

Pseudorandom number generation for computer modeling of actual shapes of spatial bar structures

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Abstract. Spatial bar structures consist of a large number of individual elements. During the assembly process these bars are consecutively connected to each other through structural joints. The actual dimensions of the bars differ from the nominal values and vary from one bar to another due to random inaccuracies in their manufacturing. These inaccuracies are regulated by a special system of tolerances, but, anyway, their accumulation during assembly leads to errors in the geometric shape of the bar structure. The errors that arise in the spatial bar structures make it difficult to connect the elements among themselves, and reduce the load-carrying capacity of structures due to the appearance of additional internal forces. Therefore, studies of possible errors in spatial bar structures help to improve their reliability.

Studies of possible errors are performed on a computer by simulating the assembly of spatial bar structures using the Monte Carlo method. This method requires the introduction of random variability in the lengths of the bars when assembled into a single spatial structure. In addition, it involves statistical analysis of the results obtained, which is why it is called the statistical computer simulation method. The reliability of computer simulation of the actual shape of the spatial bar structures can be achieved only in the case of using normally distributed random deviations in bar lengths.

To obtain normally distributed random deviations in bar lengths, special algorithmic calculators for uniformly distributed random numbers in the interval from 0 to 1 are used. One of these algorithms is investigated in this paper. It should be noted, that the numbers obtained by these algorithms are not strictly random, and therefore they are called pseudorandom. However, their sequence has the properties of randomly obtained numbers. The author also recommends an algorithm for obtaining normally distributed pseudo-random numbers from uniformly distributed pseudo-random numbers.

The process of statistical computer simulation of actual geometric shapes of spatial bar structures requires the use of a very large number of pseudo-random numbers, since multiple numerical simulation of structures is being performed. Consequently, the random nature of the pseudo-random numbers used in the simulation must be flawless. To achieve reliable results of computer simulation of the actual geometric shape of spatial bar structures, the author recommends algorithms for obtaining pseudo-random numbers both uniformly distributed from 0 to 1, and normally distributed with statistical mean $\mu = 0$ and standard deviation $\sigma = 1$. In order to confirm the quality of the pseudo-random numbers obtained by this algorithm, their sequence was subjected to statistical testing at different regions.

From different regions of the large sequence of pseudo-random numbers samples were formed with the aid of the author's computer program, which were then subjected to statistical testing. Based on the test results, histograms of the distribution were plotted, according to which the chi-square criterion was determined. The results of testing allow us to conclude that the presented algorithms for obtaining pseudo-random numbers can be recommended for computer



simulation of actual geometric shapes of spatial bar structures. With the aid of these algorithms, reliable results of such studies can be obtained.

1. Introduction

Spatial bar structures consist of a large number of individual elements. During the assembly process these bars are consecutively connected to each other through structural joints. As a result of this process, structures of different geometric patterns and spatial shapes are formed. For example, metal domes of a spherical shape can differ from each other by geometrical schemes of construction of their frameworks. Figure 1 shows framework schemes of a star grid dome (a) and a sectoral grid dome (b). The number of bars in the star dome is 568, and the number of bars in the sectoral dome is 800.

The actual dimensions of the bars differ from the nominal values and vary from one bar to another due to random inaccuracies in their manufacturing. Inaccuracies in the dimensions of bars are regulated by a special system of tolerances [1], but, although the tolerances are satisfied, the accumulation of errors during the assembly leads to errors in the geometric scheme of the bar structure. The errors that arise during the assembly of the spatial bar structures make it difficult to connect the elements among themselves, and reduce the load-carrying capacity of structures due to the appearance of additional internal forces. In addition, the design geometric shape of the structure as a whole is distorted. This leads to various defects in the construction of the enclosing structures. Therefore, studies of possible errors in spatial bar structures help to improve their reliability.

