Earth surface subsidence caused by arbitrary underground mining

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Abstract. Underworking of built-up areas, which often take place in the areas of deep mining of solid minerals, affect the surface structures located on these territories. Surface subsidence and associated with it inclination and stretching of the day surface lead to the damage of buildings, the rupture of pipelines, the disturbance of highways and railways.
Such undesirable manifestations also occur during underground construction in large cities, where they are more pronounced due to the small depths of underground work.
In many cases, the shape of the worked-out space in the plan can have an arbitrary shape unlike, say, the case of the development of coal seams, when this form is strictly regulated by the mining method used. In this case, recommendations have been developed for assessing the influence of underground excavations on the day surface, which will have to apply with large approximations for estimation the trough for an arbitrary form of mining openings during construction in urban conditions.
The conceptual approach and the numerical algorithm for estimating the parameters of the day surface subsidence for an arbitrary in the plan system of excavations are considered in the article. In this case, the entire area in which the workings are located is divided into elementary cells, and the subsidence of the day surface for the entire excavation as a whole is represented as the sum of subsidence from the set of all these elementary cells. Within each cell, subsidence is described by the Gaussian probability integral. This approach is based on the generalization of numerous observational data.

1. Introduction

To describe the shape of the day surface subsidence caused by underground mining works, one can use, with a certain restrictions on the properties of the enclosing rocks and on the waste form, a function of the form [1].

$$\eta(x,a,C,\eta_0) = -\frac{\eta_o}{2}\left[\Phi\left(\frac{a+x}{CH}\right)+\Phi\left(\frac{a-x}{CH}\right)\right], \tag{1}$$

where $\Phi(t)=\sqrt{\frac{2}{\pi}}\int_0^t e^{-\frac{z^2}{2}}dz$ - Gaussian probability integral, and η_0, a, C are constant values: η_0 - maximum subsidence of the day surface at full underworking; C - maximum inclination of the day surface; a - parameter related to the length of the waste $2L$ (a~$0.8 \div 1.0L$); H - bedding depth.

The selection of the parameters included in (1) with the aid of the developed methods [2] allows us to achieve an acceptable approximation and to construct a settling curve that practically coincides with the experimental data on vertical shifts of the benchmarks. In connection with this, in the sequel we will use this approximation (1), without emphasizing the initial admissions and the actual method of obtaining the above relation [1, 3]. In order to remove the constraints in the formulation of the problem, which are associated with the waste form configuration, we study some important properties of the dependence (1).

First, the function (1) has the additivity property, understood in the following sense

$$\eta(x,a,C,\eta_0) = n\,\eta(x,\frac{a}{n},C,\eta_0)\,. \tag{2}$$

The subsidence of the earth's surface caused by the underground mining with the characteristic $2a$ in (1) is equal to the subsidence caused by n identical workings adjacent to each other with the corresponding parameters $2a/n$. This assertion is a simple consequence of the possibility of summing the integrals included in (1).

This property can be illustrated on the graph, displaying on it the results of calculations for some arbitrarily chosen set of parameters η_0, a, C, H в (1). Let's take, for example, η_0 =1m, C = 0.2, H = 50m and construct a set of, say, four curves for the value a = 2m, shifted by 0, 4, 8, 12 m, respectively, relative to the coordinate origin. They are shown in Figure 1 under numbers 1-4. In this case, the center of the total mining is at the point x = 6 m, and the calculated total curve is shown at the number 5. Now we set a = 8m and construct a subsidence curve for this value of the parameter. It is displayed in the figure by means of points and to be compared with the already constructed total curve is shifted to the right by 6 m. Obviously, their complete coincidence, which well illustrates the earlier statement.

