

A new algorithm for size optimization of the truss structures using finite element method

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Abstract. The paper presents a new algorithm for size optimization of the trusses using finite element method. The objective function was established based on weight minimization of trusses. The constraint conditions were formulated based on the conditions that the structure satisfies the strength, stiffness and compatibility requirements, and equilibrium equations. Determination of internal forces, displacements and establishment of the equations are based on finite element method. In order to solve optimization problems using finite element method, the authors proposed a new method for seeking optimal solutions. For truss weight optimization, the authors proposed a coefficient and called this coefficient as “the correlation coefficient of internal forces among truss elements”. This coefficient shows the relationship between the internal forces of truss elements, which was used as the basis for re-selecting the dimensions of cross section of truss elements in iterative process. The paper introduces the calculation procedure and analysis algorithm for weight optimization of planar and space trusses. The examples for weight optimization of planar and space trusses were implemented in a subroutine written in Matlab software. The calculation results using the method proposed by the authors matched with the calculation results using other methods such as Hybridized Genetic Algorithm, Harmony search method, HyperWork, etc. However, using the method proposed by the authors gives less iterative circles.

1. Introduction

Trusses are common structures in construction due to their outstanding advantages in material saving and maximum utilization of structure load capacity. Optimization of trusses structure is a problem of seeking best solution in Preliminary Design stage. Depending on design variables optimization problems of trusses are classified into three categories: size optimization, shape optimization and structure optimization. In order to solve the optimization problems, the mathematical and mechanical analyses are required, in which:

- Use the mechanics theories of deformable bodies to establish equilibrium equations (in the case of that displacements are variable functions) or compatibility equations (in the case of that stresses are variable functions). From there, problems of structure analysis are determination of stresses, strains, displacements and internal forces.
- Use the mathematical optimization theories to solve structural optimization problems with constraints determined from the equilibrium or compatibility equations of structural analysis problems.



- In the early study period, solving problems of truss structure optimization was mainly based on the analytical method, so that only simple problems could be solved. The development of information technology helps the solving optimization problems to achieve high efficiency. Using numerical methods, especially finite element method allows solving problems of structural analysis with any structure sizes and shapes. Programming languages, mathematical software are powerful tools for implementation to solve optimization problems.

The authors studied to solve weight optimization problems of plane trusses using finite element method to determine internal forces, displacements and to establish constraint conditions [2]. For solving optimization problem, the authors proposed a new iterative algorithm for seeking optimal solution and establishment of calculation program of size optimization of plane trusses by Matlab software. This article presents proposed algorithm which applies in solving problem of size optimization of plane and space trusses with different selections of design variables.

2. Establishment of size optimization problems trusses based on Finite Element Method [1, 3]

2.1. Optimization objective function

In size optimization problems, optimization objective function is chosen to find minimum of truss weight G .

$$G = \sum_{i=1}^N \gamma_i L_i A_i \rightarrow \min \quad (2.1)$$

Where N is number of design variable groups. Design variable is cross-section area A , for convenience in fabrication truss elements are divided into several groups of element having the same cross-section area and different lengths; L_i , A_i is the total length and cross-section area of i^{th} element group; γ_i is specific weight of material of group i .

In the case of homogeneous structures optimization objective function is the volume of the structure as follows

$$V = \sum_{i=1}^N L_i A_i \quad (2.2)$$

2.2. Constraints

- Strength of materials constraints

$$\begin{aligned} \sigma_{\max i} &\leq [\sigma]_i^{\text{ten}} \\ \sigma_{\min i} &\leq [\sigma]_i^{\text{com}} \end{aligned} \quad (2.3)$$

Where $[\sigma]_i^{\text{ten}}$; $[\sigma]_i^{\text{com}}$ are allowable stresses in tension and compression.

Stresses and internal forces of elements are determined by finite element method

$$\begin{aligned} \sigma &= \frac{E}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} (T_e \delta_e) \\ f &= \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} (T_e \delta_e) \end{aligned} \quad (2.4)$$

In which, E is elastic modulus; A is cross-section area; L is element length; T_e is coordinate transformation matrix; δ_e is element nodal displacement vector in global coordinate system.