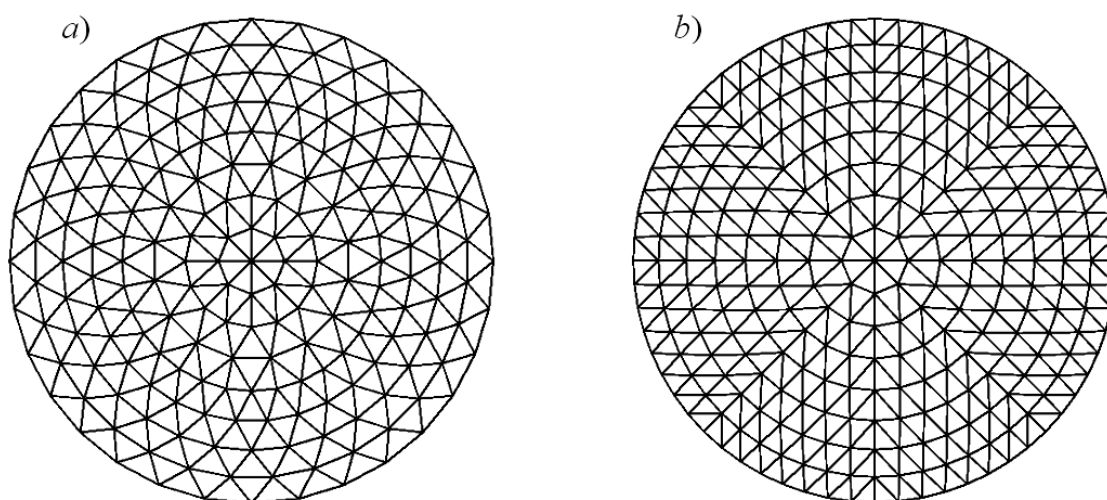


Figure 1. The schemes of the frameworks of the star grid dome (a) and the sectoral grid dome (b).

Studies of possible errors are performed on a computer by simulating the assembly of spatial bar structures using the Monte Carlo method [2]. This method requires the introduction of random variability in the lengths of the bars when assembled into a single spatial structure [3]. The actual bar size L_i^* will differ from the nominal size L_i by the amount of random deviation $\delta L(\sigma)_i$ [4]

$$L_i^* = L_i + \delta L(\sigma)_i . \quad (1)$$

In addition, the Monte Carlo method involves statistical analysis of the results obtained, which is why it is called the statistical computer simulation method. This requires a multiple imitation of the process under investigation, including modeling of all the factors affecting it. The deviation in bar lengths is a summation of a large number of mutually independent random errors, and therefore, according to the central limit theorem of probability theory (Lyapunov's theorem), the distribution of

deviations approximates to the normal law [5]. Therefore, the reliability of computer simulation of the actual shape of the spatial bar structures can be achieved only in the case of using normally distributed random deviations in bar lengths.

An important part of the numerical simulation of the assembly of spatial bar structures using the Monte Carlo method is the simulation of random variables with a normal distribution law. In addition, the system of tolerances for the dimensions of building structures used in construction is based on their normal distribution. At that, the deviation corresponding to 3σ is taken as permissible deviation of a dimension [1]. This is achieved by modeling random variables distributed uniformly between zero and one, and then converting them to normal random variables using specially chosen formulas or algorithms. The numbers obtained in this way are called pseudorandom, and the algorithms that generate them are called pseudorandom number generators. One of these algorithms is investigated in this paper. It should be noted, that the numbers obtained by these algorithms are not strictly random, and therefore they are called pseudorandom. However, their sequence has the properties of randomly obtained numbers.

