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An effective numerical method for calculating unseparated flows in building aerodynamics

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Abstract. The problem of the mixing of heated gases is of interest in connection with the development of pollution-free technologies for natural-fuel combustion in modern heat and electricity generating plants. The operating principle of the tall structures designed for this purpose, which combine a smokestack and a cooling tower, is as follows. At the base of the stack flue gas, from which the sulfur has been removed, is fed into a flow of air heated in a heat exchanger. As it moves through the stack, the gas mixes with the hot air and is carried into the atmosphere by the natural draft. The design must satisfy certain requirements. The temperature of the flue gas must not fall below a certain limit at which condensation that leads to corrosion develops. The gas outlet velocity must be higher than 4 m/s to prevent downdraft. The concentrations of pollutants released into the atmosphere must be within the permissible limits. One promising means of enhancing the efficiency of such structures is to swirl the flue gas ahead of the stack inlet. Flow swirling considerably intensifies the heat and mass transfer processes, improves the mixing of the hot gases, reduces the pollutant concentrations at the stack outlet, and prevents flow separation on the walls. The general formulation of the problem of the mixing of two nonisothermal turbulent flows is based on the complete Reynolds equations. This system closed by a turbulence model (algebraic or differential) is fairly complicated and its solution is laborious. A simplified model is based on the parabolized Navier-Stokes equations, which restricts the area of applicability to unseparated flows. However, in view of the mechanical nature of the problem considered, unseparated flows are of most interest. In this paper, an effective method of solving boundary-layer equations is discussed, in which the initial system reduces to ordinary differential equations written on the streamlines.

1. Introduction

Swirling flows are widely used in various technical applications and are observed in natural phenomena (water spouts, tornados). They have been the object of a great deal of theoretical and experimental study. The most important properties of swirling flows are described in [1–4]. Swirling flows in channels were numerically investigated in [5–8] and swirling flows in an unbounded medium in [9, 10]. The results of experiments on the stability of swirling flows and the structure of the axial recirculation zone are presented in [11-15].

We will consider the problem of mixing of hot gases in the axisymmetric pipe, whose lateral surface is specified in the cylindrical coordinate system r, φ, z by the equation $R(z)$. A swirled flow of flue gas is fed into the central part $(0 \le r \le R_1)$ of the inlet cross-section $(z = 0)$ of this pipe. An outer unswirled hot air flow is introduced into the peripheral part $(R_1 \le r \le R_0 = R(0))$. To investigate the gas dynamic processes of turbulent mixing of heated gases, we use the system of conservation equations for mixture mass, momentum, energy, and admixture mass in the form of boundary layer approximation

$$\frac{\partial(r\rho U)}{\partial z} + \frac{\partial(r\rho V)}{\partial r} = 0$$

$$\frac{\partial[(p+\rho U^2)r]}{\partial z} + \frac{\partial(\rho r U V)}{\partial r} = \frac{\partial}{\partial r}(r\tau) - \rho g r$$

$$\frac{\partial(r\rho\, UH)}{\partial z} + \frac{\partial(r\rho\, VH)}{\partial r} = \frac{\partial(rq)}{\partial r} + \rho V W^2 + \mu r\left(\frac{\partial W}{\partial r} - \frac{W}{r}\right)^2$$

$$\frac{\partial(r\rho\, UH)}{\partial z} + \frac{\partial(r\rho\, VH)}{\partial r} = \frac{\partial(rq)}{\partial r} \tag{1}$$

$$\frac{\partial(r\rho\, UE)}{\partial z} + \frac{\partial(r\rho\, VE)}{\partial r} = \frac{\partial(\mu r \gamma_\alpha)}{\partial r}$$

$$\frac{\partial(r\rho\, UW)}{\partial z} + \frac{\partial(r\rho\, VW)}{\partial r} = \frac{\partial}{\partial r}\left(\mu r \frac{\partial W}{\partial r}\right) - \frac{\rho V W}{r} - \mu\frac{W}{r^2}$$

$$h = c_p T\,,\quad H = h + \frac{U^2}{2} + gz\,,\quad q = \frac{1}{\sigma}\mu\frac{\partial}{\partial r}(h + 0.5\sigma U^2)\,,\quad \gamma_\alpha = \frac{1}{\sigma_\alpha}\frac{\partial E}{\partial r}\,,\quad \sigma = \frac{1}{\lambda}\mu c_p\,,\quad \tau = \mu\frac{\partial U}{\partial r}$$

