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About the damage to building materials and structures

Olga Vasil'eva

Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

E-mail: vasiljevaoa@mgsu.ru

Abstract. We consider the damage to building materials and structures as the initial stage of dynamic process of its destruction. For the mathematical description of the initial stage of material destruction we apply recently proposed mathematical model of a non-equilibrium phase transitions formulated in terms of the Cahn-Hillard theory of spinodal decomposition. The representation of the damage process as a non-equilibrium phase transitions is carried out on the principles of thermodynamics and taking into account the specific geometric forms of the investigated construction. This allows to describe the damage process in details. So, the changes in building material under load due to the mechanisms of creep, plasticity, ductile or brittle fracture are considered as a non-equilibrium phase transitions. A mathematical model is a one-dimensional spatial variables initial-boundary problem for a system of four differential equations of second and fourth orders. Because of the nonlinearity of the system its analytical solution is difficult. So for it solution, we apply numerical methods. For the numerical solution of the problem a finite-difference scheme of second order accuracy is used. The numerical results confirm the conclusion that the mathematical model describes the main features of the process. Numerical study of one-dimensional models allows to go to the two-dimensional case and compared obtained results with experimental data

1. Introduction

We consider the damage to building materials and structures as the initial stage of dynamic process of its destruction. For the mathematical description of the initial stage of material destruction we apply recently proposed [1] mathematical model of a non-equilibrium phase transitions formulated in terms of the Cahn-Hillard theory of spinodal decomposition [2-9].

Attempts to consider the process of destruction as a non-equilibrium phase transition have a long history (see, for example, [10]). In [1] mathematical model of initial stage of process of destruction of the structural material is proposed, its justification conducted The representation of the damage process as a non-equilibrium phase transitions is carried out on the principles of thermodynamics and taking into account the specific geometric forms of the investigated construction [1]. This allows to describe the damage process and to agree on two methodologically different approaches represent the process of destruction of the structural material [11]. So, the changes in building material under load due to the mechanisms of creep, plasticity, ductile or brittle fracture are considered as a non-equilibrium phase transitions.

This mathematical model is a system of four nonlinear partial differential equations of the fourth order [1]. This is a system of equations for functions of the temperature, the velocity, the enthalpy and

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the destruction parameter. We consider the case of the one dimensional in space variables initial-boundary problem. For the convenience of results interpretation we consider the system for dimensionless variables and model parameters. The mathematical model is a nonlinear problem, so, for its investigation we apply numerical methods. For numerical study of the boundary-value problem we propose finite-difference scheme of second order.

We study the excitation of a homogeneous one dimensional construction material, caused by temperature changes on its left border. For different values of the parameters the numerical results of temporal cross-sections of all dimensionless variables are presented. A comparison of numerical results with experimental data and results obtained using other approaches is given.

2. Formulation of the problem

We consider the mathematical model of excitation of homogeneous one dimensional construction material, caused by temperature change on its left border. The mathematical model is a nonlinear initial - boundary problem formulated for dimensionless variables [1]

$$
\begin{gathered}
\frac{d}{dt}T - \frac{1}{3l_0\alpha_l} l(T)\frac{\partial U}{\partial x} = \varepsilon\frac{\partial^2 T}{\partial x^2} \\
\frac{d}{dt}U + \frac{1}{\rho}\frac{\partial P}{\partial x} = \varepsilon\frac{\partial^2 U}{\partial x^2}, \\
\frac{d}{dt}S + \frac{1}{\rho}\nu T^{\frac{\gamma}{\gamma-1}} e^{-kS}\frac{\partial U}{\partial x} = \varepsilon\frac{\partial^2 S}{\partial x^2}, \qquad (1) \\
\frac{d}{dt}\xi + \frac{1}{2\xi T\rho S}\left(\xi^2\left(\frac{1}{3l_0\alpha_l} l(T)S\rho + \nu T^{\frac{2\gamma-1}{\gamma-1}} e^{-kS}\right) + P + S\rho\frac{1}{3l_0\alpha_l} l(T) - \nu T^{\frac{2\gamma-1}{\gamma-1}} e^{-kS}\right)\frac{\partial U}{\partial x} = \\
P = (\xi^2 + 2)ST\rho + \frac{p_0}{\rho_0}\rho - \left(\frac{3S}{l_0\alpha_l} - \frac{1}{3}E_0\left(\frac{2\beta}{\alpha_l E_0} - 1\right)\right)\frac{1}{l(T)^2} - \frac{\beta}{3l_0\alpha_l}\frac{1}{l(T)}, \\
\mu = \varepsilon^2\left(TS\partial_\xi(g_D(\xi)) - \frac{\partial^2 \xi}{\partial x^2}\right), \\
t > 0, \qquad 0 < x < 1,
\end{gathered}
$$