The process of statistical computer simulation of actual geometric shapes of spatial bar structures requires the use of a very large number of pseudo-random numbers, since multiple numerical simulation of structures is being performed. For a relatively accurate estimate of the errors in the actual shape of the bar spatial structures, at least five hundred numerical simulations should be performed [6]. In addition, to calculate one normally distributed pseudo-random number, twelve equally distributed numbers are used. For example, a study of possible errors in the assembly of dome frames, shown in Figure 1, will require the use of more than 3.4 million uniformly distributed pseudo-random numbers in the interval from 0 to 1 for the star grid dome and more than 4.8 million for the sectoral grid dome. And in more complex bar structural systems, the required number of such pseudo-random numbers exceeds 10 million. Consequently, the random nature of the pseudo-random numbers used in the simulation must be flawless.

2. Methodology

The author developed computer programs designed for the numerical analysis of the possible errors of the assembly of the bar frameworks of large-span metal domes by the Monte Carlo method. Pseudo-random numbers in these programs are generated based on a first-order recurrence formula

$$\xi_{j+1} = f(\xi_j). \quad (2)$$

where ξ_0 is the given initial value.

To obtain uniformly distributed pseudo-random numbers from 0 to 1, it is recommended to use a simple analytical dependence [7]:

$$\xi_{i+1} = D(997 \xi_i), \quad (3)$$

where: $D()$ – the function of discarding of the whole part of the product,

$$\xi_0 = 0.5284163517.$$

In the programming language C, this algorithm looks like:

```
double t,tx,ty,con;
t=5.284163517e-01;
con=997;
...
{tx=t*con;          ty=floor(tx);          t=tx-ty;}
```

In order to determine the quality of a sequence of uniformly distributed pseudorandom numbers, studies were carried out on the uniformity of filling with "random" points of a square and a cube. Using twos or triples of successive pseudorandom numbers as coordinates, we get a set of points located, respectively, inside a square or a cube with sides equal to one. The essence of such a calculation is to determine the ratio of the number of pseudo-random points that fall within a given region to their total number. The given area is a quarter-circle inscribed in a square or an eighth part of a sphere inscribed in a cube. At this, the radii of the circle and the sphere are equal to one.

From mathematics it is known that the area of the circle and the volume of the sphere, respectively, are equal to:

$$\frac{1}{4} \pi r^2, \quad \frac{1}{8} \frac{4 \pi r^3}{3}. \quad (4)$$

In this case, the ratio of the areas of a quarter of a circle with the radius $r = 1$ and a square with the side 1 is equal to $\pi/4$; and the ratio of the volumes of an eighth part of a sphere with the radius $r = 1$ and a cube with the side 1 is equal to $\pi/6$. Therefore, to calculate a number by the Monte Carlo method, one can write, respectively, for twos and for triples of uniformly distributed numbers of a pseudo-random sequence.

$$\pi^* = 4 \frac{k'}{k}, \quad \pi^* = 6 \frac{k'}{k}. \quad (5)$$

where k – total number of points;
 k' – total number of points in the given area.

Fig. 2 and Fig. 3 show a graphic representation of the "approach" of the obtained values of the number π^* to its theoretical value π with an increase in the number of used twos or triples of pseudo-random numbers. It has to be noted that the number of twos k is twice less than the volume of the sample n of uniformly distributed pseudo-random numbers ξ_i , and the number of triples k is three times less than the volume of the sample n . It can be seen from the figures that with the increase in the sample size, the accuracy of calculation is clearly increasing.