Here, the following notation is used: U, V and W are the axial, radial, and azimuthal velocity components, respectively, μ is the dynamic viscosity, g is the gravity acceleration, T is the temperature, c_p is the specific heat, h is the enthalpy, H is the total enthalpy, q is the heat flux, E_i is the admixture concentration, γ_i is the admixture mass flow-rate, σ and σ_α are Prandtl numbers, and τ is the friction force.

2. Numerical procedure

We will use Shkadov method of equal flow-rate surfaces [16, 17]. In the cylindrical coordinate system r, φ, z we define smooth lines $r = \delta_n(z)$, $n = 0,1,2,\ldots,N$, each of which is a streamline and satisfies the equation

$$U\frac{\partial \delta_n}{\partial z} = V \ \text{ for } r = \delta_n(z) \tag{2}$$

The grid of lines $\delta_n(z)$ is constructed together with the solution. Obviously, $\delta_0 = 0$ is the symmetry axis and $\delta_N = R(z)$ is the pipe wall. The gas dynamic functions can be calculated on the intermediate lines

$$r = \delta_{n+1/2}(z) = 0.5(\delta_n + \delta_{n+1})\,,\ n = 0, 1, 2, \ldots, N-1.$$

Each equation from the system (1) can be written in the form [18]

$$\frac{\partial(r\rho\, UA)}{\partial z} + \frac{\partial(r\rho\, VA)}{\partial r} = \frac{\partial Q}{\partial r} - \varepsilon_A \omega r \tag{3}$$

$$A=\{1,U,H,E,W\},\ Q=\{0,r\tau,rq,r\mu\gamma_\alpha,\mu r\partial W/\partial r\}$$

$$\varepsilon_A=1,\ \omega=\frac{\partial p}{\partial z}+\rho gz\quad(A=U)$$

$$\varepsilon_A=1,\ \omega=-\frac{\rho VW^2}{r}-\mu\left(\frac{\partial W}{\partial r}-\frac{W^2}{r}\right)\quad(A=H)$$

$$\varepsilon_A=1,\ \omega=\frac{\rho VW}{r}+\mu\frac{W}{r}\quad(A=W)$$

$$\varepsilon_A=0\quad(A=1,E)$$

Integrating equation (3) with respect to r from $r=\delta_n$ to $r=\delta_{n+1}$ and taking into account equation (2), we obtain

$$\frac{d}{dz}\int_{\delta_n}^{\delta_{n+1}}(r\rho UA)\,dr=-Q\Big|_{\delta_n}^{\delta_{n+1}}-\varepsilon_A\,\omega\frac{1}{2}(\delta_{n+1}^2-\delta_n^2),\qquad\frac{d}{dz}\int_{\delta_n}^{\delta_{n+1}}(r\rho U)\,dr=0.\tag{4}$$

Integrals are approximated by finite-difference expressions

$$\int_{\delta_n}^{\delta_{n+1}}(r\rho UA)\,dr=0.5(\delta_{n+1}^2-\delta_n^2)\,(\rho UA)_{n+1/2}\,.$$

Considering as unknown functions

$$f_{n+1/2}=0.5(\delta_{n+1}^2-\delta_n^2)\,,\ n=0,1,2,\ldots,N-1,$$

we obtain expressions for $\delta_n(z)$

$$\delta_1^2=2f_{1/2},\quad\delta_2^2=2(f_{1/2}+f_{3/2}),\ \ldots,\ \delta_N^2=2\sum_{n=1}^{N}f_{n-1/2}\,.$$