with initial and boundary conditions

$$
\begin{aligned}
& T(0,x) = T_0, \quad U(0,x) = U_0, \quad S(0,x) = S_0, \quad \xi(0,x) = \xi_0, \qquad (2) \\
& T(t,0) = T_0 + V_T t, \qquad T_x(t,1) = 0, \\
& U_x(t,0) = 0, \qquad U_x(t,1) = 0, \\
& S_x(t,0) = 0, \qquad S_x(t,1) = 0, \\
& \xi_x(t,0) = 0, \qquad \xi_x(t,0) = 0.
\end{aligned}
$$

Here $T(t,x)$ is the temperature at the point x at time t, $U(t,x)$ – velocity, $S(t,x)$ - enthalpy, $\xi(t,x)$ - parameter destruction,

IOP Conf. Series: Materials Science and Engineering **365** (2018) 042024 doi:10.1088/1757-899X/365/4/042024

$$l(T) = l_0(1+\alpha_l(T-T_0)),\ \rho = \frac{1}{l(T)^3}$$

and l_0, p_0, ρ_0, α_1, E_0 V_T, $\varepsilon = Re^{-1}$ - model parameters.

For numerical investigation of initial-boundary problem (1), (2) the finite-difference scheme of the second order is used.

3. Results of numerical investigation

We consider the problem (1) – (2) for the $l_0 = 1$, $p_0 = 1$, $\rho_0 = 1$, $\alpha_1 = 0.4$, $E_0 = 0.7$, $V_T = 2$, $\varepsilon = 0.015$, $T_0 = 2.5$, $U_0 = 0$, $S_0 = 2$, $\xi_0 = 0.45$ $\gamma = 1.4$. Figures 1- 10 present the numerical solution components of the problem. Fig. 1 - 4 show the destruction parameter $\xi(t_i, x)$, the values of the time variable $t_1 = 0.0005$, $t_2 = 0.001$, $t_3 = 0.002$, $t_4 = 0.003$. On the Fig. 1 - 3 we can see the distribution of oscillations. Fig. 4 demonstrates the interaction of oscillations.

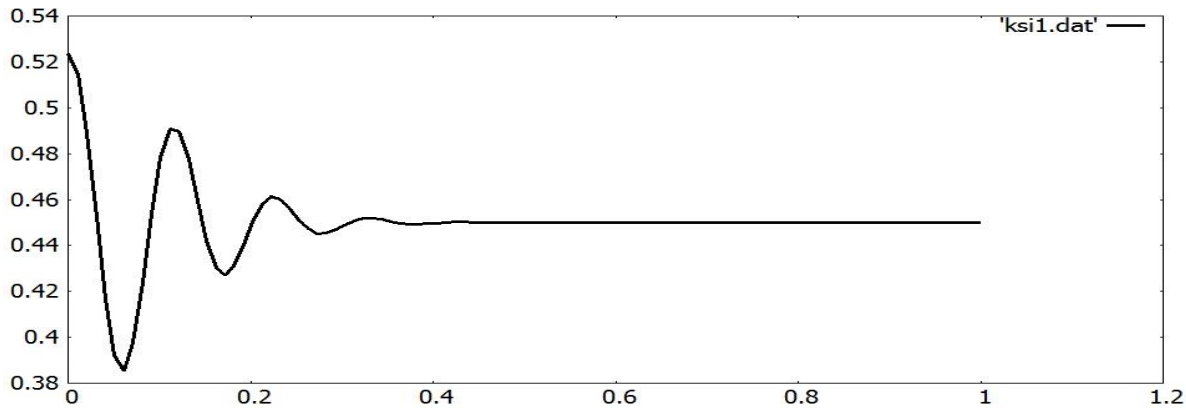

Figure 1. The fourth component of numerical solution of problem (destruction parameter) $\xi(t_1,x)$

