IOP Conf. Series: Materials Science and Engineering **365** (2018) 042031 doi:10.1088/1757-899X/365/4/042031

Interaction of thin-walled prismatic shell with elastic foundation

Andrey Leontiev and Ilia Efremov

Moscow State University of Civil engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

E-mail: an_leontiev@mail.ru

Abstract. The problem of static analysis of prismatic shells consisting of rectangular plates rigidly interconnected along the longitudinal edges is considered. Shells freely lie on the surface of an elastic foundation or embedded in elastic medium. It is accepted that at the ends of the shell located diaphragm rigid in its plane and flexible out of the plane. According to applied bounding conditions the method of single trigonometric series is used which resulted in conversion of two-dimensional problem to one-dimensional. To solve one-dimensional problem uses the method of displacement, in accordance with which all the hard nodes of the system (the junction of the horizontal and vertical plates) are fixed against possible displacement. For ease of algorithmization of the calculation the plates are considered as generalized finite elements. To determine the nodal forces that correspond to the four degrees of freedom considered plane stress of the plate and her bending. The example of calculation of prismatic shell which is in two-parameter elastic medium and loaded on the upper plane by uniformly distributed load is given. It is shown that the account of friction forces leads to a significant reduction of the moments due to the fact that it reduced the total sediment of the shell.

1. State of the problem

The objects of research are prismatic shells consisting of rectangular plates rigidly interconnected at longitudinal edges and freely supported by the surface of elastic foundation (Fig. 1) or embedded in elastic medium (Fig. 2).

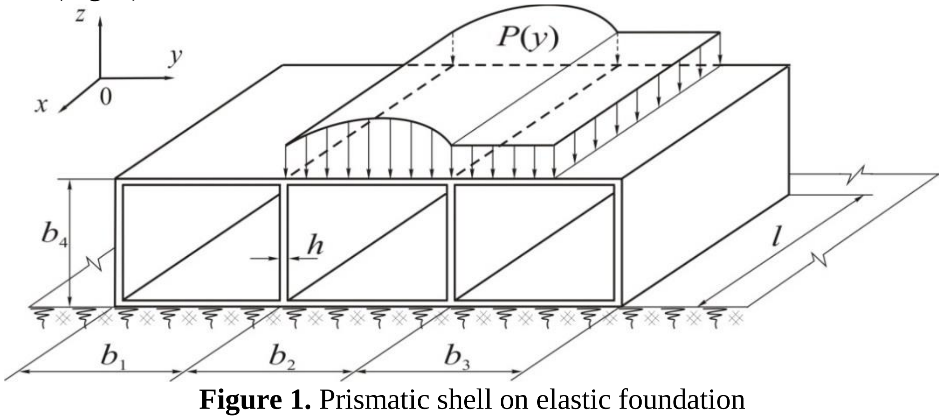

Figure 1. Prismatic shell on elastic foundation

It's is assumed that at shell edges there are diaphragms, rigid in its plane and flexible out of the plane, which do not restrain its free longitudinal movement in elastic medium. For description of the foundation the two-parameter model has been used. Approximately taken into account the forces of friction between the base and the shell.

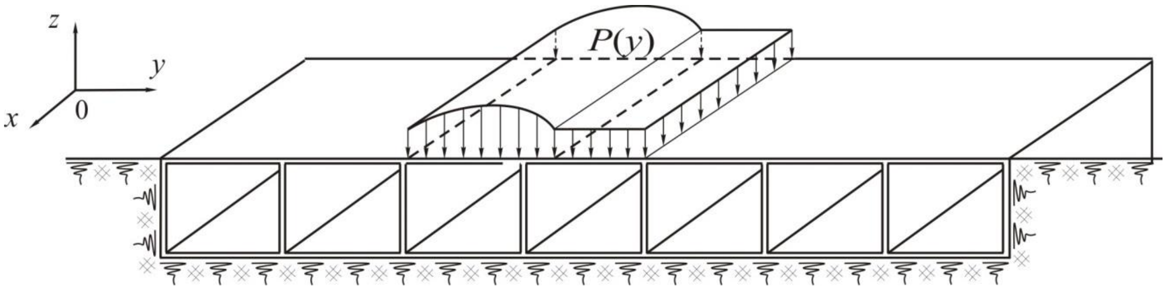

Figure 2. Prismatic shell embedded in an elastic medium

Under the action of the load, which we assume constant, as all other conditions along the *Ox* axis, a shell, embedded into flexible medium, will receive rigid movements B_i (i = 1,2,3), as it's shown at the Fig. 3, and bending deformation of its component plates. At the same time a shell is affected by reactive pressures q from flexible medium directed along the normal to a shell surface, and also by tangential forces r, proportional to corresponding displacements υ of the points of shell surface: $r = k_{s*}\upsilon$.

