

Bending of a wavy plate of a periodic profile on an elastic foundation

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Abstract. As a roofing material profiled sheets are most often used in large areas in industrial and civil construction. The weight of snow on the roof, especially in snow-covered regions of Russia, can exceed the weight of the roof itself, so it is necessary to take into account the snow load in the winter period in calculations for strength and rigidity. Nowadays the steel with polymer coatings, which give the sheets more decorative, is increasingly used in individual and low-rise buildings. To increase the rigidity of metal sheets, they undergo profiling, wavy shaping. In this paper, we consider the bending of a wavy plate on an elastic foundation rigidly clamped along the edges. Plate in the plan has a rectangular form. The plate material is isotropic. For the calculation scheme of the receiving orthotropic plate, take different cylindrical stiffness in two mutually perpendicular directions. The elastic foundation is adopted by Winkler, so it is believed that the reaction of the base is directly proportional to the deflection of the plate at each point. To determine the deflection, the Bubnov-Galerkin method is used. To solve the problem, we use special orthonormal Legendre polynomials satisfying the boundary conditions. Investigations of the stress-strain state of a wavy plate loaded with a distributed load depending on the amplitude, the length of the half-wave and the modulus of subgrade reactions, as well as the stress-strain state of the flat plate, were carried out.

1. Introduction

During designing and manufacturing building structures, the strength increasing and at the same time facilitating the elements of various structures is achieved using wavy thin plates replacing orthotropic materials, [1-5]. For example, the wavy profile plates are used as a roof material and. They are also widely used in fencing structures. As a roofing material profiled sheets are most often used in large areas in industrial and civil construction. At present, in connection with the use of steel with polymer coatings, which give the sheets more decorative, the latter are increasingly used in individual and low-rise buildings (cottages, small shops, gas stations, kiosks).

To increase the rigidity of metal sheets, they undergo profiling, i.e. giving a wavy form. Profiled or, as they are called, corrugated sheets, the corrugated board are made of galvanized steel with or without a polymer coating. The waves on the sheets can be high and low and have a trapezoidal, sinusoidal or rounded shape. Sheets of a height of more than 20 mm are constructive elements, their application must be confirmed by calculations for strength and deflection.

Wavy or corrugated plates are used in various areas. In [6] the composite wavy plate is used for breakwaters. Numerical analyses and experiments are presented to study the characteristics of the reflection due to composite wavy plate breakwaters. It's shown that theoretical and experimental



studies have been conducted on a fixed horizontal plate barrier consisting of single, twin and multiple plates located at different submerged depths.

Analytic solution of heat flow between two wavy plate fins was presented in [7]. The geometric features of the wavy plate fins are described by a sinusoidal 3 parameters function. In [8-10], the analytical solutions were proposed for various engineering examples under creep conditions.

2. Materials and methods

The paper considers a rectangular wavy profile plate loaded with a distributed load perpendicular to the median plane (Fig. 1). The waveform of the plate has the periodic function $z = f \cdot \sin \frac{\pi x}{l}$.

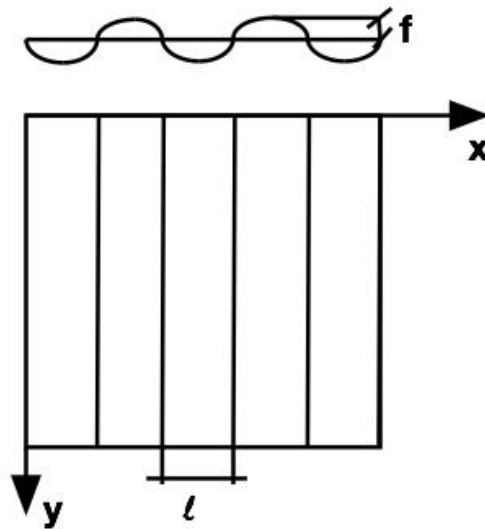


Figure 1. The scheme of the wavy plate

A flat plate is considered to be structurally orthotropic with various cylindrical bending rigidity in the direction of the generatrix and the guiding wave. The differential equation of the bending of a corrugated plate on an elastic base has the form [11]

$$D_1 \frac{\partial^4 w}{\partial x^4} + 2D_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 w}{\partial y^4} = q(x, y) - k w(x, y). \quad (1)$$

where $D_1 = \frac{l}{s} \cdot \frac{Eh^3}{12(1-\nu^2)}$, $D_2 = 0.5 Eh f^2 \cdot \left[1 - \frac{0.81}{1+2.5 \cdot \left(\frac{f}{2l}\right)^2} \right]$, $D_3 = 2D_k = \frac{s}{l} \cdot \frac{Eh^3}{12(1+\nu)}$,

$q(x, y)$ - external load distributed over the surface of the plate;

E and ν - elastic constants of plate material, h - plate thickness, $s = l \left(1 + \frac{\pi^2 f^2}{4l^2} \right)$ - arc length, f - wave amplitude, k - elastic foundation modulus.