- Displacements constraints

$$y_p \leq [y] \quad (2.5)$$

Where $[y]$ is allowable displacement with different positions of loads;

Displacement y is determined by finite element method

$$y = K^{-1} \cdot P \quad (2.6)$$

In which, K is stiffness matrix of truss; P is nodal load vector of truss

- Equilibrium constraints satisfies the equation (2.6) in solving problem using finite element method

3. The method of correlation between internal forces of optimum elements for weight optimization of trusses

Results of determination of displacements, stresses, internal forces of truss elements can be expressed as functions of design variables A as follows:

$$y_i = a_i \cdot A^{-1}; \quad \sigma_i = b_i \cdot A^{-1}; \quad f_i = c_i \quad (3.1)$$

Where a , b , c are constants determined from equations using finite element method

According to strength of material constraints (elements with the same strength of material) it is received:

$$\frac{f_1}{A_1} = \frac{f_2}{A_2} = \dots = \frac{f_i}{A_i} \quad (3.2)$$

From (3.1) and (3.2) can derive the relationship:

$$n_i = \frac{f_i}{f_1} = \frac{A_i}{A_1} = \frac{c_i}{c_1} \quad (3.3)$$

According to (3.3) the relationship between design variables A_i can be calculated from the correlation of elements internal forces. Thereof, the authors proposed a new method to solve problems of weight minimization of trusses with constraints established using finite element method. The basis content of the method can be expressed in algorithm of seeking of optimum values of design variables by correlation between elements internal forces. The authors call this method as “the method of correlation coefficient between element internal forces”. The order of solving problems of weight optimization of trusses by method of correlation between element internal forces includes following steps:

Step 1: Preliminary selection of cross-section sizes for design variable groups. Displacements y , stresses σ , elements internal forces f are determined using finite element method in the case of that the elements have the same design variables A . Determine $A_{\min}^{(i)}$ for design variable groups according to the constraints of strength material and displacement of all elements in groups as follows:

$$\begin{cases} y_i \leq [y] \\ \sigma_i \leq [\sigma] \end{cases} \Rightarrow A_{\min}^{(i)} = \max \left\{ \frac{a_i}{[y]}, \frac{b_i}{[\sigma]} \right\} \quad (3.4)$$

Preliminary determination of truss volume: $V_1 = \sum_{i=1}^N L_i A_i^{(i)}$

Step 2: Select cross-section sizes one more time $A_j = n_j A_1$ of design variable group “ j ” according to correlation coefficient between elements internal forces of design variable groups

$$\bar{n}_j = \frac{\sum f_j}{N \cdot f_1} \quad (3.5)$$

Where $\sum f_j$ is total element internal force in the same group; N is number of elements in the same group.

In principle, internal forces can be selected corresponding to $A_{\min}^{(i)}$ of any element as standard value f_1 . Groups of elements which have value approximately null should not selected to avoid errors

Determine truss volume: $V_2 = \sum_{i=1}^N L_i A_i^{(i)}$

Determine the percentage of decrease of cross-section area.

$$\frac{V_1 - V_2}{V_1} 100\% \quad (3.6)$$

These steps are implemented iteratively to n circle until archieving required percentage of decrease of cross-section area. The iterative method according to correlation between element internal forces can be represented in the form of a diagram as in Figure 1.

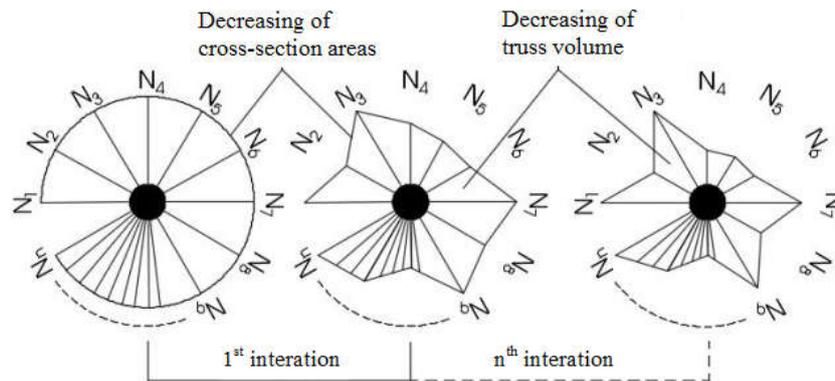


Fig. 1. Iterative diagram using correlation coefficient between element internal forces

4. Establishment of calculation programme for weight optimization of trusses using Matlab software

Based on the proposed above method of correlation coefficient between element internal forces the block diagram of algorithm is established (Fig.2) for weight optimization of plane and space trusses with any design variable group using Matlab software.