Taking this into account we deduce from (4) the system of ordinary differential equations on each line $r=\delta_{n+1/2}(z)$ [19]

$$U\dot{U}=\frac{1}{\rho f}R_u-(1-\frac{1}{\gamma})\pi_T\frac{1}{\rho}\dot{p}-\pi_g$$

$$U\dot{T}=\frac{1}{\rho f}R_T-(1-\frac{1}{\gamma})U\frac{1}{\rho}\dot{p}+\frac{1}{\rho}\frac{\pi_w^2}{\pi_T}G_T$$

$$U\dot{E}=\frac{1}{\rho f}R_E\tag{5}$$

$$U\dot{W}=\frac{1}{\rho f}R_w+\frac{1}{\rho}G_w$$

$$\frac{\dot{f}}{f}=-\frac{\dot{p}}{p}+\frac{\dot{T}}{T}-\frac{\dot{U}}{U}$$

Here a dot denotes differentiation with respect to z. The system of equations (5) is written in the dimensionless form. The quantities U, T, ρ, E, p and W are scaled by their maximum values U_1, T_1, ρ_1, E_1, p_1 and W_1 in the inner jet at the pipe inlet and f is scaled by R_0^2. The three dimensionless parameters in (5)

$$\pi_g = R_0 g / U_1^2, \quad \pi_w = \frac{W_1}{U_1}, \quad \pi_T = c_p T_1 / U_1^2$$

are the Froude number, the swirl parameter, and the analog of the Mach number M.

In the approximation considered the pressure is determined by the equation

$$\frac{\partial p}{\partial r} = \frac{\gamma}{\gamma-1}\frac{\pi_w^2}{\pi_T}\rho\frac{W^2}{r}$$

which can, after integration, be written in the form:

$$p(z,r) = p^w(z,r) + p_0(z), \quad p^w(z,r) = \frac{\gamma}{\gamma-1}\frac{\pi_w^2}{\pi_T}\int_0^r \rho\frac{W^2}{r}dr$$

In order to find $p(z,r)$, we calculate $p^w(z,r)$ from the mean value theorem and $\dot{p}^w(z,r)$ from the recurrence relations

$$\dot{p}_1^w = \alpha_{1/2} f_{1/2}, \quad \dot{p}_{n+1}^w = \dot{p}_n^w + \alpha_{n+1/2} f_{n+1/2}, \quad n = 1,2,\ldots,N-1$$

$$\dot{p}_{n+1/2}^w = 0.5(\dot{p}_n^w + \dot{p}_{n+1}^w), \quad n = 0,1,2,\ldots,N-1$$

$$\alpha_{n+1/2} = \frac{\gamma}{\gamma-1}\frac{\pi_w^2}{\pi_T}\frac{\rho f}{r^2}\left(2W\dot{W} - W^2\frac{\dot{U}}{U}\right)\Bigg|_{\delta_{n+1/2}}$$

and determine $p_0(z)$ by integrating the equation

$$\dot{p}_0\sum_{n=0}^{N-1} g_{n+1/2} + \sum_{n=0}^{N-1}\dot{p}^w g_{n+1/2} = R\dot{R} - \sum_{n=0}^{N-1}\left(\pi_g\frac{f_{n+1/2}}{U^2} + \frac{R_T}{\rho UT} + \frac{f_{n+1/2}}{\rho UT}\pi_w^2\frac{G_T}{\pi_T} - \frac{R_u}{\rho U^2}\right) \tag{6}$$

$$g_{n+1/2} = -\frac{f_{n+1/2}}{\gamma(p^w + p_0)} + \left(1-\frac{1}{\gamma}\right)\pi_T f_{n+1/2}\frac{1}{\rho U^2}$$

Density on each line is calculated by the formula

$$\rho(z,\delta_{n+1/2}) = \frac{p(z,\delta_{n+1/2})}{T(z,\delta_{n+1/2})} \tag{7}$$

Equations (5) and (6) contain the dissipative terms

$$R_u = [r\mu\frac{\partial U}{\partial r}], \quad R_w = [r\mu\frac{\partial W}{\partial r}], \quad R_E = \frac{1}{\sigma_\alpha}[r\mu\frac{\partial E}{\partial r}]$$

$$R_T = \frac{1}{\sigma}[r\mu\frac{\partial T}{\partial r}] + \frac{1}{\pi_T}\left([r\mu U\frac{\partial U}{\partial r}] - U[r\mu\frac{\partial U}{\partial r}]\right) \tag{8}$$

$$G_T = \mu\left(\frac{\partial W}{\partial r} - \frac{W}{r}\right)^2 + \frac{\rho V W^2}{r}, \quad G_w = -\frac{\rho V W}{r} - \mu\frac{W}{r^2}$$

where the quantity in brackets means $[Q] = Q_{n+1} - Q_n$.

