

Refinement of the accounting methodology of bi-moments transfer at the junctions of the I-section bars

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Abstract. Modern construction standards require an accounting of bi-moments when determining normal stresses in the cross-sections of the bars. This factor may first be essential to provide the required safety and reliability of thin-walled bars of an open profile. The bars restrained torsion theory was sufficiently considered in literature, however, in recent years the classic concepts about interaction of such bars in frame systems have been challenged by some authors. Within the framework of the finite element method such problem may be solved on the basis of a combined approach, when the bar finite elements are imposed out of the area of joined nodes and strains at the joints are described with the help of plate finite elements. At the same time, calculation models become more complicated in comparison to using only the bar finite elements. In this paper the detailed studies of interaction of the bars with I-sections are performed on the basis of mathematical modeling of their strains with the help of plate finite elements. The methodology of these studies contains definition of bi-moments in several bars cross-sections, including the sections adjacent to the joints. Different variants of relative positions of the bars are considered. It has been determined that in some instances the condition of bi-moments balance is provided in assembly joints with a high degree of accuracy. At the same time, in some schemes of the bars connection such a balance is being largely damaged. The paper gives the explanation for the phenomenon based on the features of the transfer of the internal forces factors in the connection joints of the bars. It also represents a way of obtaining imbalance on bi-moments in accordance with the real work condition of these thin-walled constructions. The obtained results will make it possible to increase the accuracy of calculations of the thin-walled constructions formed by the bars with I-sections within the bars calculation models. The considered approach to the analysis of the interaction of the thin-walled bars may be the basis for development of the calculation algorithms of thin-walled bar systems with other forms of cross-sections.

1. Introduction

The construction rules [1] require an accounting of bi-moments when determining normal stresses in the cross-sections of thin-walled bars exposed to torsion. The bars for restrained torsion can be effectively calculated using Vlasov's torsion theory [2]. The finite-element modeling of strains of thin-walled bars was considered in [3-11]. The paper [12] proposes a scheme for calculating the plate-bar structures by the finite element method, in which the restrained warping in the bars is taken into account. At the same time, the problem of calculating thin-walled frames taking into account restrained torsion has not been properly developed yet. The use of the assumption about the equality of



bi-moments and warping measures, considered in the book [13], at the joints of pairs of bars when calculating systems of thin-walled bars in many cases may give significant errors [5, 7]. The combined approach, when the bar finite elements are used outside the zone of joined nodes, and the joint strains are described with the help of the shell finite elements [7], greatly complicates the calculation schemes. Researches on this issue must, first of all, give an answer to the possibility of using the condition of the balance of bi-moments in the joints of the bars in bar calculation schemes. Based on these data, the theories of calculation of structural systems of this type can be improved. This paper analyzes the regularity of the transfer of bi-moments at the nodes of the connection of the I-beams and proposes an approach to correcting the solution of the problem in those cases where the balance of bi-moments can be largely upset. A bar finite element in which restrained torsion is described with the help of the shear-less theory and the bars connection scheme is adopted in accordance with the working methodology [14] is used. In this case, there is a possibility to apply corrective bi-moments expressed through eccentrically transferred moments in joined nodes.

2. The research of bi-moments transfer at the bars joints

We considered several variants of connection of the I-beams. In particular, the examples presented in Figure 1 were analyzed, where F is a rigid attachment. The moments $M_1 = M_2 = 10$ kNm of the applied force couples were set. Structural schemes for these examples are shown in Figures 2a and 2b, where α , β and γ are the bars. The rib stiffeners R were taken according to the requirements of construction standards [1]. The ribs thickness is 7 mm. For both objects, I-beams N 20 were taken as per GOST 26020-83.