The plate is rigidly constrained along the contour, the deflection $w(x, y)$ and the rotation angles $\frac{\partial w(x, y)}{\partial x}$, $\frac{\partial w(x, y)}{\partial y}$ at the edges of the plate are zero

$$\begin{aligned} w(0, y) = \frac{\partial w(0, y)}{\partial x} = 0; \quad w(a, y) = \frac{\partial w(a, y)}{\partial x} = 0; \quad w(x, 0) = \frac{\partial w(x, 0)}{\partial y} = 0; \\ w(x, b) = \frac{\partial w(x, b)}{\partial y} = 0; \end{aligned} \quad (2)$$

3. Results

We transform to the dimensionless variables ξ, η by the following substitution

$$x = \frac{a}{2}(\xi + 1); \quad y = \frac{b}{2}(\eta + 1) \frac{y}{b}; \quad -1 \leq \xi \leq 1; \quad -1 \leq \eta \leq 1; \quad \lambda = \frac{a}{b}.$$

As a result, we obtain (1) in the following form

$$D_1 \frac{\partial^4 w(\xi, \eta)}{\partial \xi^4} + 2D_3 \lambda^2 \frac{\partial^4 w(\xi, \eta)}{\partial \xi^2 \partial \eta^2} + D_2 \lambda^4 \frac{\partial^4 w(\xi, \eta)}{\partial \eta^4} = \left(\frac{a}{2}\right)^4 [q(\xi, \eta) - kw(\xi, \eta)] \quad (3)$$

The decision is made in the form of a double row

$$w(\xi, \eta) = \sum_{m=1}^M \sum_{n=1}^N A_{mn} Q_{m+3}(\xi) Q_{n+3}(\eta), \quad (4)$$

where A_{mn} - unknown coefficients, $Q_i(\xi)$ - special orthonormal polynomials of degree i , which can be represented as a combination of three classical Legendre polynomials [11], satisfy the boundary conditions

$$Q_i(\pm 1) = \frac{\partial Q_i(\pm 1)}{\partial \beta} = 0, \quad \beta = \xi, \eta$$

and the normalization conditions

$$\int_{-1}^1 \frac{\partial^4 Q_i(\xi)}{\partial \xi^4} Q_j(\xi) d\xi = \delta_{ij}, \quad \delta_{ij} - \text{Kronecker symbol.}$$

Using the Bubnov-Galerkin method, we obtain a system of linear algebraic equations with respect to unknowns A_{mn}

$$\begin{aligned} \sum_{m=1}^M \sum_{n=1}^N B_{kl}^{mn} A_{mn} &= q_{kl}, \\ B_{kl}^{mn} &= D_1 \int_{-1}^1 \frac{\partial^4 Q_{m+3}(\xi)}{\partial \xi^4} Q_{k+3}(\xi) d\xi \int_{-1}^1 Q_{n+3}(\eta) Q_{l+3}(\eta) d\eta \\ &\quad + 2D_3 \lambda^2 \int_{-1}^1 \frac{\partial^2 Q_{m+3}(\xi)}{\partial \xi^2} Q_{k+3}(\xi) d\xi \int_{-1}^1 \frac{\partial^2 Q_{n+3}(\eta)}{\partial \eta^2} Q_{l+3}(\eta) d\eta \\ &\quad + D_2 \lambda^4 \int_{-1}^1 Q_{m+3}(\xi) Q_{k+3}(\xi) d\xi \int_{-1}^1 \frac{\partial^4 Q_{n+3}(\eta)}{\partial \eta^4} Q_{l+3}(\eta) d\eta \\ &\quad + k \left(\frac{a}{2}\right)^4 \int_{-1}^1 Q_{m+3}(\xi) Q_{k+3}(\xi) d\xi \int_{-1}^1 Q_{n+3}(\eta) Q_{l+3}(\eta) d\eta \\ q_{kl} &= \left(\frac{a}{2}\right)^4 \int_{-1}^1 Q_{k+3}(\xi) d\xi \int_{-1}^1 q(\xi, \eta) Q_{l+3}(\eta) d\eta; \quad k, l = 1, 2, 3 \dots \dots \end{aligned}$$