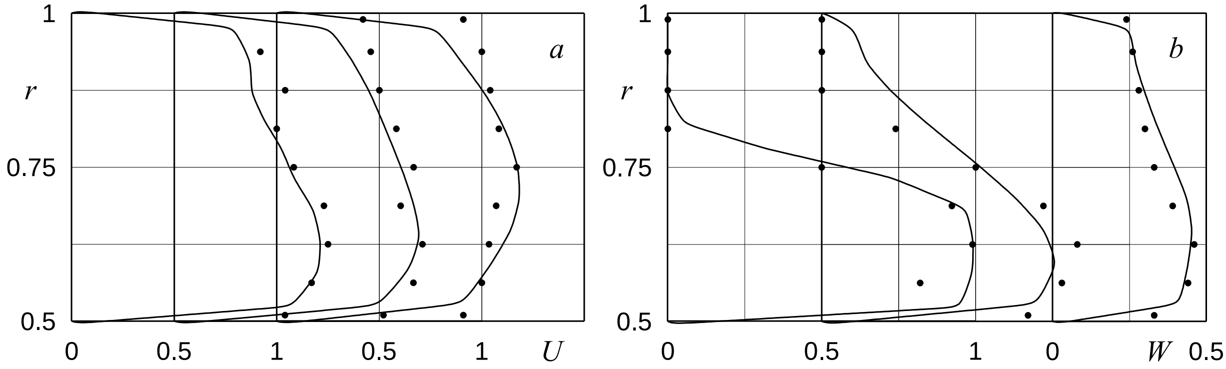

Figure 1. Comparison of the calculated profiles of the axial (*a*) and azimuthal (*b*) velocities with experiments [22] for *z*=0.06, 1.56, 4.56.

The boundary conditions on the flow axis for the unknown quantities $A = \{U, T, E, W\}$ of system (5) follow from the symmetry conditions. In the wall region we assume that the boundary layer is thin and the uniform flow zone extends to the wall. Therefore, we have

$$\frac{\partial A}{\partial r} = 0 \text{ for } \delta = 0,\ \delta = R(z)$$

The system of equations (5)–(8) must be closed by specifying a turbulence model. We will use an algebraic model based on the Prandtl mixing length l_i which for swirling flows is linked with the turbulent viscosity ν_{ti} as follows:

$$l_i^2 = \nu_{ti}\left\{\left(\frac{\partial V_z}{\partial r}\right)^2 + \left[r\frac{\partial}{\partial r}\left(\frac{V_\varphi}{r}\right)\right]^2\right\}^{-1/2} \tag{9}$$

where the subscript i has the values z for the axial and φ for the azimuthal direction. The dimensionless empirical constants were taken to be equal to $l_z / R_0 = 0.068$ and $l_\varphi / R_0 = 0.034$. These values were obtained experimentally [20] for swirling flows of the gas-curtain type. Investigations presented in [21] showed that the numerical results based on these values of l_z and l_φ are in good agreement with the experimental data.

3. Testing of the method and numerical results

This method of calculating swirled flows was tested using the data of experiments [22] on the mixing of two coaxial isothermal flows in the absence of admixtures. The flow in an annular channel $(0.5 \le r \le 1)$, which the inner flow entered pre-swirled and the outer flow unswirled, was considered. At $z = 0$, the velocity distribution was specified as follows:

$$U = 1.2,\ W = 1,\ 0.5 \le r \le 0.75;\quad U = 0.8568,\ W = 0,\ 0.75 \le r \le 1 \tag{10}$$

Here, U is divided by the mean-flow velocity and W by the maximum azimuthal velocity at the channel inlet. In this case, the inner to outer mass flow-rate ratio was equal to unity and the swirl parameter $\pi_w = 0.833$. On the channel walls, the boundary conditions for the velocity were determined in accordance with a logarithmic law.