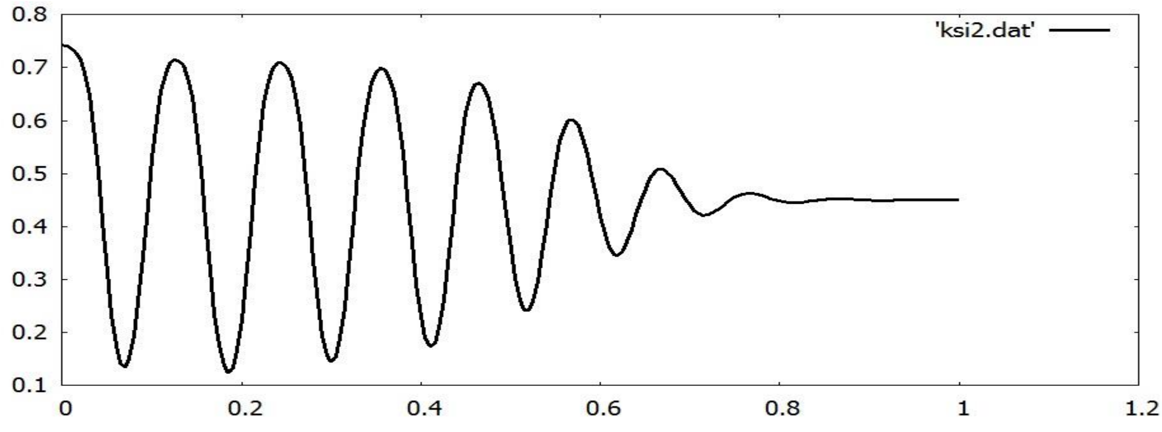

Figure 2. The fourth component of numerical solution of problem (destruction parameter) $\xi(t_2,x)$

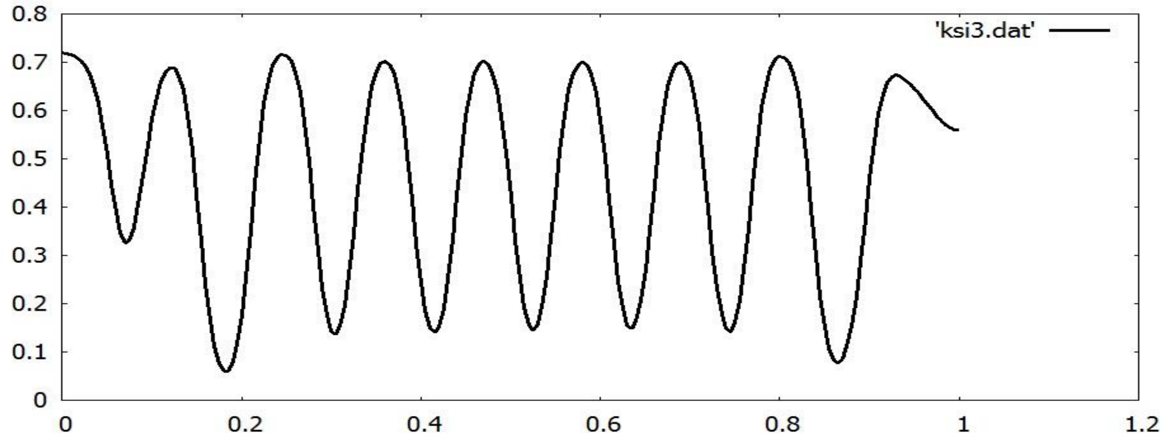

Figure 3. The fourth component of numerical solution of problem (destruction parameter) $\xi(t_3,x)$