The bending moments arising in the plate are determined by the formulas

$$\begin{aligned} M_x(\xi, \eta) &= -4 \frac{D_1}{a^2} \left(\frac{\partial^2 w(\xi, \eta)}{\partial \xi^2} + \nu \lambda^2 \frac{\partial^2 w(\xi, \eta)}{\partial \eta^2} \right); \\ M_y(\xi, \eta) &= -4 \frac{D_2}{a^2} \left(\lambda^2 \frac{\partial^2 w(\xi, \eta)}{\partial \eta^2} + \lambda^2 \frac{\partial^2 w(\xi, \eta)}{\partial \xi^2} \right). \end{aligned}$$

4. Discussion

A numerical study of the stress-strain state of a wavy plate was carried out for the following data: the size of the plate on the x -axis $a=2$ m, the size of the plate on y -axis $b=1$ m, $q=100$ kN/m², $E=2.0 \cdot 10^5$ MPa, $h=0.015$ m, $\nu=0.3$

From the spatial pattern of the distribution $M_x(x, y)$, $M_y(x, y)$, $w(x, y)$ in the plate, it is seen that the maximum bending moments arise in pinching, and the maximum deflection in the center of the plate figure 1,2,3.

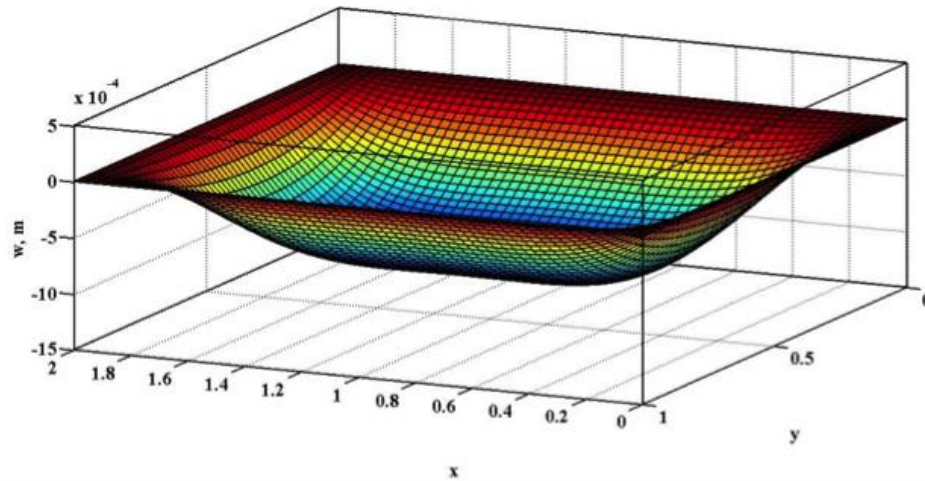


Figure 2. Dependence of deflections $w(x, y)$ of a wavy plate

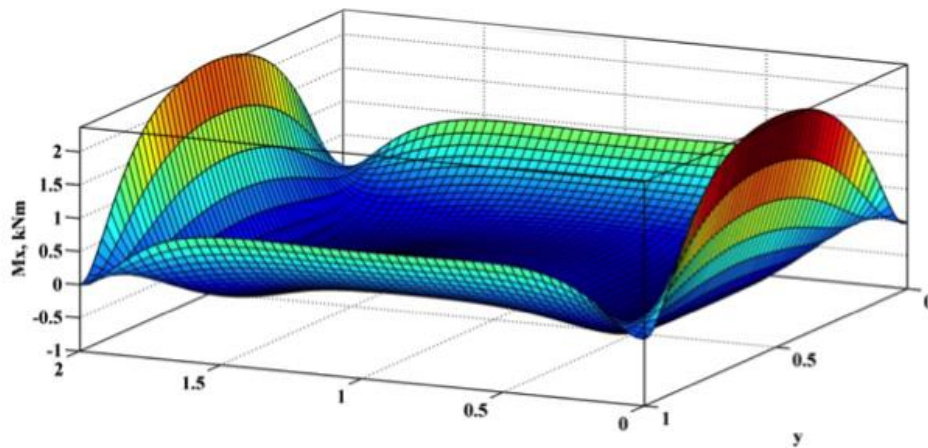


Figure 3. Dependence of bending moments $M_x(x, y)$ of wavy plate

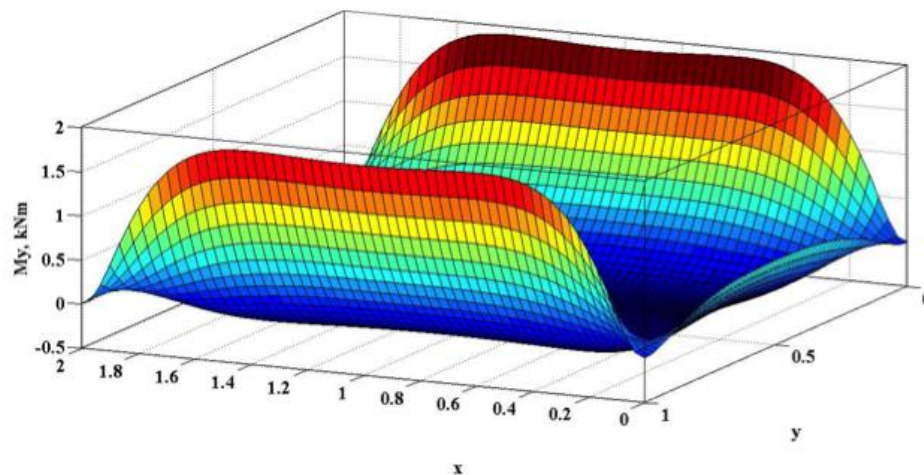


Figure 4. Dependence of bending moments $M_y(x, y)$ of wavy plate

A numerical study of stress-strain state for a reinforced concrete I-beam of fiber foam concrete was carried out. The elastic moduli for tension and compression were obtained experimentally [12-14].

Table 1. The results of calculating the plate for $k=100$ MPa/m

$k=100$ MPa/m (Sandy-clay soil); $N=14$; $M=14$							