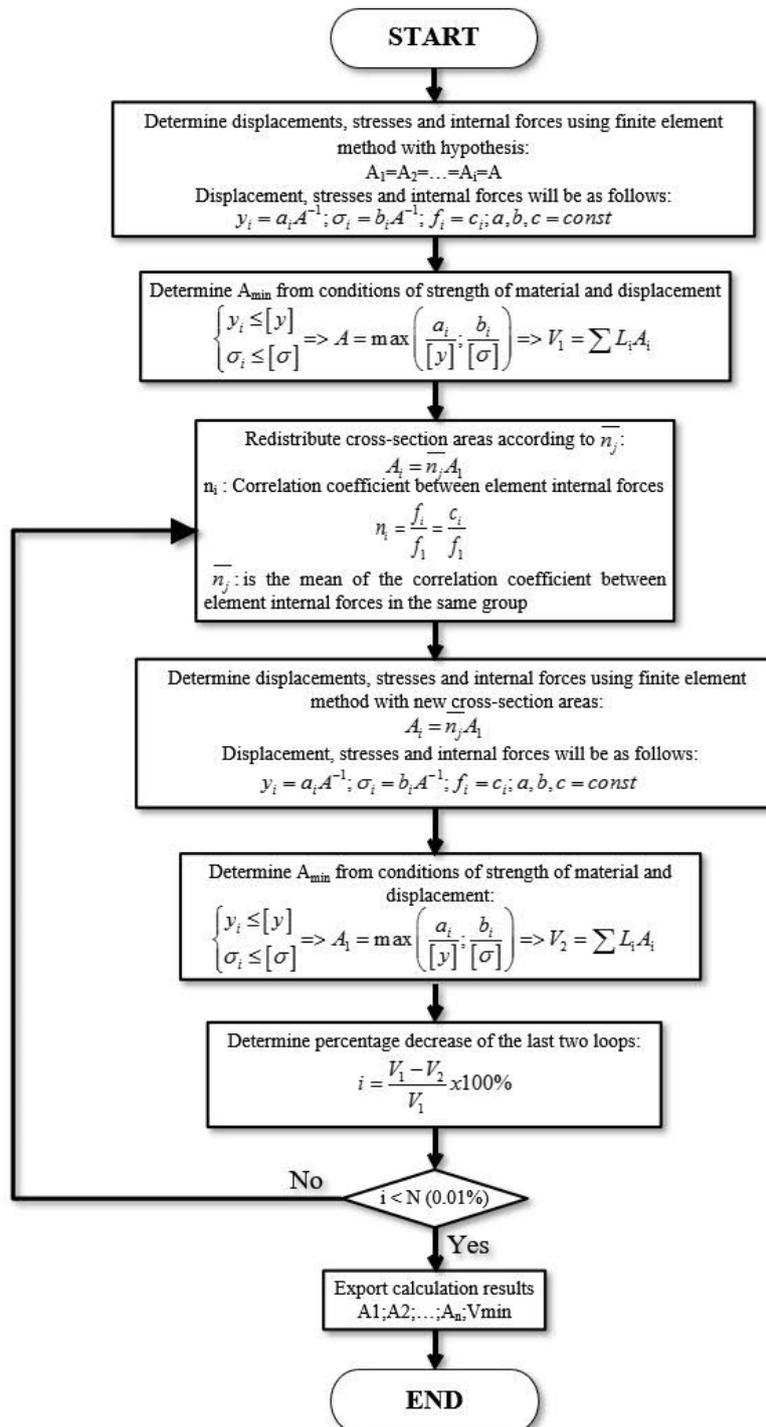


Figure 2. Block diagram of algorithm for weight optimization using method of correlation coefficient between element internal forces

5. Examples

The authors used the above established programme for optimization to solve several examples of determining optimum weight of plane and space trusses with any number of design variable group. Calculated results were compared with results implemented by other method in published studies [4-13].

5.1. Example of weight optimization of plane truss

Optimization of plane truss with following parameters (Fig. 3):

Specific weight of material $\rho = 2,77 \times 10^{-5} (\text{N}/\text{mm}^3)$; Elastic modulus $E = 69.000 (\text{N}/\text{mm}^2)$; Allowable stress $[\sigma] = \pm 172 (\text{N}/\text{mm}^2)$; Allowable displacement $d = \pm 50,8 (\text{mm})$; Loads $P_{2Y} = P_{3Y} = 445.000 (\text{N})$.