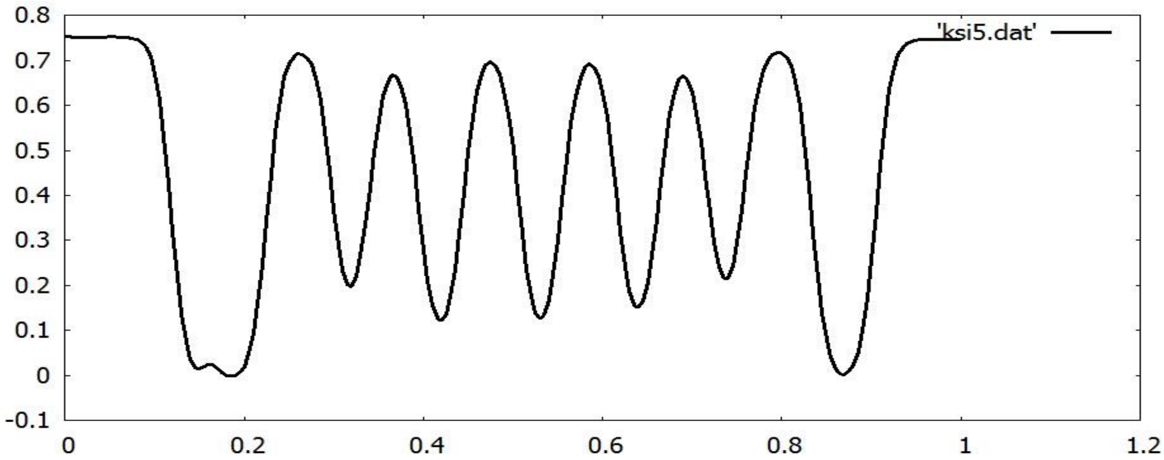

Figure 4. The fourth component of numerical solution of problem (the destruction parameter) $\xi(t_4,x)$

Fig. 5, 6 show the temperature $T(t_i,x)$, the values of the time variable $t_2 = 0.001$and $t_4 = 0.003$.

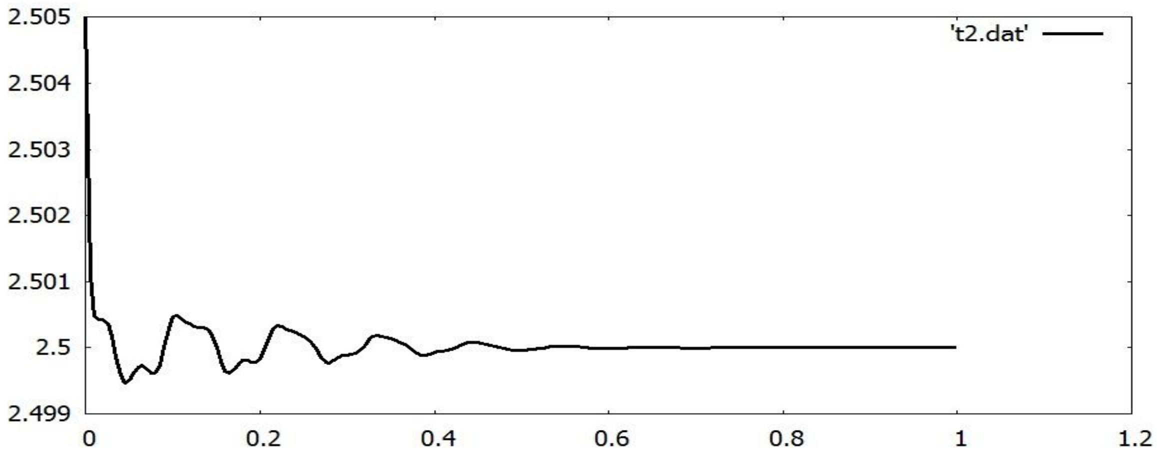

Figure 5. The first component of numerical solution of problem (the temperature) $T(t_2,x)$

