulfillment of all the necessary conditions for the BVP testifies to the correctness of the results obtained.

In a number of practical problems, the author succeeded in successfully implementing the parametrization of a set of control functions that ensures the continuity of residuals and the stability of solution of BVP [4]. We note that in the case of a small number of variable parameters, it becomes possible to construct simple methods for calculating the optimal control functions and the corresponding motions of the mechanical system under study.

2. Methods

The investigation of a mechanical system with the purpose of revealing its limiting possibilities begins with setting up a number of optimal control problems for different required values of integral and terminal functionals of the problem, including functionals that specify constraints on the current values of the phase coordinates. At the first stage, using the direct methods [2-4], we seek an approximate optimal control for this series of problems. At the second stage, we seek the qualitative structure of the optimal control law and the possibility of parametrization of a family of optimal control functions are investigated. At the third stage, we seek the possible variants of control parameterization are analyzed (for example, by means of relay functions with a predetermined number of switching operations or by means of relay functions combining with pieces of touch with to phase constraints) for the purpose of their use in solving the BVP.

3. The results of application of technology calculations to solving the practical problems

3.1. The problem of optimal control of the movement of the bucket of a dragline excavator
Various formulations of this problem were investigated in [5-7].

3.1.1. Formulation of the problem. The motion of the model "the arrow on the turntable platform and suspended bucket" (Fig. 2) is described under some   assumptions by the following system of differential equations [2]:

$$d^2\sigma/dt^2 + g\sigma/l = d^2\varphi/dt^2, \quad J_0 d^2\varphi/dt^2 = M \qquad (1)$$

where φ - the angle of rotation of the platform of the excavator - dragline around the vertical axis, σ - the bucket deviation angle from the plane of the boom, J_0 - the moment of inertia of the system relative to the axis of rotation of the platform, M - the driving force moment of the platform turning relative to the stationary base of the excavator, l - conditional length of the suspension , g - acceleration of gravity, t - time of motion.
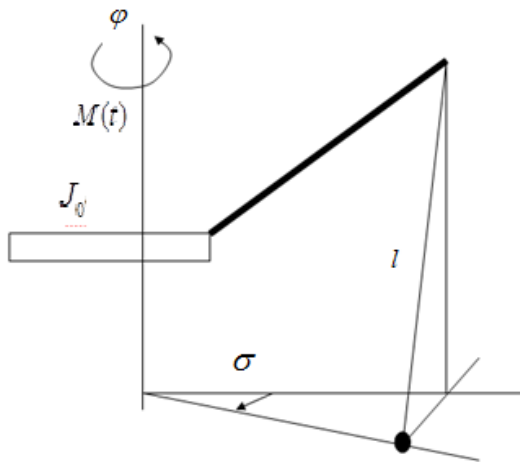
Fig. 1. Model "the arrow on the turntable platform and suspended bucket ".

The system of equations (1) is correct under the assumption that the mass of the ladle is negligibly small in comparison with the total mass of the boom and of the moving platform, the bucket cannot oscillate in the plane of the boom, the angle characterizing the deviation of the ladle from the plane of the boom is small [5,6].

As a control function, we choose $M(t)$ - the driving moment of the platform rotation relative to the fixed base. The length of the bucket suspension is assumed constant.

We will consider the motion of the model "an arrow on a turntable platform and a bucket suspended from an arrow" on a fixed interval of time $t \in [0,T]$. Let the system in the initial time $t = 0$ is in stationary state:

$$\sigma(0) = 0, \ \varphi(0) = 0, \ \dot{\sigma}(0) = 0, \ \dot{\varphi}(0) = 0. \tag{2}$$