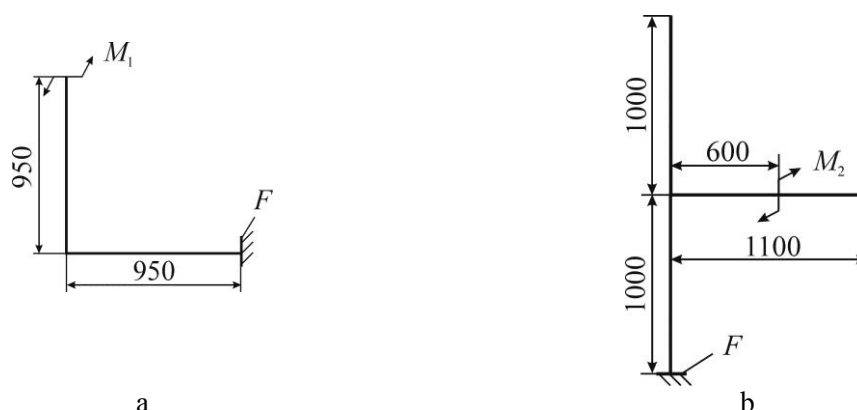


Figure 1. Bar systems considered in the examples 1 (a) and 2 (b).

A finite-element modeling of the stress-strain state of the thin-walled systems under analysis was carried out based on spatial plate schemes. Autodesk NEi Nastran software package (license of the Bryansk State Engineering Technological University N PR-05918596) was used for this purpose. The visualization of these models is shown in Figure 3. For examples 1 and 2, 8400 and 13400 four-node rectangular finite elements were introduced respectively.

After calculations on the basis of plate schemes for a number of cross-sections of the bars, bi-moments were determined by numerical Gaussian integration using the following dependency:

$$B = \sum_{i=1}^n \left(t_i L_i \sum_{j=1}^m a_j \sigma_j^{(i)} \omega_j \right), \quad (1)$$

where n is a number of plate finite elements, separated by the cross-section, t_i is the thickness of the finite element i , L_i is the length of the segment along which the middle plane of the finite element i is intersected by the cross-section, m is a number of points of Gaussian integration [15], a_j is the

coefficient of integration for the point j , $\sigma_j^{(i)}$ is the membrane normal stress for the given cross-section in the finite element i for the integration point j , ω_j is the principal sectorial coordinate, corresponding to the integration point j .

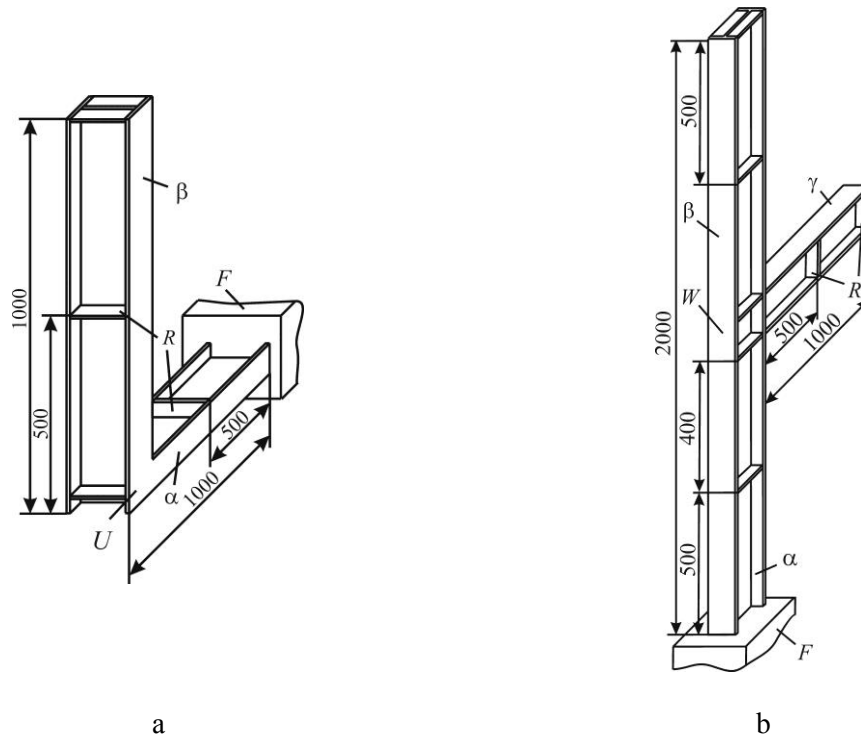


Figure 2. Structural schemes of the examples 1 (a) and 2 (b).