The problem was solved with seven options of dividing design variables groups as below.

- Option 0: A1; A2; A3; A4; A5; A6; A7; A8; A9; A10;
- Option 1: A1=A5; A8=A9; A3=A4=A6=A10; A2; A7.
- Option 2: A1=A5=A8=A9; A3=A4=A6=A10; A2; A7;
- Option 3: A1=A2=A4=A5; A3=A6; A7=A8=A9=A10.
- Option 4: A1=A2; A4=A5; A3=A6; A7=A8; A9=A10;
- Option 5: A1=A5; A6; A7; A8=A9; A3=A4=A10; A2.
- Option 6: A1=A2=A4=A5; A3=A6=A7=A8=A9=A10;

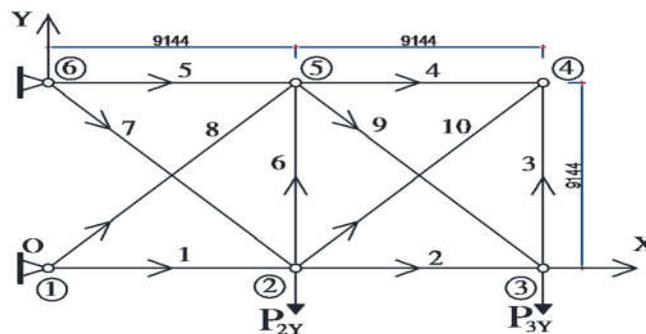


Figure 3. Plane truss under static load

Calculated results using method of correlation coefficient between element internal forces

The results of weight optimization of plane truss using method of correlation coefficient between element internal forces according to options of dividing design variable groups are shown in Table 1. The diagram of weight relationship depends on the number of iterations according design variable groups is presented in Figure 4.

Order	Calculated results of cross – section according to option of design variable groups (in ²)							Order	Calculated results of cross – section according to option of design variable groups (in ²)						
	[0]	[1]	[2]	[3]	[4]	[5]	[6]		[0]	[1]	[2]	[3]	[4]	[5]	[6]
1	28,9	28,78	23,94	22,08	25,4	29,34	23,72	7	8,24	8,55	9,99	18,74	18,78	5,34	15,76
2	0,01	14,34	14,51	22,08	25,4	14,67	23,72	8	20,43	20,33	23,94	18,74	18,78	23,92	15,76
3	28,35	0,05	0,01	3,19	3,47	0,01	15,76	9	20,43	20,33	23,94	18,74	18,78	23,92	15,76
4	14,44	0,05	0,01	22,08	18,86	0,01	23,72	10	0,02	0,05	0,01	18,74	18,78	0,01	15,76
5	0	28,78	23,94	22,08	18,86	29,34	23,72	Inter	20	7	7	4	3	11	2
6	0,01	0,05	0,01	3,19	3,47	0	15,76	W	5065	5107	5197	7230	7231	5902	7765

Table 1. Calculated results of weight optimization of plane truss according to options of design variable groups.

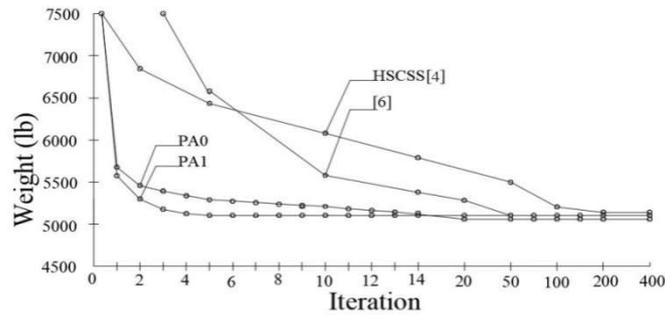


Figure 4. Diagram of weight relationship of plane truss depends on the number of iterations using various optimization methods.