It is required to construct a control function $M(t)$ restricted by magnitude M_{13}:

$$|M(t)| \le M_{13}, \ 0 \le t \le T, \tag{3}$$

for transferring the system in a fixed time T from the stationary state (2) to the desired final stationary state:

$$\sigma(T) = 0, \ \dot{\sigma}(T) = 0, \ \dot{\varphi}(T) = 0, \tag{4}$$

and providing the maximum deviation of the boom from the initial position:

$$\varphi(T) \rightarrow \max \tag{5}$$

At the same time, the maximum angular velocity of the boom rotation must not exceed a predetermined value C_1:

$$\max_{t} \left| \dot{\varphi}(t) \right| \le C_1 \tag{6}$$

3.1.2. Qualitative analysis of the structure of optimal control. The optimum control function can be either a relay control function that takes the maximum (minimum) possible values $M(t) = M_{13}$ either $M(t) = -M_{13}$, or a relay function conjugate with the driving segments over the phase constraints (6) at $M(t) = 0$.

Since the right-hand sides of the system of equations (1) are independent of σ, and the initial and final conditions are symmetrical (the system from one state of rest is transferred to another state of rest with the invariable suspension length), the control function $M(t)$ whose graph in the plane $(t, M(t))$ is symmetric with respect to straight $t = T/2$.

For case of constant control values: $M(t) = M_{13}$, $M(t) = -M_{13}$, $M(t) = 0$ the equations (1) can be integrated analytically. Phase portraits $(\sigma(t), \dot\sigma(t))$ and $(\varphi(t), \dot\varphi(t))$ are given in [5].

Depending on the specified final value of the driving time T for working cycles with a boom turn in the range $0° - 75°$, the integral of the normalized optimal control function $M(t)/J_0$ is:

$$\dot\varphi(t) = \int_0^t (M(t)/J_0)\,dt, \quad 0 \le t \le T, \tag{7}$$

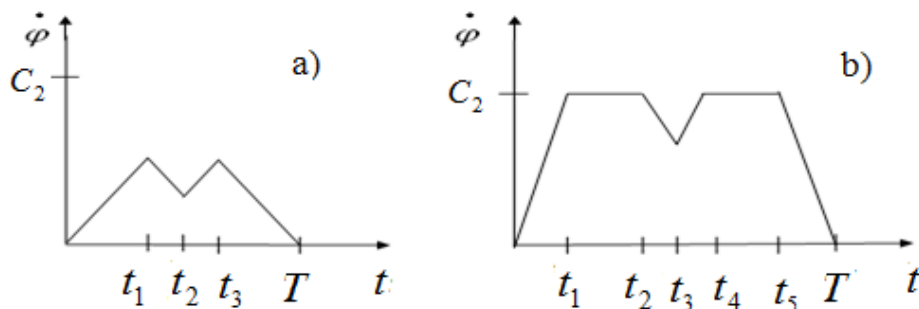and it belongs to one of the set of functions shown in Fig. 2.

Fig.2. To parametrization of a set of control functions.

3.1.3. Algorithms for the method of parametrization of a set of control functions. The set of control functions shown in Fig. 2a is parametrized by one parameter $\xi = t_2$. As a residual we choose $z = \sigma(T)$. Then ABVP reduces to solving one nonlinear equation $z(\xi) = 0$. Moments of time t_1 and t_3, uniquely determined from the restrictions on the time of motion T, the angular velocity of the turn of the platform and the symmetry conditions of the required control function. The constraints $\dot\varphi(T) = 0$ and $\dot\sigma(T) = 0$ will be satisfied automatically by virtue of symmetry [5, 6].

The set of control functions, shown in Fig. 2b, is parametrized by one parameter $\xi = t_3$. As a residual we choose $z = \sigma(T)$. Then ABVP reduces to solving one nonlinear equation $z(\xi) = 0$. Moments of time t_1 and t_2 uniquely determined from the restrictions on the time of motion T, the angular velocity of the turn of the platform and the symmetry conditions of the required control function. The constraints $\dot\varphi(T) = 0$ and $\dot\sigma(T) = 0$ will be performed automatically [5,6].

The cases of parametrization by two and three parameters are described in [5, 6].