$f, \text{ cm}$	$l, \text{ cm}$	$y = b/2, 0 \leq x \leq a$			$x = a/2, 0 \leq y \leq b$		
		$w_{max}, \text{ cm}$	$M_{xmax}, \text{ kNm}$	$M_{ymax}, \text{ kNm}$	$w_{max}, \text{ cm}$	$M_{xmax}, \text{ kNm}$	$M_{ymax}, \text{ kNm}$
1	5	0.108	2.53	0.424	0.108	0.976	1.819
1.25		0.101	2.43	0.702	0.101	0.738	2.37
1.5		0.0925	2.28	1.06	0.0918	0.573	2.97
1.8		0.0808	2.07	1.62	0.0800	0.427	3.70
2		0.0728	1.93	2.06	0.072	0.351	4.17

Table 1 shows that the increase in the wave amplitude doubles the rigidity of the wavy plate by a factor of 1.5, reduces by 30% the bending moment $M_x(x, y)$ and increases the bending moment $M_y(x, y)$ by three times.

Table 2. The results of calculating the plate for $k=500$ MPa/m

$k=100$ MPa/m (limestone); $N=14$; $M=14$							
$l, \text{ cm}$	$f, \text{ cm}$	$y = b/2, 0 \leq x \leq a$			$x = a/2, 0 \leq y \leq b$		
		$w_{max}, \text{ cm}$	$M_{xmax}, \text{ kNm}$	$M_{ymax}, \text{ kNm}$	$w_{max}, \text{ cm}$	$M_{xmax}, \text{ kNm}$	$M_{ymax}, \text{ kNm}$

4		0.1073	2.46	0.456	0.1069	0.907	1.866
5		0.1079	2.53	0.424	0.1076	0.976	1.819
6	1	0.1083	2.56	0.406	0.1079	1.017	1.792
7		0.1085	2.58	0.395	0.1081	1.044	1.775
8		0.1086	2.60	0.389	0.1083	1.062	1.76

Table 2 shows that the change in the wavelength of the sinusoidal plate practically does not affect the rigidity of the plate, it gives a slight increase in $M_x(x, y)$ and a decrease in $M_y(x, y)$

Table 3. Dependence of maximum deflections $w(x, y)$, bending moments $M_x(x, y)$ and $M_y(x, y)$ on elastic foundation modulus k