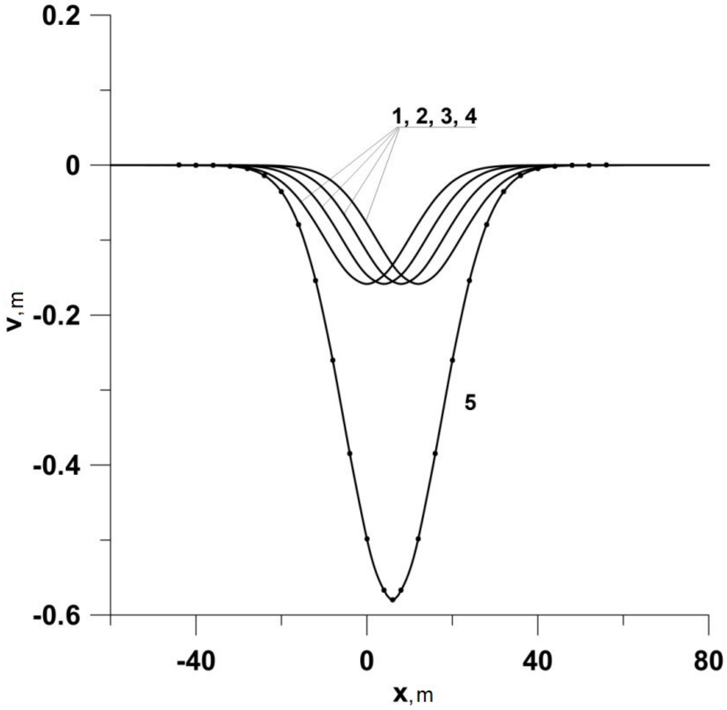

Figure 1. The additivity property for (1).

2. Mathematical algorithm

This fact gives grounds in the course of the subsidence calculations, caused by some underground working, to divide this excavation, if necessary, into several parts and then calculate the subsidence for all these parts separately, and afterwards summarize them all. Moreover, it is principally possible here to sum up the subsidence from several different workings provided the correct choice of the corresponding parameters a for each of them.

Secondly, we consider the form of the function (1) for small values of the parameter a, which we now denote by l_x, and some arbitrary set of other parameters.

To this end, we expand the function (1) into series in a and preserve the terms of the first order. It is obvious that $v(x)|_{l_x=0}=0$ and therefore

$$v(x,l_x)=\frac{\partial v(x)}{\partial a}\Big|_{l_x=0}\, l_x\,. \tag{3}$$

Simple calculations lead to

$$v(x,l_x)=-\eta_o\frac{l_x}{CH}\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\left(\frac{x}{CH}\right)^2}\,. \tag{4}$$

Obviously, if the centre of elementary working with the parameter l_x is located not at the origin but at the point ξ, then instead of (4) it is necessary to write

$$v(x,l_x,\xi)=-\eta_o\frac{l_x}{CH}\sqrt{\frac{2}{\pi}}e^{-\frac{1}{2}\left(\frac{x-\xi}{CH}\right)^2}=F(x,\xi)l_x\,. \tag{5}$$

The resulting function can be used as a so-called influence function. In this case, the subsidence caused by the underground working with the parameter $2a$ will be expressed by an integral of $F(x,\xi)$ by ξ in the range $2a$ (from $-a$ to a).

In carrying out numerical calculations, integration over the region, as a rule, is replaced by the summation of elementary regions effects, i.e. expressions of (5) type. It is important to estimate the maximum elementary region size that is admissible from the point of view of an acceptable approximation accuracy for the obtained solution. As an example, Figure 2 illustrates the accuracy of the approximation (4) as a function of the quantity l_x, for the previously chosen set of values for the parameters η_0, C, H, where $\Delta=\frac{\eta(x,a,C,\eta_0)-V(x,l_x)}{V(x,l_x)}100\%$. For $l_x<2$ the maximum error is less than 2%.

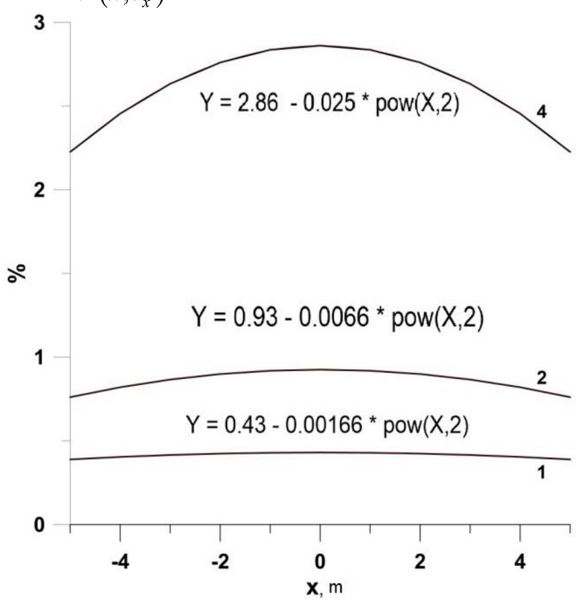

Figure 2. The accuracy of the approximation (4) as a function of the quantity l_x.