Calculations have shown that the control functions constructed in this way and the corresponding trajectories of (1) together with the reconstructed conjugate variables satisfy the necessary conditions for the extremum in the form of the maximum principle of L.S. Pontryagin.

Note that the described algorithms, simultaneously, are the simple methods for calculating optimal trajectories and control functions.

A qualitative analysis of the necessary conditions of extremum for problem (1) - (6) showed that they are equivalent to the corresponding conditions of extremum for the problem of the fastest moving of the excavator - dragline bucket to a given point with finite damping of bucket oscillations [5].

3.2. *The problem of constructing the optimal foot motion of the Walking Machine (WM)*

Some variety problems of controlling the motion of WM and its individual parts were considered in [8-17].

3.2.1. Formulation of the problem. To describe the motion of the leg of the WM, we introduce the right coordinate system OXYZ with axes fixedly oriented in space. The OZ axis is directed vertically upwards, the axes OX and OY are in the horizontal plane. The leg of the WM consists of two links - the thigh and the shin. The thigh is connected to the body of the WM by means of a hinge with two degrees of freedom. The connection between the thigh and the shin is carried out by means of a hinge with one degree of freedom. The plane passing through the shin and thigh will be called the plane of the foot. As generalized coordinates, we choose the angles $\{\alpha_i, i=1,2,3\}$ shown in Fig. 3.
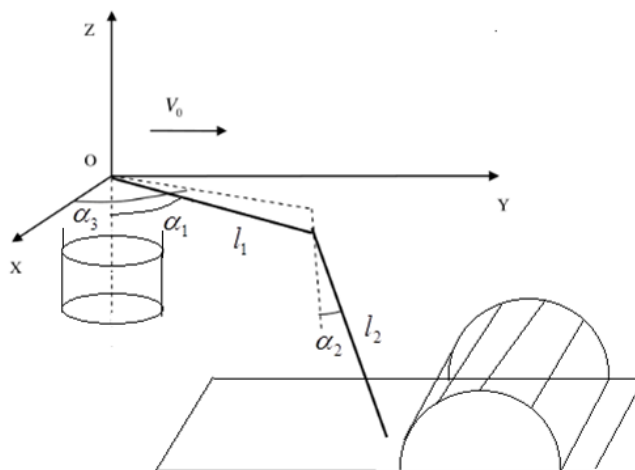

Fig. 3. Kinematics of the leg, an obstacle in the form of a circular half-cylinder, a technological restriction in the form of a circular cylinder.

As control functions $\{u_i, i=1,2,3\}$ we choose the force moments in the hinges of the foot. The motion of the leg in the transfer phase is described by the following system of differential equations [11, 12]:

$$dx_i / d\tau = f(x_1,...,x_6,u_1,u_2,u_3,T), \ dx_{i+3}/d\tau = x_i, \ i=1,2,3, \ 0 \le \tau \le 1, \qquad (8)$$

where τ - the dimensionless time, T - the duration of the stepping cycle, $\{x_{i+3}=\alpha_i, \ x_i=da_i/dt, i=1,...,3\}$ - the angular coordinates and the angular velocities of the leg links, respectively.

At the moment $\tau = 0$ of the beginning of the movement, the angular coordinates and velocities are determined by $\{x_i(0) = x_{i0}, \quad i = 1,...,6\}$, and the velocity of the foot at the initial instant of time is directed vertically upwards [11,12].

The body of the WM performs rectilinear uniform motion with speed V_0. The contact of the foot with the supporting surface (reference plane) is considered point. In the future, the end of the foot will be called a foot.

Let us list the restrictions on the phase-coordinates [11,12]. These restrictions are described using differentiable functional by sense of Gato [2]:

A) During the movement of the foot, it could not fall below the reference plane;

B) When moving a leg through an obstacle, the foot must not fall into the obstacle (an obstacle having the shape of a circular half cylinder, located perpendicular to the direction of movement of the WM (Fig. 3));

C) During the movement, the leg should not be "strongly compress": (the foot should not intersect the surface of the circular cylinder with the vertical axis passing through the point of the hanging of the leg (Fig.3).This limitation is related to the technological limitations of the design of the WM.)