$k, \text{MPa/m}$	l, cm	f, cm	$y = b/2, 0 \leq x \leq a$			$x = a/2, 0 \leq y \leq b$		
			w_{max}, cm	M_{xmax}, kNm	M_{ymax}, kNm	w_{max}, cm	M_{xmax}, kNm	M_{ymax}, kNm
100			0.1079	2.53	0.424	0.1076	0.976	1.819
200			0.0557	1.771	0.297	0.0545	0.672	1.253
400	5	1	0.0275	1.223	0.205	0.0266	0.474	0.884
600			0.0180	0.985	0.165	0.0174	0.388	0.723
800			0.0134	0.846	0.142	0.0130	0.336	0.627

Table 3 shows the elastic foundation modulus k of the base strongly affects the stress-strain state of the wavy plate. So the increase in the elastic modulus of the base reduces the rigidity of the wavy plate by eight times, the bending moments $M_x(x, y)$, $M_y(x, y)$ almost three times less.

The obtained results indicate the need to investigate the stress-strain state of a wave plate on an elastic base not only on the basis of the value and type of the applied bending load and plate thickness but also to take into account the influence of the wave dimensions and the bedding coefficient of the elastic base. The proposed method makes it possible to investigate the stress-strain state of a wavy plate under the action of any flexural distributed loads, for the arbitrary support of a wave plate on an elastic and without an elastic base and for any dimensions of the wafer wave.

Consider a plate of waved steel sheet (figure 5). The stress-strain state in the case of roof coverage without a bumper was conducted, i.e. the plate lies on the elastic base, rigidly pinched or hingedly sprawled.

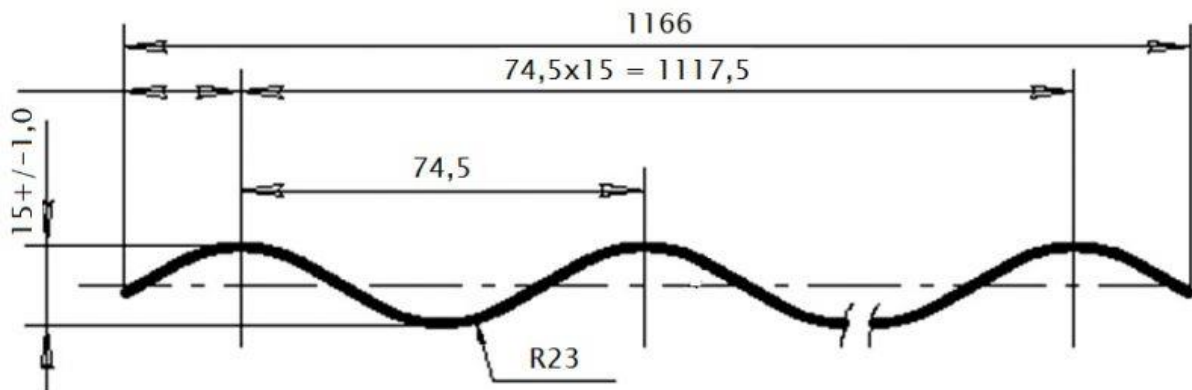


Figure 5. The scheme of waved steel plate

Table 4. Dependence of maximum deflections $w(x,y)$, bending moments $M_x(x,y)$ and $M_y(x,y)$ on N and M

$k=100$ MPa/m (Sandy-clay soil);									
N	M	l , cm	f , cm	$y = b/2, 0 \leq x \leq a$			$x = a/2, 0 \leq y \leq b$		
				w_{max} , cm	M_{xmax} , kNm	M_{ymax} , kNm	w_{max} , cm	M_{xmax} , kNm	M_{ymax} , kNm
8	8	5	1	0.1079	2.5258	0.4238	0.1076	0.9763	1.8199
10	10			0.1079	2.5257	0.4238	0.1076	0.9758	1.8191
12	12			0.1079	2.5257	0.4238	0.1076	0.9757	1.8180
14	14			0.1079	2.5257	0.4238	0.1076	0.9756	1.8186

5. Conclusions

An analytical solution for calculating the stress-strain state of corrugated plates lying on the elastic Winkler base is proposed. Examples from building practice show the influence of various characteristics of corrugated plates on their stress-strain state. The impact of Winkler's foundation characteristics is investigated for various examples.

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