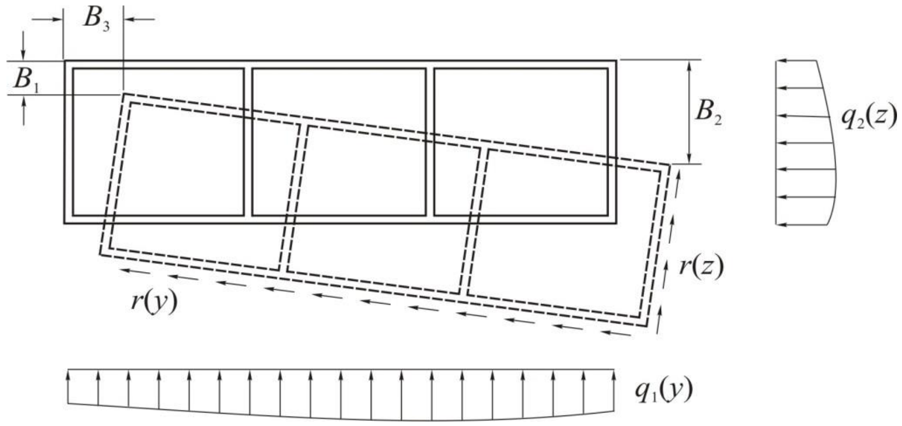

Figure 3. Displacement of the shell and reactive resistance of foundation

2. Analytical method of solving

The theory of calculating of prismatic shells, is devoted [1]. The method of single trigonometric series applied to longitudinal coordinate x allows transforming two-dimensional problem to one-dimensional [2], [3]. For solution of the one-dimensional problem displacement method is used according to which all the rigid system joints (joint lines of horizontal and vertical plates) are fixed to prevent possible displacements (Fig. 4, *a*). Each joint has four degrees of freedom: three linear displacements and angular rotation (Fig. 4, *b*). The matrix of coefficients of fundamental equations system can be interpreted as the matrix of stiffness of the structure and so finite element approach can be applied for its arraying. According to this approach rectangular plates composing a shell are treated as generalized finite elements.

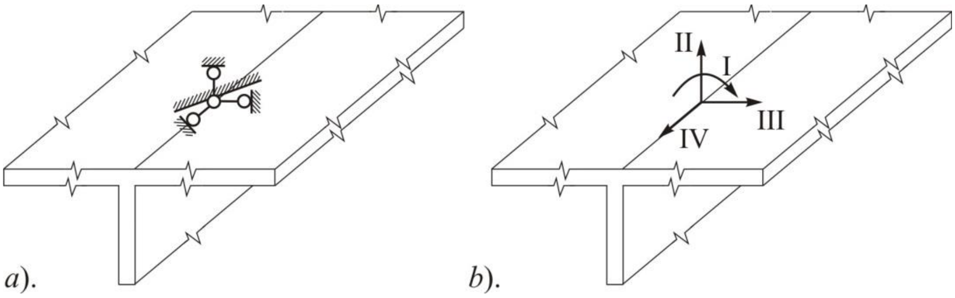

Figure 4. Degrees of freedom of rigid system joints

For calculation of nodal forces, corresponding to III и IV degrees of freedom, the state of the plane stress is analysed. In accordance with the bounding conditions at the lateral edges (υ = 0, σ_x = 0), displacements in the plane of the plate are represented by series:

$$u(x,y)=\sum_n U_n(y)\cos\lambda_n x, \quad \upsilon(x,y)=\sum_n V_n(y)\sin\lambda_n x, \quad \text{where } \lambda_n=\frac{n\pi}{l}. \tag{1}$$