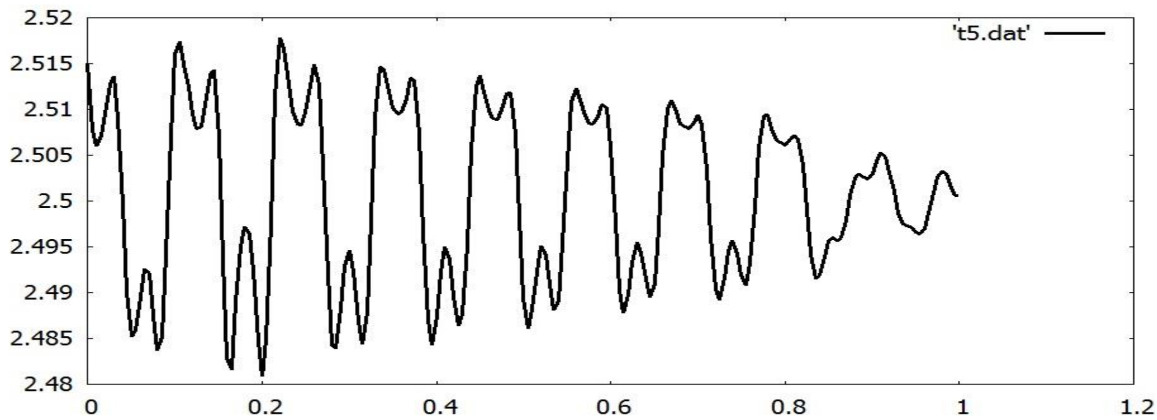

Figure 6. The first component of numerical solution of problem (the temperature) $T(t_4,x)$

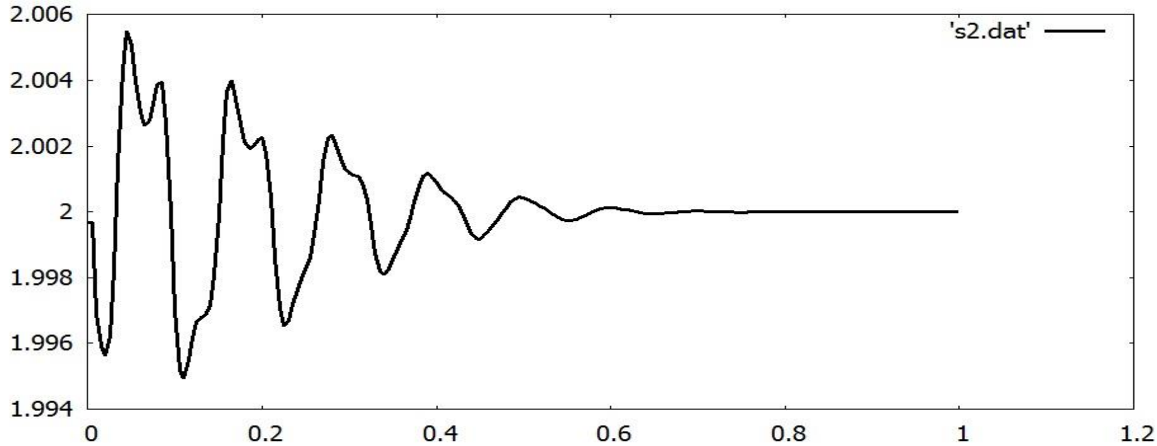

Figure 7. The third component of numerical solution of problem (the enthalpy) $S(t_2,x)$

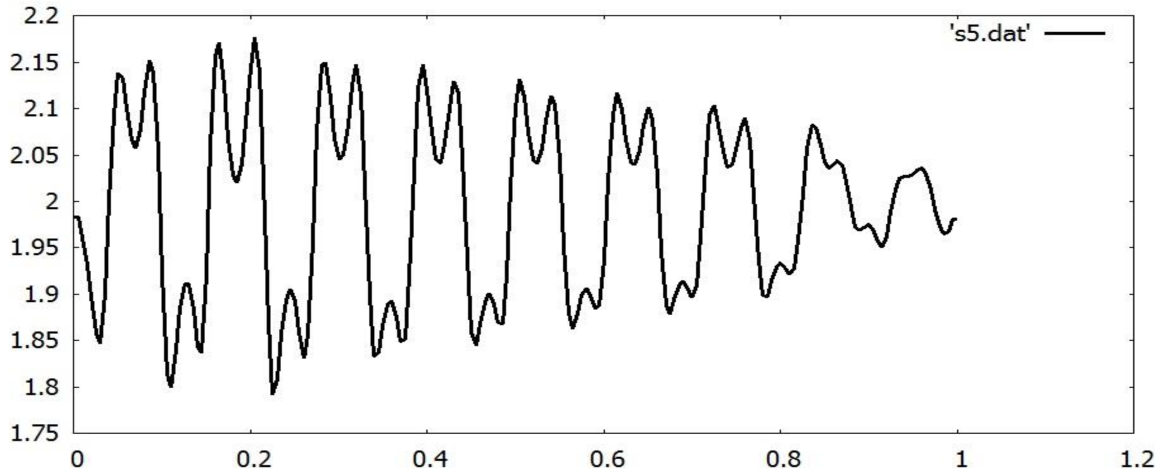

Figure 8. The third component of numerical solution of problem (the enthalpy) $S(t_4,x)$