5.2. Example of weight optimization of space truss

Optimization of space truss with following parameters (Fig. 5):

Specific weight of material $\rho = 0,1(\text{lb}/\text{in}^3)$; Elastic modulus $E = 10000(\text{ksi})$; Allowable stress $\sigma = \pm 40(\text{ksi})$; Allowable displacement $d = \pm 0,35(\text{in})$; Loads $P_{1X} = 1(\text{kip})$; $P_{1Y} = P_{1Z} = -10(\text{kip})$; $P_{2Y} = P_{2Z} = -10(\text{kip})$; $P_{6X} = 0,6(\text{kip})$; $P_{3X} = 0,5(\text{kip})$; $P_{6X} = 0,6(\text{kip})$.

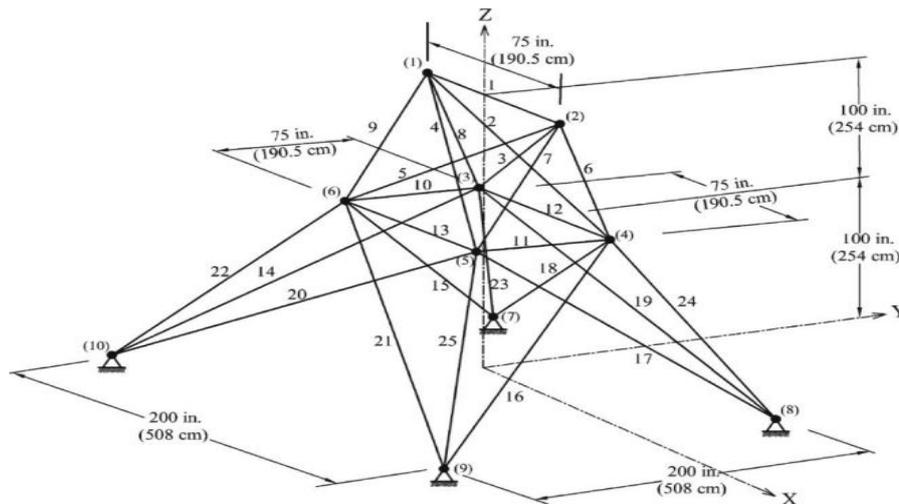


Figure 5. Space truss with 25 elements

The problem was solved with five options of dividing design variables groups as below. Option 0: group not divided, 4 division options of groups are shown in Table 2.

Options of dividing groups				
Order	[Option 1]	[Option 2]	[Option 3]	[Option 4]
1	A1	A1,A2,A3,A4,A5	A1	A1- A9
2	A2,A3,A4,A5	A6,A7,A8,A9	A2,A3,A4,A5	A10-A13
3	A6,A7,A8,A9	A10,A11	A6,A7,A8,A9	A14-A25
4	A10,A11	A12,A13	A10,A11,A12,A13	
5	A12,A13	A14,A15,A16,A17	A14,A20,A22	

6	A14,A15,A16,A17	A18,A19,A20,A21	A15,A18,A23	
7	A18,A19,A20,A21	A22,A23,A24,A25	A16,A21,A25	
8	A22,A23,A24,A25		A17,A19,A24	

Table 2. Options of dividing groups of design variable of space truss

Calculated results of weight optimization of space truss according to the options and other studies are shown in Table 3. Diagram of weight relationship depends on the number of iterations is presented in Fig. 6-7

Order	CROSS-SECTION AREA OF ELEMENTS BY STUDIES (in ²)							
	[7]	[4]	[13]	[Op0]	[Op1]	[Op2]	[Op3]	[Op4]
1	0,1	0,1	0,1	0	0	0,119	0,002	2,318
2	0,72	0,12	1,8	0,104	0,216	0,119	0,216	2,318
3	0,72	0,12	1,8	0,317	0,216	0,119	0,216	2,318
4	0,72	0,12	1,8	0,555	0,216	0,119	0,216	2,318
5	0,72	0,12	1,8	0,317	0,216	0,119	0,216	2,318
6	3,34	3,53	2,3	2,051	3,536	3,605	4,054	2,318
7	3,34	3,53	2,3	4,824	3,536	3,605	4,054	2,318
8	3,34	3,53	2,3	2,226	3,536	3,605	4,054	2,318
9	3,34	3,53	2,3	4,629	3,536	3,605	4,054	2,318
10	0,1	0,1	0,2	0	0	0	0,012	0,035
11	0,1	0,1	0,2	0	0	0	0,012	0,035
12	1,82	1,87	0,1	1,084	1,98	1,982	0,012	0,035
13	1,82	1,87	0,1	2,62	1,98	1,982	0,012	0,035
14	0,67	0,791	0,8	0,527	0,786	0,782	2,473	2,206
15	0,67	0,791	0,8	1,036	0,786	0,782	1,544	2,206
16	0,67	0,791	0,8	0,453	0,786	0,782	2,644	2,206
17	0,67	0,791	0,8	1,119	0,786	0,782	1,401	2,206
18	0,32	0,152	1,8	0,096	0,068	0,112	1,544	2,206
19	0,32	0,152	1,8	0,053	0,068	0,112	1,401	2,206
20	0,32	0,152	1,8	0,136	0,068	0,112	2,473	2,206
21	0,32	0,152	1,8	0,194	0,068	0,112	2,644	2,206
22	3,47	3,97	3	5,201	4,003	3,949	2,473	2,206
23	3,47	3,97	3	2,678	4,003	3,949	1,544	2,206
24	3,47	3,97	3	2,265	4,003	3,949	1,401	2,206
25	3,47	3,97	3	5,671	4,003	3,949	2,644	2,206
Inter	20000	1000	100	15	15	13	10	5
W	466,8	467,7	546	468,1	467,6	466,4	584,5	675,8