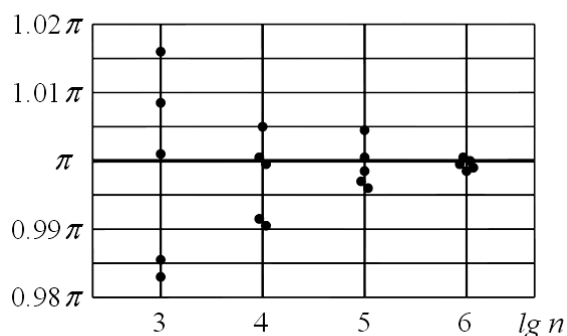


Figure 2. Accuracy of calculation of π using twos of pseudorandom numbers ξ_i

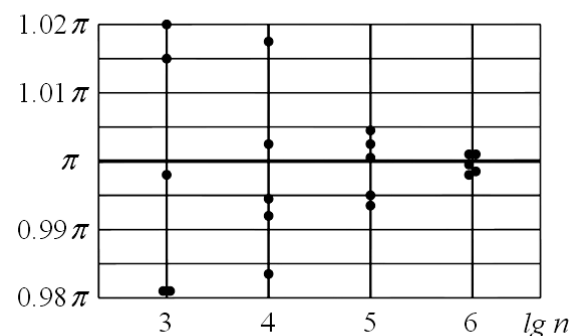


Figure 3. Accuracy of calculation of π using triples of pseudorandom numbers ξ_i

The following formula is used [2, 3] to obtain from uniformly distributed pseudo-random numbers ξ_i normally distributed pseudo-random numbers ζ_j with mathematical expectation $\mu = 0$ and standard deviation $\sigma = 1$:

$$\zeta_j = \sqrt{\frac{12}{n}} \cdot \sum_{i=1}^n \left(\xi_i - \frac{1}{2} \right). \quad (6)$$

In this case, the number of uniformly distributed numbers ξ_i can be limited to 12 ($n = 12$), which is quite sufficient for practical tasks. Therefore, the author also recommends an algorithm for obtaining normally distributed pseudo-random numbers from uniformly distributed pseudo-random numbers. In the programming language C, the function norm() is written to obtain normally distributed pseudo-random numbers:

```
double sm,t,tx,ty,con;          int k;
    t=5.284163517e-01;
    con=997;
    ...
    norm( );
    ...
void norm( )
{
    sm=0;
    for (k=1; k<=12; k++)
    {
        tx=t*con;          ty=floor(tx);          t=tx-ty;
        sm=sm+tx;
    }
    sm=sm-6;
    return;
}
```

Obviously, the pseudo-random numbers obtained in this case are not actually random. The properties of the aggregate of the actual random numbers are judged from the results of statistical checks that are performed with special techniques [8 – 11]. To achieve reliable results of computer simulation of the actual geometric shape of spatial bar structures, pseudo-random numbers must be the same as random ones.

In order to confirm the quality of the pseudo-random numbers obtained by this algorithm, their sequence was subjected to statistical testing [12 – 15] at different regions. Using the computer program specially developed by the author, from different regions of the large sequence of pseudo-random numbers samples were formed, which were then subjected to statistical testing. During the testing, the histograms of the distribution of numbers ζ_j were constructed and compared with the density of the normal distribution with $\mu = 0$ and $\sigma = 1$ which is described with the formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \quad (7)$$

The comparison was carried out according to the chi-square criterion [8, 12] with a division of the interval from -4σ to $+4\sigma$ into 16 classes, the standard deviation σ , and the mathematical expectation

μ . In this case, the number of actually used classes in the calculation of the chi-square criterion (χ^2) was 12, hence the number of degrees of freedom $\nu = 11$. The chi-square criterion was calculated by the formula

$$\chi^2 = \sum_{i=1}^k \frac{(m_i - np_i)^2}{np_i}, \quad (8)$$

where: n – sample size;

k – number of classes used;

m_i – the number of sample numbers in the i -th class;

p_i – probability of falling into the i -th class according to the normal distribution law.

Samples of normally distributed numbers ζ_j were tested, the accumulation of which was carried out from the 1st, 10000th, 100000th, 1000000th, 5000000th and 10000000th uniformly distributed number ξ_i of pseudo-random sequence. At each stage, three consecutively accumulated samples of volume (n) were tested, with 1000 normally distributed numbers ζ_j in each.

The results of testing of normally distributed numbers generated by the function $\text{norm}()$ are shown in the table. Comparing the obtained values of the chi-square criteria (χ^2) with the chi-square distribution, it can be noted, that the sequence of pseudo-random numbers under study is in good agreement with the normal distribution law.