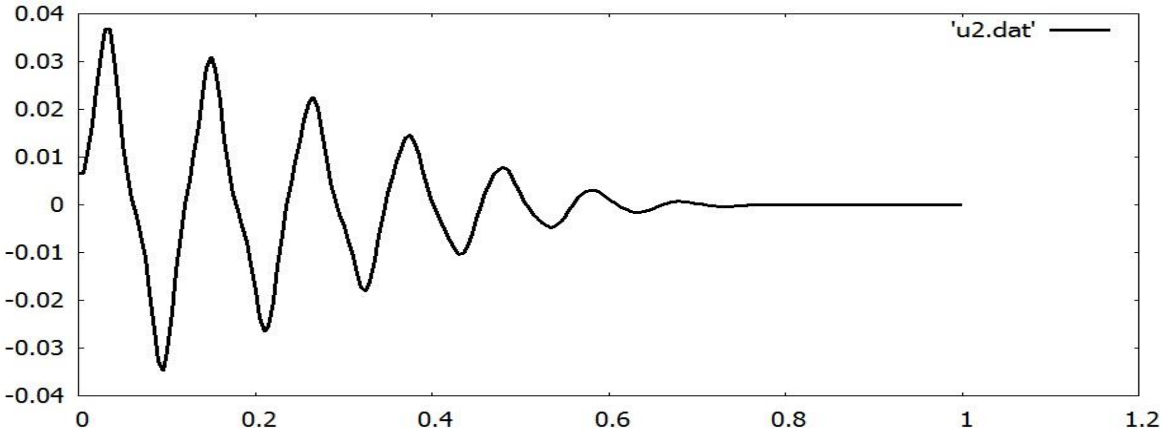

Figure 9. The second component of numerical solution of problem (the velocity) $U(t_2,x)$

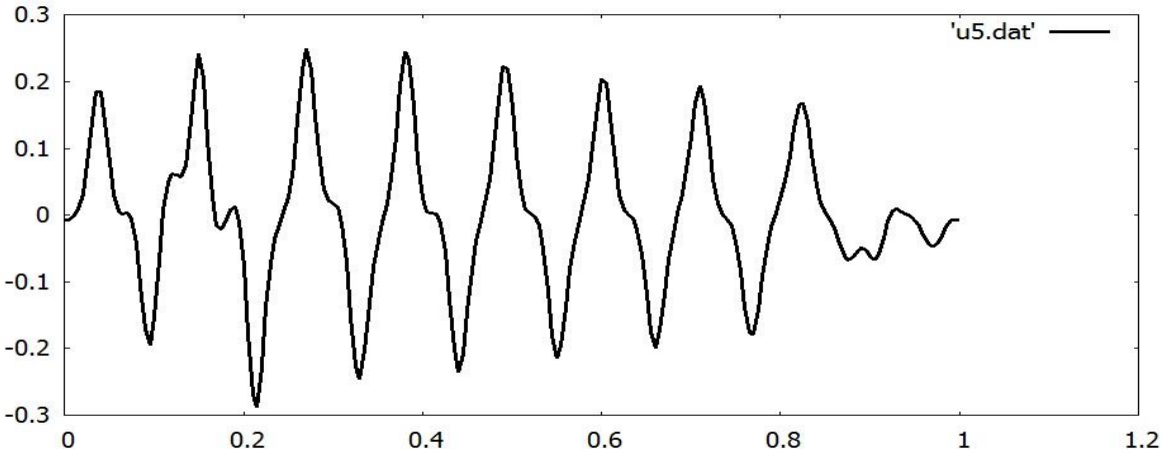

Figure 10. The second component of numerical solution of problem (the velocity) $U(t_4,x)$

Figures 7 – 10 demonstrate the third and second components of numerical solution (the enthalpy and the velocity) $S(t_i,x)$, $U(t_i,x)$, the values of the time variable $t_2 = 0.001$and $t_4 = 0.003$.

4. Conclusion

We present the results of numerical investigation of the initial stage of material destruction. The model is one - dimensional, however, there is a good agreement with available experimental data. The account of material of the building structures heterogeneities will more accurately describe the initial stage of destruction. Numerical study of one-dimensional model allows to go to investigation of the two-dimensional case and compared obtained results with experimental data.

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