The profiles of the axial and azimuthal velocities found from the numerical solution of problem (5) – (8) with conditions (10) are presented in figure 1. The data of experiments [22] are indicated by the points. A comparison shows fairly good agreement between theory and experiment and confirms

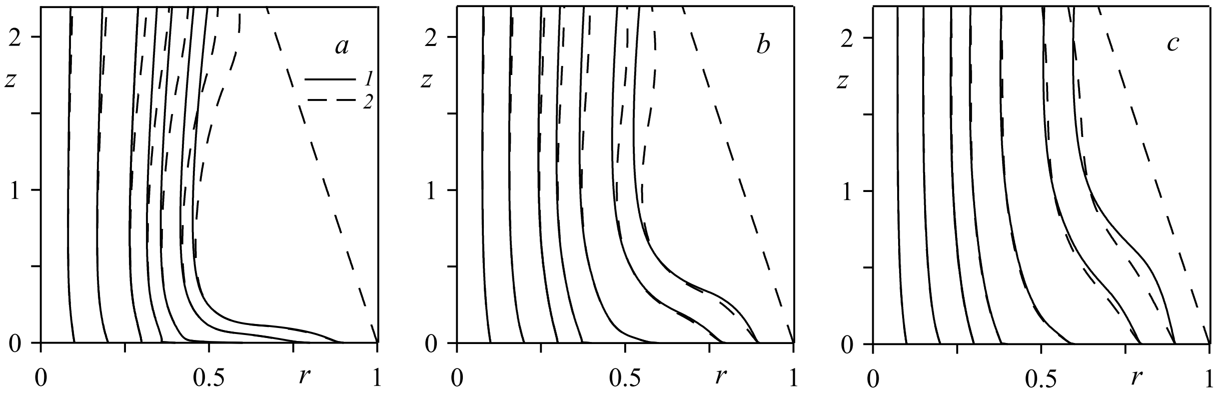

Figure 2. Streamlines for T_2=0.8, r_1=0.33, U_2=0.1, 0.2, 0.3 (*a, b, c*), 1 – $R(z)$=1; 2 – $R(z)$=1–0.15z

the possibility of using the mathematical model to describe the process of mixing of two turbulent flows in the presence of swirling.

Typical values of the parameters for tall structures are as follows: base diameter is 90 m, height is 100 m, flow-rate of the flue gases in the inner flow is 300 m^3/s at a gas temperature of 120°C, and air flow-rate in the outer flow is 5000 m^3/s at a gas temperature of 70°C. Therefore, in the calculations the values of the dimensionless quantities are equal to $\sigma_\alpha = \sigma = 0.72$, $\pi_g = 6.45$, $\pi_T = 5754$.

Most of the calculations have been performed for the following distributions over the inlet cross-section $z = 0$

$$\begin{aligned} &U(r) = U_1 = 1,\ W(r) = W_1(r),\ T(r) = T_1 = 1,\ E(r) = E_1 = 1,\ 0 \le r \le r_1 \\ &U(r) = U_2, \quad W(r) = W_2, \quad T(r) = T_2, \quad E(r) = E_2, \quad r_1 < r \le 1 \end{aligned} \tag{11}$$

where $r_1 = R_1 / R_0$. The solution domain is determined by the length $z_0 = 2.2 R_0$ and the lateral surface is either assumed to be cylindrical $R_0 = 1$ or specified by the equation $R(z) = 1 - 0.15z$.

A graphic representation of the flow pattern inside the ventilation pipe is shown at figure 2. The streamlines converge fairly rapidly toward the center with increase of the distance z. This information can be used in profiling the stack walls in order to reduce the dimensions of the structure, cut costs, and make the structure more stable. In all cases, the temperature decreases with increase of z. A favourable effect is detected in the variation of the admixture concentration along the flow axis which is almost halved at the channel outlet as compared with its initial value.

4. Conclusions

The method developed makes it possible to seek the optimal flow regimes in tall stack and tower structures for ejecting pollutant-containing smokes and gases into the atmosphere with a view to minimizing environmental damage.

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