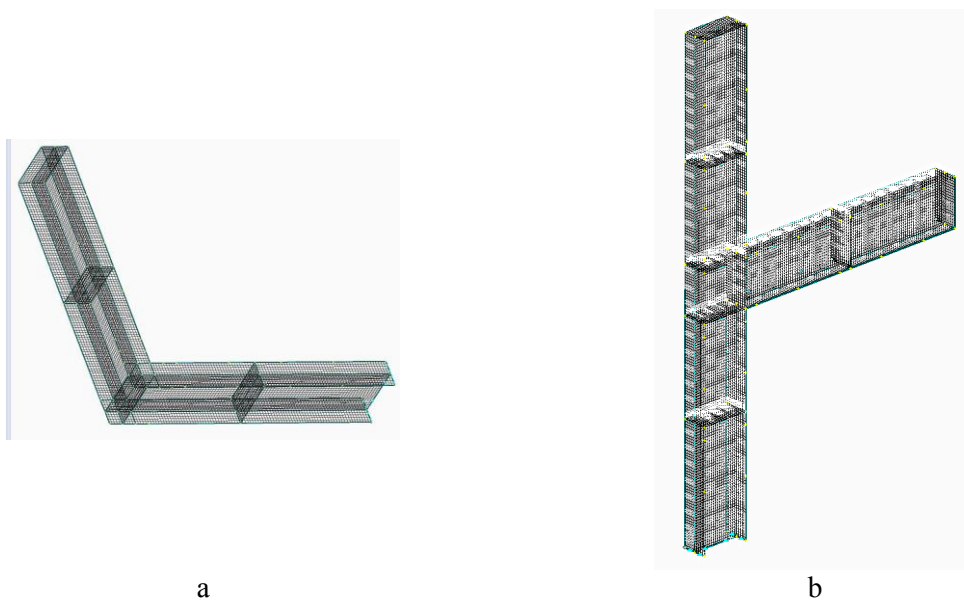


Figure 3. Finite element plate models in the examples 1 (a) and 2 (b).

When plate four-node finite elements are used, a sufficiently high accuracy for estimating bi-moments can be achieved on the basis of the second order of Gaussian integration. The curves of bi-moments obtained, using equation (1), are shown in the figure 4, where the distribution of bi-moments between the considered cross-sections is linearized. In the example 1, in the cross-sections of the bars α and β adjacent to the joined node U , modulus of bi-moments were: $B_\alpha = 2484 \text{ Nm}^2$, $B_\beta = 2579 \text{ Nm}^2$. Value B_α differs from value B_β by less than 4%. In the example 2, for the cross-sections adjacent to the joined node W , the sum modulus of bi-moments in the bars α and β $B_{\alpha+\beta} = 2348 \text{ Nm}^2$, which exceeds the corresponding bi-moment $B_\gamma = 1791 \text{ Nm}^2$ of the γ bar by more than 31%. That is, for the first example, the assumption about the balance of bi-moments in the joined node may be made for practical purposes, and for the second example such an assumption can give significant errors in the results. Similar differences in providing the approximate balance of bi-moments were also obtained when considering some other examples.