Table 1. The results of testing pseudo-random normally distributed numbers.
Sample size $n = 1000$

The number of the initial uniformly distributed number ξ_i	Criteria χ^2	Standard deviation σ	Mathematical expectation μ
1	6.071	0.975	-0.005
	3.118	0.990	-0.002
	5.360	1.037	0.001
10000	6.513	1.035	0.019
	13.146	1.020	0.065
	11.595	1.020	0.009
100000	10.381	0.975	-0.001
	7.016	0.979	-0.013
	13.768	0.989	-0.018
1000000	11.899	1.019	-0.053
	4.803	0.992	-0.008
	15.981	1.018	0.007
5000000	3.259	0.978	-0.002
	12.066	0.970	-0.014
	12.431	1.037	-0.054
10000000	3.478	0.999	0.004
	4.919	0.996	0.030
	7.566	1.014	-0.005

Fig. 4 shows a histogram of the distribution of normally distributed pseudo-random numbers ζ_j constructed for a sample with a minimum value of chi-square criteria (χ^2). Fig. 5 shows a histogram of the distribution of normally distributed pseudo-random numbers ζ_j constructed for a sample with a maximum value of chi-square criteria (χ^2).

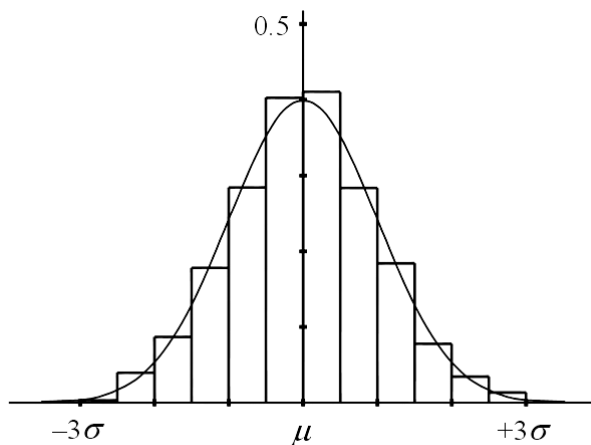


Figure 4. The histogram of the distribution of the generated numbers ζ_j , obtained from the 2nd sample, starting with the 1st number ξ_i ($\chi^2 = 3.118$).

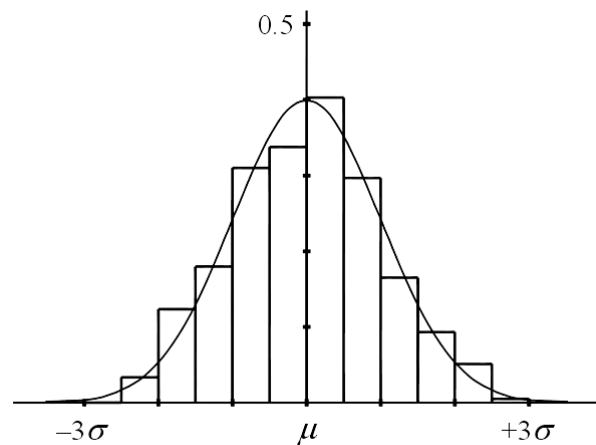


Figure 5. The histogram of the distribution of the generated numbers ζ_j , obtained from the 3rd sample, starting with the 1000000th number ξ_i ($\chi^2 = 15.981$).

3. Conclusions

The results of testing a large number of pseudorandom uniformly distributed ξ_i and normally distributed ζ_j numbers generated by the above algorithms lead to the conclusion that they fully correspond to the predetermined distribution laws over the entire length of the pseudo-random sequence.

Therefore, the presented algorithms for obtaining pseudo-random numbers can be recommended for computer simulation of actual geometric shapes of spatial bar structures. With the aid of these algorithms, reliable results of such studies can be obtained.

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