In general, such an estimate can be obtained by retaining terms of a higher order in the expansion of the function (1). This is the case

$$v(x,l_x) = -\eta_o \frac{l_x}{CH}\sqrt{\frac{2}{\pi}} e^{-\frac{1}{2}\left(\frac{x}{CH}\right)^2}\left[1+\frac{1}{6}\frac{l_x^2 x^2}{(CH)^4}\right]. \tag{6}$$

For the legitimacy of applying relation (4), it is necessary to require the satisfaction of inequality

$$\frac{1}{6}\frac{x^2}{(CH)^4} l_x^2 << 1, \tag{7}$$

that for $C \cong 0.2$ and $x \cong H$ leads to the estimation

$$l_x \approx 0.01H . \tag{8}$$

Thus, (1) and (4) practically coincide in the region $-H < x < H$, if the parameter l_x in (4) (or, which is the same, a in (1)) is taken equal $\approx 0.01H$. The resulting relations ultimately enable to estimate the order of magnitude of the parameter a for elementary underground working at numerical integration of (5) over the region.

The above considerations have revealed the fundamental possibility of using an approach based on the construction of the influence function. However, all that has been said so far has little practical significance, since the use of relation (1) is incomparably more efficient than the integration (summation) of the influence functions. The situation drastically changes for the two-dimensional case, when the use of the influence function makes it possible to include in consideration an arbitrary number of two-dimensional regions with any configuration in the plan.

Further, construct the influence function for the two-dimensional case. To do this, we start from the relation describing the subsidence of the earth's surface for a rectangular development, which is a generalization of (1) [4]

$$\eta(x) = -\frac{\eta_o}{4}\left[\Phi\left(\frac{a_x + x}{CH}\right) + \Phi\left(\frac{a_x - x}{CH}\right)\right] * \left[\Phi\left(\frac{a_y + y}{CH}\right) + \Phi\left(\frac{a_y - y}{CH}\right)\right] \tag{9}$$

In this case, a_x and a_y have the same meaning as a in (1).

Proceeding as above, in deriving (5), we obtain the relation

$$v(x,y,l_x,l_y,\xi,\eta) = -\eta_o \frac{2 l_x l_y}{C^2 H^2} e^{-\frac{1}{2}\left[\left(\frac{x-\xi}{CH}\right)^2 + \left(\frac{y-\eta}{CH}\right)^2\right]} = F(x,y,\xi,\eta) l_x l_y \tag{10}$$

The idea of the approach developed below is to summarize the influence of individual small parts of the form (10) on which an arbitrary development is broken.

In view of the discussion above, it is fair to set the following algorithm for calculating the earth's surface subsidence:

1. All excavations are divided into rectangular areas of size $l_x * l_y$, taking into account the constraints (7), (8), and otherwise completely arbitrary. It is important to ask as far as possible matching the boundaries of the original area and the borders made up of rectangles. In this case, a common coordinate system is defined and each rectangle is linked by its centre to this system.

2. In the same coordinate system, an arbitrary grid of points is defined, in which the earth's surface subsidence will be calculated.

3. All mentioned above rectangular areas are run through the cycle and for each of them the corresponding subsidence is calculated and summed in at all grid points. In this way, a grid function is constructed that describes the subsidence in the region overlapped by the grid accepted in the calculations.

As an example, confirming the above considerations, let us consider the test problem of calculating the surface subsidence for a rectangular in plan waste, in which the length of one side considerably exceeds the length of the other. This case is illustrated in Figure 3, which depicts an "effective" area with dimensions a_x = 50m, a_y = 10m. In addition, we accept the following parameter values: η_0 = 1m, C = 0.2, H = 100m. We consider the settling profile in the vertical section A-A passing through the centre of the longest side of the waste perpendicular thereto.