As a minimized functional, we choose the integral functional, which characterizes the energy costs arising from the use of electric drives for control in the hinges of the leg of the WM.

$$F_0[u(.)] = T \cdot K \int_0^1 \sum_{i=1}^3 u_i^2(t)\,dt \to \min , \tag{9}$$

where K is the coefficient.

At the final moment of time T, the following conditions should be met (provide the driving the foot to a given point of the reference plane with a given vertical velocity):

$$\{x_i(T) = x_{iK}, i = 1,...,6\} \tag{10}$$

3.2.2. Structure of the optimal control law. Fig. 4 shows the dependence of consumed energy of the time of foot transfer, corresponding to the optimal law of motion.
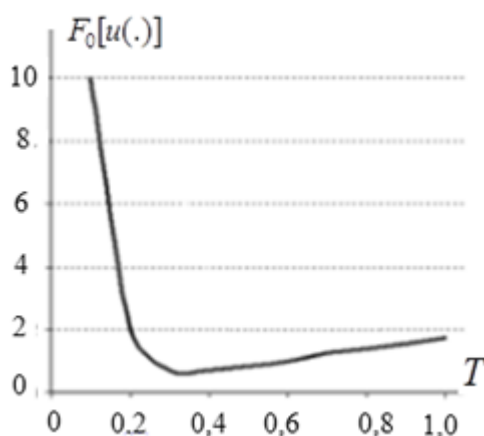

Fig. 4. Dependence of consumed energy on the dimensionless duration of the stepper cycle

The presence of a pronounced minimum agrees well with the results of [8-10]. Note that for sufficiently large values of motion time, the energy expenditure T is close to linear, and for $T \to 0$ the energy expenditure increases indefinitely.

Based on the results of calculations, the structure of the optimal law of movement of the leg of the WM is revealed. Let us describe its structure qualitatively. At the beginning of the stepper cycle, the foot breaks away from the reference point with a vertical speed. Further, the foot moves at a small height above the reference plane, while the length of the projection of the radius-vector of the foot onto the reference plane decreases. The leg is trying to creep up to the vertical. However, reaching the limit given by the circular cylinder with the vertical axis of the foot begins to move along the surface of the cylinder. At the same time, the plane of the foot turns. Next, the leg comes to restriction in the form of a half cylindrical pipe which lying across its path. Next, the foot move along the surface of the obstacle in the form of a half-cylindrical pipe which lying across its path. Further, the foot breaks away from its surface and continues to move along the surface of a circular cylinder with a vertical axis. Then the foot smoothly goes away from the boundary of the vertical cylinder, descends to a small height and moves approximately staying at this height. The length of the projection, the radius of the foot vector on the reference plane gradually increases. Finally, the foot drops vertically to the desired trace point.

When solving the corresponding ABVP, the family of control functions was parametrized by the moments of entry into the phase constraints and the moments of leaving the indicated limitations.

4. Discussion

Despite the currently available software tools for modeling mechanical systems and software for researching high dimension systems [18, 19], the technology of studying controlled mechanical systems described in the article remains relevant, because the technology allows to consider a wide set of restrictions on the parameters of motion of mechanical systems. Such set of restrictions cannot be taken into account in the standard software for modeling and optimization.

5. Orders

The developed technology for constructing optimal motions of controllable mechanical systems was effectively used over a number of years to solve practical problems of optimal control of the trajectories of the spacecraft when entering the atmosphere [3,4]. The developed technology can be used for solving optimization problems in the field of construction and housing and communal sectors.