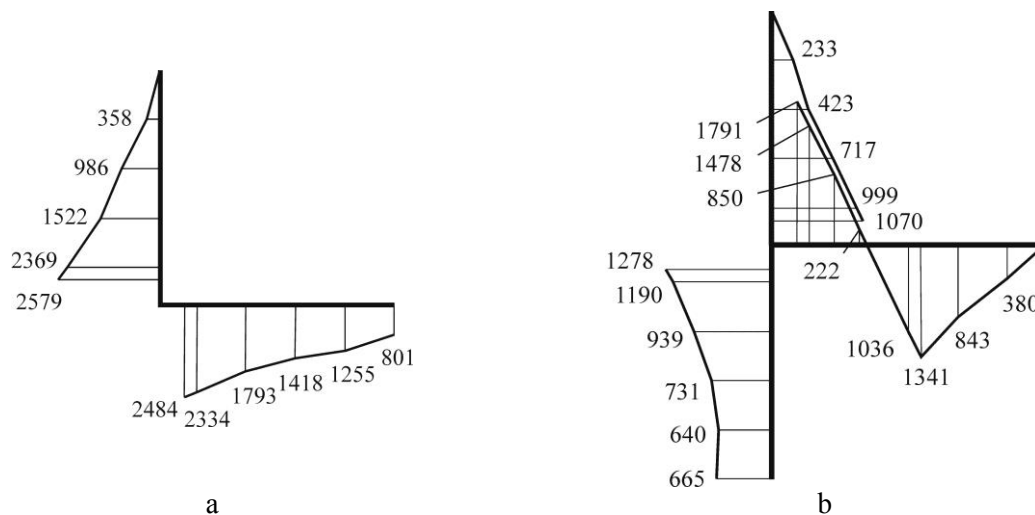


Figure 4. The results of bi-moments calculation for the examples 1 (a) and 2 (b) (Nm^2).

3. Accounting for the bi-moments imbalance

Based on the analysis of the stress-strain state of thin-walled systems for the examples 1 and 2, as well as other models, we propose the following explanation of the possible appearance of significant imbalances in bi-moments. Let us assume that a force couple with M_A moment, which in the cross-section L adjacent to the joined node is actually took in a plane spaced from the axis of the bar at some distance d , acts upon a particular I-beam from another beam (Figure 5). Let us introduce a self-balanced system of force couples acting in the principal central plane Cxz of the bar with the moments M_C , M_B provided that $M_C = M_B = M_A$. At the same time, the moment M_B bends the bar, and we consider the system of moments M_A, M_C as a bi-moment, the numerical value of which is

$$B^* = M_A d. \quad (2)$$

We will treat this bi-moment as the main reason for obtaining an imbalance on the bi-moments in the node. To take this phenomenon into account, let us introduce the following approach for the calculation of flat frames formed by the I-section beams. At the first stage we perform the calculation of the frame assuming that balance of bi-moments obtained in the system of cross-sections adjacent to the joined nodes is provided. In this case, the connection of the bars is carried out using inter-bar ties within the approach of the paper [14]. Further, based on the obtained moments transferred in the joined nodes, we add bi-moments B^* , and repeat the calculation. If the introduction of additional bi-moments somewhat changes the transferred moments, the procedure for determining and accounting

for bi-moments B^* may be repeated several times until the required convergence of the calculation process is provided. Usually, for practical purposes, only 1-3 steps of correction of the obtained solutions are required. In the case of using the finite element method, such corrections lead to an insignificant increase in the complexity of calculations, since they do not require a change in the global stiffness matrix of the finite element model. As calculations show, for the I-section connection scheme of example 2, $d=0.6h$ can be approximately taken, where h is half the height of the cross-section (see Figure 5).

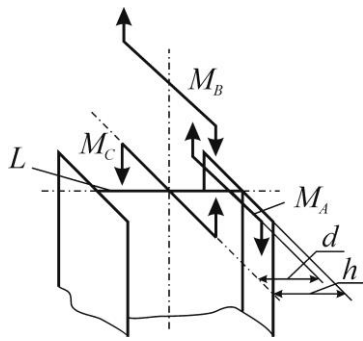


Figure 5. The eccentric transfer of the moment to the bar.

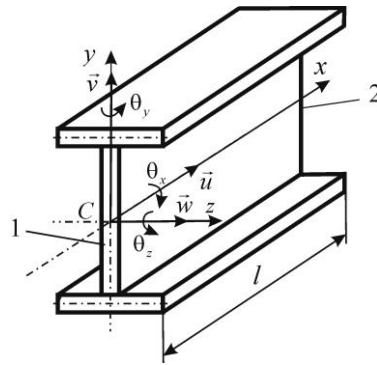


Figure 6. The bar finite element.