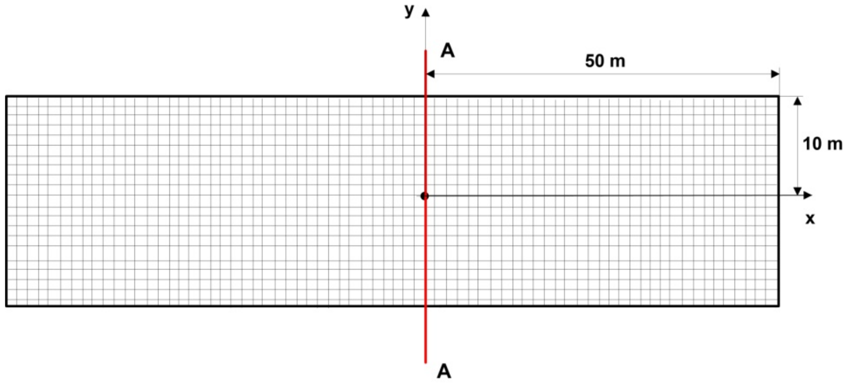

Figure 3. Configuration of the calculation area for the test task.

The solution of this problem can be obtained in at least three ways. First, we note that this section is located far from the short sides of the waste, i.e. outside the zone of their influence. In connection with this, the waste in the x direction can be assumed to be infinite and therefor the conditions for the applicability of relation (1) are satisfied.

Secondly, we can apply relation (9) for $x = 0$, which is just the case for the configuration in question.

3. Example of the developed algorithm application

Finally, we can apply the algorithm developed above. To do this, divide the rectangle into elementary rectangular cells of the size $l_x = 2$m and $l_y = 2$m, which are also shown in Figure 3. As a computational grid, we take a set of points uniformly distributed along the line A-A.

The performed calculations showed the complete identity of the results obtained by all three methods.

Let us now consider a model problem for two excavations for which the configuration is shown in Fig. 4.

As before we assume: $\eta_0 = 1$m, $C = 0.2$, $H = 100$m, $l_x = 2$m, $l_y = 2$m. The same figure shows the breakdown (partially) of regions into elementary cells, which number in this case turned out to be 829. As a computational grid, was taken a grid uniform in x and y with a step of 5 meters in the range $0 < x < 120$ m, $0 < y < 80$ m and with a number of nodes in it - 384.

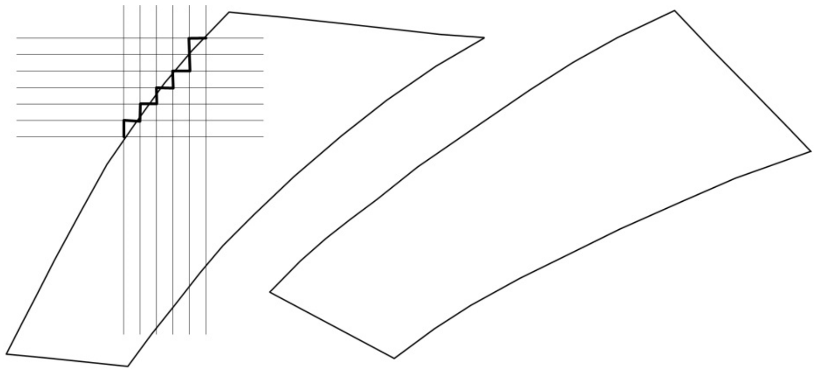

Figure 4. Two excavations and their division (partial) into unit cells.

Calculations carried out using the developed algorithm made it possible to construct a mesh function, which was later used, in particular, for constructing subsidence contours of the earth's surface. These isolines are shown in Figure 5 in relation to the contour of the area.

In conclusion, let us consider a cross section of the so built trough. The choice of its location is evident from the figure. Namely, it must be chosen in the place where the isolines are, as far as possible, straight and the section is taken perpendicular to these lines. In addition, it passes through (or near) the point of maximum subsidence. This ensures that the conditions of applicability of relation (1) are satisfied (see [1]).