The problem reduces to solution of homogeneous differential equation:

$$F_n^{IV}-2\lambda_n^2 F_n^{II}+\lambda_n^4 F_n=0, \tag{2}$$

general integral of which is represented as follows:

$$F_n\ (y)=C_{1n}\mathrm{sh}\lambda_n y+C_{2n}\mathrm{ch}\lambda_n y+C_{3n}\lambda_n y\mathrm{ch}\lambda_n y+C_{4n}\lambda_n y\mathrm{sh}\lambda_n y\ . \tag{3}$$

Herewith generalized displacements $U_n(y)$ and $V_n(y)$ can be calculated with resolution function $F_n(y)$ from the formula:

$$U_n=F_n^{II}-\frac{1-\nu}{2}\lambda_n^2 F_n, \qquad V_n=\frac{1+\nu}{2}\lambda_n F_n^{I}, \tag{4}$$

where ν – the Poisson's ratio of the material.

When calculated plates bending, vertical displacements are represented in the following form:

$$w(x,y)=B+\theta y+\sum_n W_n(y)\sin\lambda_n x, \tag{5}$$

where B and θ – constants, characterizing plate displacements as of a rigid body.

After representation of loads into series

$$p(x,y)=\sum_n p_n(y)\sin\lambda_n x \tag{6}$$

the solution of the problem is reduced to integration of ordinary differential equation:

$$W_n^{IV}-2r_n^2 W_n^{II}+s_n^4 W_n=-\frac{4k}{n\pi}\frac{B+\theta y}{D}+\frac{p_n(y)}{D}, \tag{7}$$

where $r_n^2=\lambda_n^2+\dfrac{t}{D}$, $s_n^4=\lambda_n^4+\lambda_n^2\dfrac{t}{D}+\dfrac{k}{D}$, k and t – quotients of elastic foundation, D – cylindrical stiffness of a plate.

General integral of equation (7) has the form:

$$W_n(y)=C_{1n}S_{1n}(y)+C_{2n}S_{2n}(y)+C_{3n}S_{3n}(y)+C_{4n}S_{4n}(y)+W_n^k(y)+W_n^p(y), \tag{8}$$

where $S_{1n}(y)=\mathrm{ch}\alpha_n y\sin\beta_n y$, $S_{2n}(y)=\mathrm{ch}\alpha_n y\cos\beta_n y$, $S_{3n}(y)=\mathrm{sh}\alpha_n y\cos\beta_n y$,

$S_{4n}(y)=\mathrm{sh}\alpha_n y\sin\beta_n y$, $\alpha_n=\sqrt{\dfrac{s_n^2+r_n^2}{2}}$, $\beta_n=\sqrt{\dfrac{s_n^2-r_n^2}{2}}$, $W_n^k(y)=-\dfrac{4k}{n\pi s_n^4}\dfrac{A+\theta y}{D}$, and integral $W_n^p(y)$ depends on the law of distribution of load along axis *y*.

In the right-hand side of the equation system describing the deformation state of a plate, constant values B_i, will be included with accuracy to which reactions of elastic foundation are calculated. It's worth mentioning that for calculation of these constant values application of trigonometric series assumes immobility of end diaphragms. Determining reactions in these ties and equating them equal to zero allows calculating constant values B_i.

For realization of described finite-element algorithm it was made a program for analysis of various multi-element systems of surface and underground structures. With the use of this program the series of examples have been implemented in which is considered influence of such parameters as relative length of construction, density and cohesion of elastic foundation, force due to friction, on the values of structure internal forces.

3. Examples of calculation

At the Fig. 5, *b*, as an example, there were given the results of analysis of simply connected prismatic shell embedded into elastic two-parameter medium and loaded on the top plane by uniformly distributed load (Fig. 5, *a*).