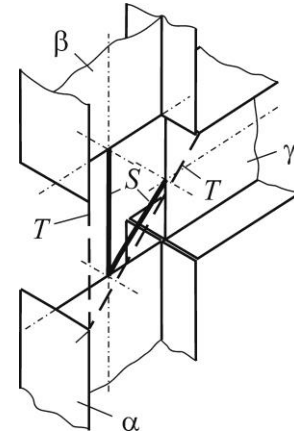


Figure 7. Connections in the joined node.

While implementing this approach, we use the bar finite element constructed according to the Vlasov's torsion theory. We assume that in the general case bars can be exposed to tension-compression, transverse bending in two principal planes, and restrained torsion. In this case, we consider the following procedure for approximating the displacements for a bar of constant I-cross-section. Let us introduce a two-node bar finite element with nodes $i = 1, 2$, located at the centers of gravity of the extreme cross-sections (Figure 6). The vector of generalized displacements of the bar is represented as follows:

$$\{\delta_i\} = \{u_i \quad v_i \quad w_i \quad \theta_{xi} \quad \theta_{yi} \quad \theta_{zi} \quad (\partial\theta_x/\partial x)_i\}^T, \quad (3)$$

where u_i , v_i and w_i are the projections of the displacement vector of the center of gravity of the cross-section for the node i on the axes Cx , Cy and Cz respectively, θ_{xi} , θ_{yi} and θ_{zi} are the rotation angles of the cross-section of the node i relative to these axes, $(\partial\theta_x/\partial x)_i$ is the measure of warping in the node i .

The vector of generalized strains of the bar is represented as follows:

$$\{\varepsilon_e\} = \left\{ \varepsilon_x \quad \chi_y \quad \chi_z \quad \frac{\partial\theta_x}{\partial x} \quad \frac{\partial^2\theta_x}{\partial x^2} \right\}^T = \left\{ \frac{\partial u}{\partial x} \quad -\frac{\partial^2 w}{\partial x^2} \quad -\frac{\partial^2 v}{\partial x^2} \quad \frac{\partial\theta_x}{\partial x} \quad \frac{\partial^2\theta_x}{\partial x^2} \right\}^T, \quad (4)$$

where ε_x is a relative linear strain along the axis of the bar, χ_y and χ_z are bending strains of the bar relative to the axes Cy and Cz .

We will record the vector of generalized stress corresponding to the $\{\varepsilon_e\}$ vector in the form of:

$$\{\sigma_e\} = \{N \quad M_y \quad M_z \quad M_\theta \quad B\}^T, \quad (5)$$

where N is a normal force, M_y and M_z are bending moments relative to the axes Cy and Cz , M_θ is a moment of pure torsion.

Taking into account the equalities (4) and (5), we represent the elasticity matrix of the finite

element, defined by the relation $\{\sigma_e\} = [D_e]\{\varepsilon_e\}$ [15], in the form of:

$$[D_e] = \begin{bmatrix} EA & 0 & 0 & 0 & 0 \\ 0 & EI_y & 0 & 0 & 0 \\ 0 & 0 & EI_z & 0 & 0 \\ 0 & 0 & 0 & GI_t & 0 \\ 0 & 0 & 0 & 0 & EI_\omega \end{bmatrix}, \quad (6)$$

where E , G are elastic modulus and material shear modulus respectively, A is a cross-sectional area of the bar, I_y and I_z are inertia moments of the cross-section relative to the axes Cy and Cz , I_t is a torsional constant, I_ω is a principal sectorial moment of inertia of the section.

We will approximate the displacement u by linear law, and displacements v , w and the rotation angle θ_x with the help of the third degree polynomials. The nodal displacements vector $\{\delta_e\}$ of a finite element is considered as a union of vectors $\{\delta_1\}$ and $\{\delta_2\}$. Then, taking into account formulas (3), (4), we will represent the strain matrix $[B_e]$ of the finite element, defined by the expression $\{\varepsilon_e\} = [B_e]\{\delta_e\}$ [15] in the form of:

$$[B_e] = \begin{bmatrix} -\frac{1}{l} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{l} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varphi_1 & 0 & \varphi_2 & 0 & 0 & 0 & 0 & -\varphi_3 & 0 & \varphi_4 & 0 & 0 \\ 0 & -\varphi_1 & 0 & 0 & 0 & -\varphi_2 & 0 & 0 & -\varphi_3 & 0 & 0 & 0 & -\varphi_4 & 0 \\ 0 & 0 & 0 & \xi_1 & 0 & 0 & \xi_2 & 0 & 0 & 0 & \xi_3 & 0 & 0 & \xi_4 \\ 0 & 0 & 0 & \varphi_1 & 0 & 0 & \varphi_2 & 0 & 0 & 0 & \varphi_3 & 0 & 0 & \varphi_4 \end{bmatrix}, \quad (7)$$

where

$$\varphi_1 = -\frac{6x}{l^2} + \frac{12x^3}{l^3}, \varphi_2 = -\frac{4}{l} + \frac{6x}{l^2}, \varphi_3 = \frac{6x}{l^2} - \frac{12x^3}{l^3}, \varphi_4 = -\frac{2}{l} + \frac{6x}{l^2},$$

$$\xi_1 = -\frac{6x}{l^2} + \frac{6x^3}{l^3}, \xi_2 = 1 - \frac{4x}{l} + \frac{3x^2}{l^2}, \xi_3 = \frac{6x}{l^2} - \frac{6x^2}{l^3}, \xi_4 = -\frac{2x}{l} + \frac{3x^2}{l^2}, \quad (8)$$

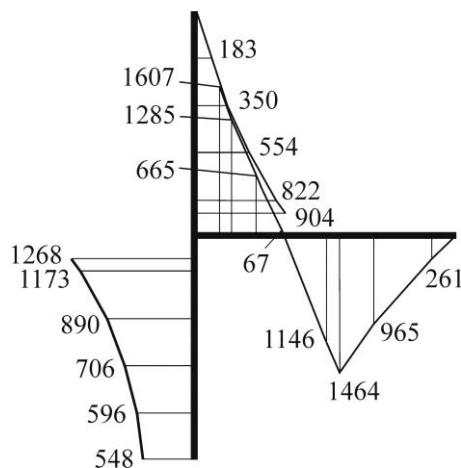


Figure 8. The curve of bi-moments for the example 2, obtained with the help of the proposed procedure (Nm^2).

take into account the imbalance in bi-moments quite effectively: in figure 4b the difference $B_{\alpha+\beta} - B_\gamma$

l is the finite element length.

Let us consider the results of using the proposed approach by the example of calculation using the bar model of the construction of the example 2. The bars connection scheme in the W joined node is shown in Figure 7, where S are additional elements rigidly connected to the bars in all degrees of freedom, T are hinged beam connections. The I-bars were divided into 17 bar finite elements. In this case, only one step was required to correct the calculation results, since the moments transferred in the joined node did not practically change. Figure 8 shows the resulting bi-moments in the bar system. A comparison of the curves of Figures 8 and 4b indicates a satisfactory agreement of the solutions obtained on the basis of the proposed procedure and the plate calculation scheme. The correction of the results allowed us to

turned out to be 557 Nm^2 , in figure 8 – 565 Nm^2 . The deviation in the bar model from the plate one by the maximum value of the bi-moment is less than 11%.

4. Discussion

In this paper we are restricted to the consideration of frame structures made of I-beams. At the same time it is assumed the system includes only transverse stiffness ribs installed in accordance with the regulatory requirements. The extension of the proposed approach to the thin-walled structures that do not satisfy these conditions requires further research. In particular, the use of transverse ribs of heavier thickness or inclined ribs may become an additional factor of the bi-moments imbalance. It seems that it is reasonable to take these factors into account in the first stage of the calculation, and then make corrections in accordance with the methodology presented in this paper.

5. Conclusions

A procedure of accounting the possible imbalance of bi-moments at joined nodes of the I-cross-section bars has been developed. The executed developments are based on the analysis of strains of bar systems using refined plate finite element models and taking into account the revealed regularities for bar calculation schemes. The fairly high for practical purposes accuracy of the proposed approach is illustrated by the example of the three-bar structure.

Acknowledgements

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