Table 3. Results of weight optimization of space truss according to the various options and methods

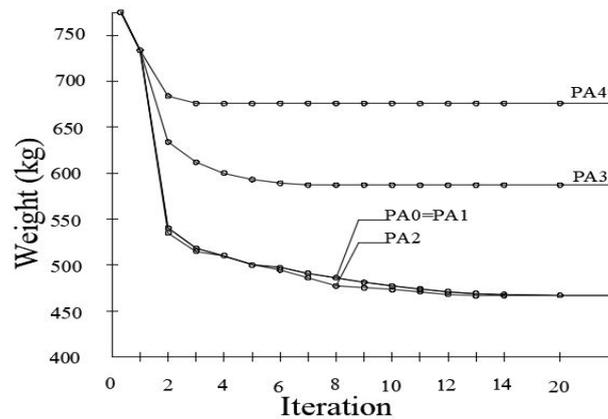


Figure 6. Diagram of weight relationship of space truss depends on the number of iterations with the options of design variable group

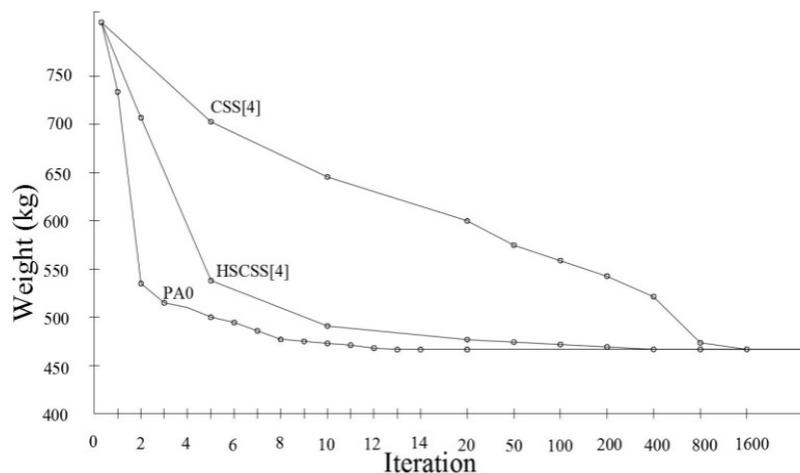


Figure 7. Diagram of weight relationship of space truss and number of iterations by various methods

5.3 Comments

On the basis of the results of weight optimization of plane and space trusses shown in paragraph 4, the following comments can be given:

- The method of correlation coefficient between element internal forces established by the authors gives the results of weight optimization of plane and space trusses with a negligible difference (0.1%) compared with the results of optimization by other methods.
- Using the method of correlation coefficient between element internal forces requires significantly less number of iterations than other methods. This is a remarkable advantage in optimization problems of large-scale space trusses.

The division of design variables groups affects the results and the convergence rate of the problem. In order to reduce the truss weight, it is necessary to select the element with the same internal force in the same group of design variables. The option of the independent variables (option 0) gives results in the smallest weight and the slowest convergence.

6. Conclusions

The method of correlation coefficient between element internal forces established by the authors is a simple and effective method for weight optimization of plane and space trusses. In the next studies, the

authors will continue to develop the established algorithms to solve problems of shape and structure optimization of trusses.

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