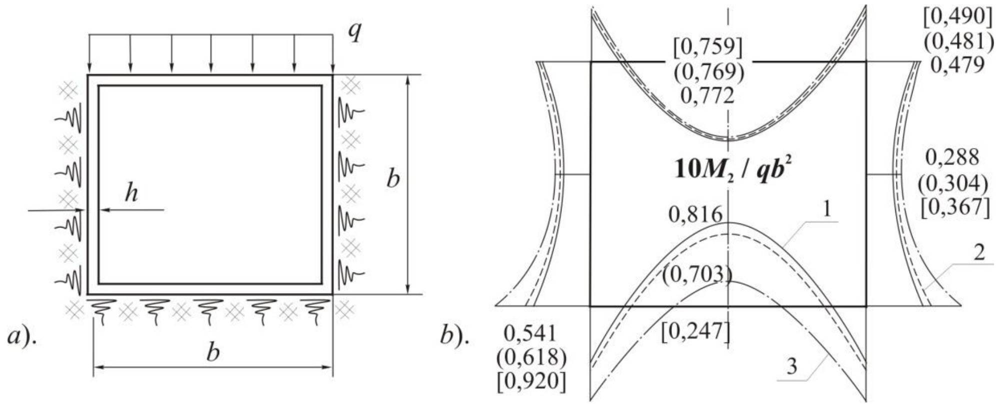

Figure 5. The results of analysis of simply connected prismatic shell

Hereby the following giving data are taken for further analysis: l/b=4, h/b=0.1, $k=50D/b^4$, which corresponds to moderately firm grounds. The second bed quotient was assumed to be equal to: $t=\bar{t}D/b^2$. The numbers in the Fig. 5, *b* mean: 1 – $\bar{t}$ = 0, 2 – $\bar{t}$ = 1, 3 – $\bar{t}$ = 5. The analysis was made with nine terms of series.

From the diagrams of the transverse bending moment M_2, shown at the Fig. 5, *b*, it's seen that the value of second bed quotient *t* affects the results of analysis substantially: for very cohesive soils (at $\bar{t}$ =5) the values of bending moment in the centre of lower plate differ more than three times from those that correspond to Winkler foundation, and at the lower corners such a difference amounts to nearly 50%.

In the above-mentioned example friction between vertical shell walls and flexible medium has not been taken into account. For this purpose you can enter a quotient:

$$\alpha=1-\frac{k_s}{k}\frac{A_{sh}}{A_{pl}}, \tag{9}$$

IOP Conf. Series: Materials Science and Engineering **365** (2018) 042031 doi:10.1088/1757-899X/365/4/042031

where A_{sh} – total area of vertical shell surfaces, A_{pl} – area of a lower plate. As it's seen, at the value of α = 1 friction is not taken into account. For the case of reinforced concrete shell and real soil ground this quotient can vary in the range of 0.7 ÷ 0.9.

At the Fig. 6 there are given bending moment diagrams in the lower plate with the allowance for friction forces. Here the values of the Fig. 5 are repeated with dotted graphs and figures in brackets for t = 0 and $\bar{t}$ =5 (α = 1), and with full line graphs and figures there are shown the values of the same situations at α = 0.7.

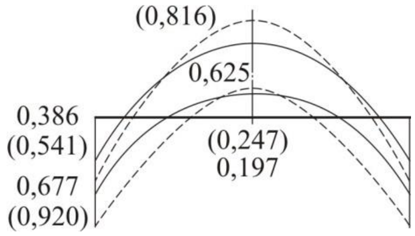

Figure 6. Bending moment diagrams in the lower plate with the allowance for forces

It's seen that taking account of friction forces comes to substantial decrease in moments due to shell general sediment.

The following example shows a multiply connected shell embedded in an elastic two-parameter environment (Fig. 7).

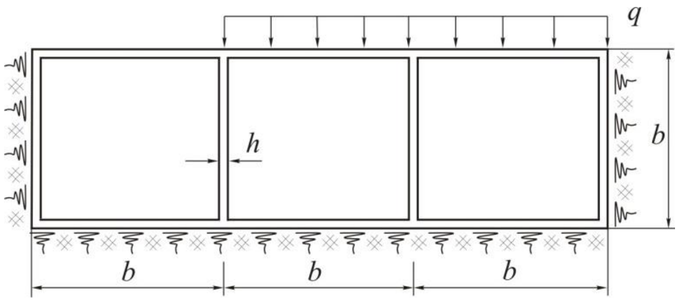

Figure 7. Multiply connected shell embedded in an elastic two-parameter environment

In Fig.8 and 9 shows some results for this example.