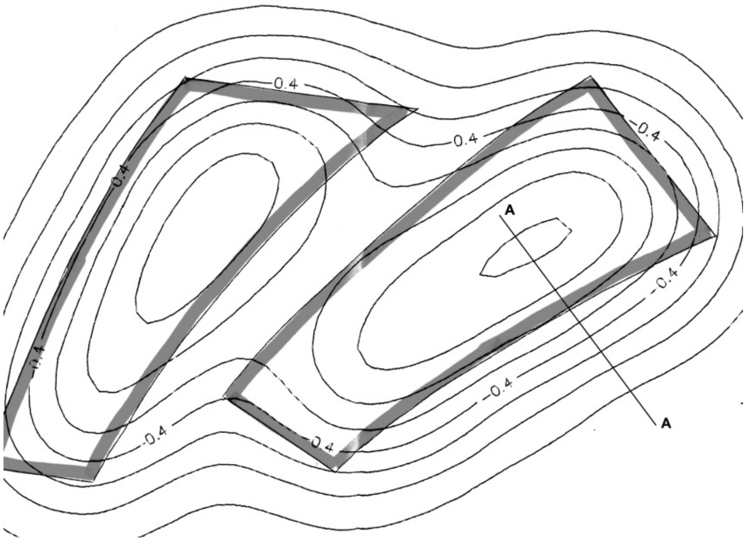

Figure 5. Subsidence contours of the earth's surface.

The profile of the subsidence surface along A-A is shown in Figure 6. Let us analyse it with the help of an approximate technique for determining the parameters, in particular *C*, set forth in [4].

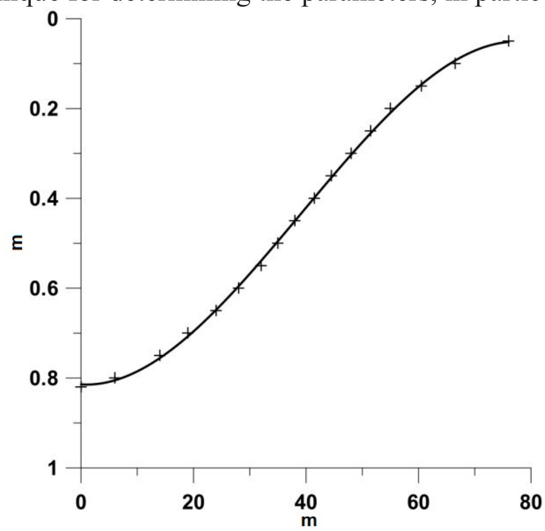

Figure 6. The profile of the subsidence surface along A-A.

For this, we normalize the values of the subsidence both in v and in x and construct the corresponding dimensionless profile (Figure 7). After this, we find the slope of the curve at the point $\bar{x} = 1$ and the corresponding value of *C*. This yields $C = 0.19$, which is very close to the predetermined initial value of - 0.2

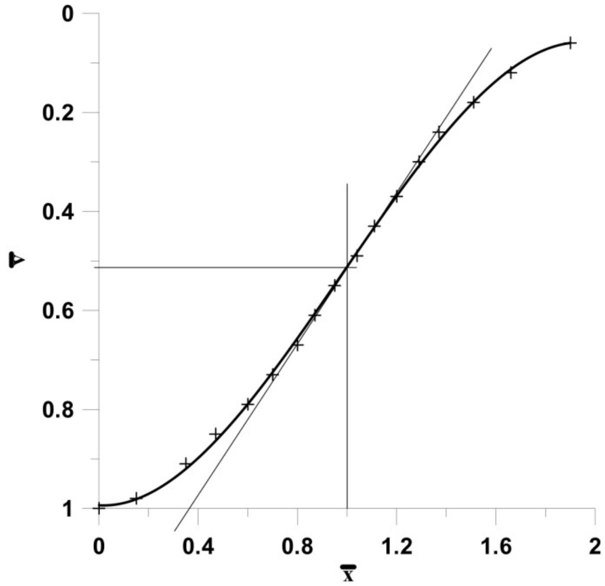

Figure 7. Determination of the maximum angle of inclination of the earth's surface (*C* in (1)).

Conclusions

Thus, the above analysis makes it possible to count that for the determination of the parameter *C* in (1) or (9) with one or another way [2, 4, 5] for any configuration of excavations, practically any profile obtained according to the measurements of bench marks station can be used, if it satisfies the above requirements. As a rule, they are realized in the central part (or near it) of a rectilinear sufficiently long excavation boundary.

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