References

[1]   Pontryagin L S, Boltyansky V G, Gamkrelidze R V and Mishchenko E F 1961 *Mathematical theory of optimal processes* Moscow Science Press p 391
[2]   Fedorenko R P 1978 *Approximate solution of optimal control problems* Moscow Science Press p 488
[3]   Golubev Yu F, Seregin I A and Khayrullin R Z 1991 Method of floating nodes Technical Cybernetics Journal **2** pp 33-40
[4]   Khayrullin R Z 1994 Area of manoeuvring of the spasecraft when entering the atmosphere with near-circular velocity and a large angle of entrance. Moscow Preprint of Institute of applied mathematics 63 p 28
[5]   Khayrullin R Z, Pevzner L D and Goryunov V Yu 1998 Optimal motion control of the bucket of the dragline. Moscow Preprint of Institute of applied mathematics Russian Academy of Sciences 72 p 28
[6]   Khayrullin R Z 2010 To study maneuvering capabilities of the dragline Vestnik MGSU Journal **4** pp 49-53
[7]   Khayrullin R Z 2015 To the construction of the synthesis of quasi-optimal motion control of excavator bucket dragline Natural and Technical Sciences Journal **3(81)** pp 22-24
[8]   Okhotsimsky D E and Golubev Yu F 1984 *Mechanics and movement control of the automatic walking machine* Moscow Science Press p 310
[9]   Lapshin V V 1993 Model estimates of the energy consumption of the walking machine Proceedings of the Russian Academy of Sciences Mechanics of a rigid body Journal **1** pp 65-74

[10] Lapshin V V 1983 Dynamics and control of the movement of the jumping machine Proceedings of the USSR Academy of Sciences Mechanics of solid Journal **5** pp 42-51

[11] Golubev Yu F, Degtyareva E V and Khayrullin R Z 1990 The method of sequential linearization in problems of searching for the optimal stepping cycle of a walking machine Proceedings of the Russian Academy of Sciences, Technical Cybernetics Journal **2** pp 214-223

[12] Golubev Yu F, Degtyareva E V and Khayrullin R Z 1989 Optimal control of the movement of the foot of the walking machine in the transfer phase Moscow Preprint of the Institute of Applied Mathematics of the Russian Academy of Sciences **59**, 1989, 28 p.

[13] Okhotsimsky D E, Platonov A K, Pavlovsky V E and Golubev Yu F 1982 Synthesis of the walking robot's movement in overcoming isolated obstacles. Moscow Information and Control Systems of Robots Journal Moscow Institute of Applied Mathematics Press pp 186-200

[14] Golubev Yu F 2003 Robot-balancer on the cylinder Proceedings of the Russian Academy of Sciences Applied mathematics and mechanics Journal **67**(4) pp 603-619

[15] Golubev Yu F and Koryanov V V 2005 Construction of motions of an insectomorphous robot overcoming the combination of obstacles by means of Coulomb friction forces Proceedings of the Russian Academy of Sciences Theory and control systems Journal **3** p143-155.

[16]   Golubev Yu F and Koryanov V V 2014 Climbing of an insectomorphous robot through a freely rolling ball Proceedings of the Russian Academy of Sciences Theory and control systems Juornal **5** pp 116-125.

[17] Golubev Yu F and Melkumova E V 2010 *Equilibrium of a biped robot on a rough horizontal cylinder with allowance for reactions along its axis* Moscow MAKS Press, p 61 (ISBN 978-5-317-03047-6)

[18]   Efimov G B and Pogorelov D Yu 1993 Universal mechanism - a software for simulation the dynamics of systems of many solids Preprint of the Institute of Applied Mathematics of the Russian Academy of Sciences **77** p 28

[19] Golubev Yu F and Pogorelov D Yu 1998 Computer simulation of walking robots Fundamental and Applied Mathematics Journal **4**(2) pp 525-534

[20] Pogorelov D Yu 2011 Algorithms for modeling the dynamics of systems of bodies with a large number of degrees of freedom Nizhny Novgorod Bulletin of the Nizhny Novgorod University named after N. I. Lobachevsky **4**(2) pp 278-279