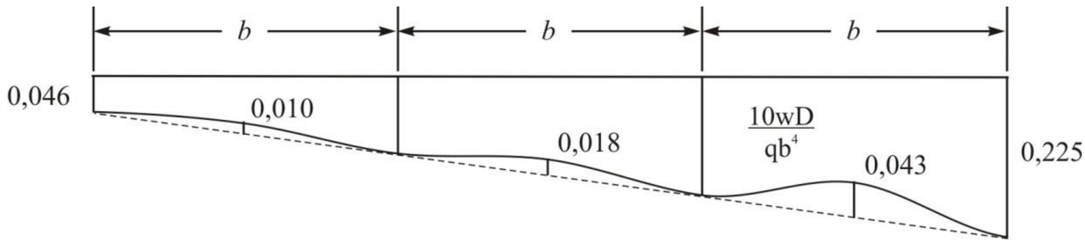

Figure 8. Deflection of the bottom plate of the shell at l/b=2, t = 0, □ = 1

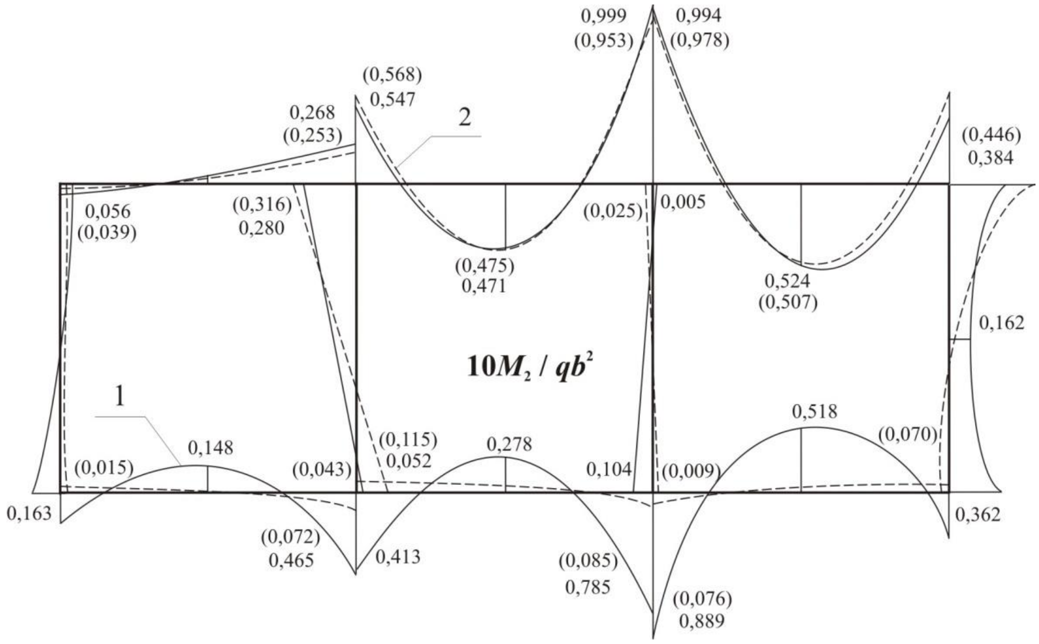

Figure 9. Bending moment diagrams with (1) and without (2) considering the displacement of diaphragms at *l*/*b*=2

4. Summary

Given in the article the method of calculation of prismatic shell located on an elastic foundation, allows determining the internal forces in the shell. It should be noted that the problems of calculation of structures on elastic foundation does not lose their relevance. The development of new models makes it possible to find engineering solutions that allow for analysis at the pre-project stage and to conduct a simpler analysis of the stress-strain state of structures and the ground base [4-6].

References

[1] Vlasov V 1949 *General theory of shells and its applications in engineering* (Moscow: Gostechizdat) p 784

[2] Leontiev N, Leontiev A and Monsef B 2000 Revisiting the analysis of underground structures as thin-shell dimensional systems *Theoretical Foundations of Construction Book of Reports* (Warsaw) pp 67–70

[3] Leontiev N, Leontiev A and Monsef B 2001 The calculation of thin-walled spatial systems that interact with an elastic medium *Theoretical and experimental studies of strength and stiffness of structural elements Book of Reports* (Moscow) pp 46–51

[4] Andreev V and Barmenkova E 2012 The modeling of the real building objects by using the model of a two-layer beam of variable rigidity on an elastic basis *Applied Mechanics and Materials* **204-208** pp 3596-3599

[5] Andreev V and Barmenkova E 2015 On Taken into account the joint work structures and foundations *Proceeding of 7th International conference on contemporary problems of architecture and construction* pp 465-470

[6] Andreev V, Matveeva A and Barmenkova E 2013 The calculation of the two-layer beam model on an elastic basis with variable modulus of subgrade reaction *Applied Mechanics and Materials* **351-352** pp 566-569