Interconnection of maximum deflections and frequencies of free vibrations of composite two-layeroval plates on pliable connections

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Abstract. The paper considers the relationship between the maximum deflections and the frequencies of natural oscillations of oval isotropic composite plates, as well as the dependence of the stiffness coefficient of the seam on the stiffness of the shift connections and the frequency of the natural oscillations of the plate. At the first stage, the system of differential equations of transverse bending of a plate under the action of a uniformly distributed load and the differential equation of free transverse oscillations of the plate is solved. The composite plate at this stage is represented in the form of a solid structure with cylindrical stiffness D_s, which is equivalent to the cylindrical rigidity of the composite plate on the pliable connections. After the transformations, the following regularity is obtained, connecting the maximal deflection of the plate with its frequency of transverse oscillations. Oval two-layer composite plates with a ratio of small to major axis from 1.0 to 0.2 with transverse connections and shift connections of variable stiffness were studied over the entire surface of the plate for rigid pinching and pivoting along the contour. The determination of oscillation frequencies and deflections was carried out by a numerical method. As a result of the research it was established that the oval composite plates, regardless of the support scheme and the stiffness of the shift connections, comply with the regularities (1) with an accuracy of 5%. Based on the results of the study, the graphs of the dependence of the maximum deflection and the natural oscillation frequency of the plate on the stiffness of the shift connections at hinged support and pinching along the contour are constructed. On the second stage of the research, based on A.P. Rzhanitsyn's theory of compound rods, an analytical dependence of the stiffness of the seam on the rigidity of the shift connections was obtained. Based on the results of numerical studies, the dependence of the frequency of natural oscillations on the stiffness coefficient of the seam was obtained. The graphs of the dependence of the frequency of the natural oscillations of the composite plate and the stiffness coefficient of the seam upon the stiffness of the shift connections were constructed.

Key words: composite plate, maximum deflection, frequency of free vibrations, stiffness coefficient of the seam, shear connections.

1. Introduction

A large number of works are devoted to the calculation of solid and composite plates [1, 2, 3, 4, 5, 6, 7, 8, 9]. In [10-13], composite plates of square and circular shapes were studied, depending on the number of symmetrically and uniformly located shear connections, and the stiffness coefficients of the seams were determined as a function of the frequency of free vibrations of composite plates. The authors also studied the stiffness coefficients of the seam for triangular composite plates [14, 15]. In this paper, the stiffness coefficients of seams were investigated as a function of the frequency of free vibrations of oval composite plates with a different number of uniformly and symmetrically located shear connections.

The determination of static and dynamic characteristics reduces to determining the deflections and frequencies of system vibrations in solving the relevant differential equations. The functional connection between the maximum deflection and the frequency of the fundamental tone of free transverse vibrations of elastic isotropic plates was proved by V.I. Korobko [5].

The differential equation of the plate transverse deflection has the form:

$$D\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2 \partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)-q(x,y)=0\,. \tag{1}$$

With the use of biharmonic operators, the equation takes the form:

$$D\nabla^2\nabla^2 W-q(x,y)=0\,, \tag{2}$$

where W = W (x, y) is the deflection function of the plate at the transverse deflection; $\Delta^2\Delta^2$ is a biharmonic operator; $D = EH^3/(12(1-\nu^2))$ is cylindrical stiffness of the plate; q(x, y) is the law of the transverse load change.

The differential equation of plate free vibrations:

$$D\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2 \partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+m\frac{\partial^2 W}{\partial t^2}=0\,; \tag{3}$$

$$D\nabla^2\nabla^2 W+m\frac{\partial^2 W}{\partial t^2}=0. \tag{4}$$

where W = W (x, y, t) is the deflection function of a freely oscillating plate; m is the mass per unit area of the plate; E, ν are respectively the modulus of elasticity of the material and the Poisson's ratio.

If the vibrations are harmonic

$$W = W(x,y)\cos(\omega t)\,, \tag{5}$$

then equation (1) can be transformed to the following form:

$$D\nabla^2\nabla^2 W-m\omega^2 W=0$$

or

$$D\nabla^2\nabla^2 W-\beta^2 W=0,$$

where $\beta^2 = m\omega^2/D$ is the eigenvalue of the differential equation of oscillations of the plates.

Let us represent the deflection function as a product of the maximum deflection W_0 by the unit function f (x, y) and substitute it in the differential equations of transverse deflection and free vibrations of the plates:

$$W(x,y) = W_0 f(x,y)\;;$$

$$\begin{cases} DW_0\nabla^2\nabla^2 f-q(x,y)=0, \\ D\nabla^2\nabla^2 f-\omega^2 mf=0. \end{cases} \tag{6}$$

It should be noted that the precise solution of these differential equations is valid only in the frequent cases of plate forms and boundary conditions, therefore, in practice, approximate methods of solution are mainly used.

If we assume that the plate is under a uniformly distributed load q, then having integrated equations (6) over the entire area of the region, and having performed the necessary transformations, we will get:

$$W_0=\frac{q}{D}\frac{A}{\iint\limits_A \nabla^2\nabla^2 f dA}, \qquad \omega^2=\frac{D}{m}\frac{\iint\limits_A \nabla^2\nabla^2 f dA}{\iint\limits_A f dA}. \tag{7}$$

The deflection function W(x, y) can approximately be put down in a one-parameter form in the polar coordinate system:

$$W(x,y)=W_0 f(x,y)=W_0 g\left[\frac{t}{r(\varphi)}\right]=W_0 g(\rho)\,, \tag{8}$$

where r = r(φ) is the equation of the contour of the plate in the polar coordinate system, t and φ are polar coordinates, ρ = t/r(φ) is the dimensionless polar coordinate.

This function describes a surface which level lines are similar to the region contour and are similarly located. The representation of the function of deflections in this form is justified by the fact that through it we can write down the exact solution to the problem of transverse deflection of a rigidly pinched elliptical plate under the action of a uniformly distributed load. Since just in a single case it is possible to represent the real deflection function in the form of a one-parameter function (8), further results are of an approximate nature.

We transform the integrals in (7), taking into account the deflection function in form (8).

$$\iint_A f dA = \int_0^{2\pi}\int_0^r g(\rho) t dt d\varphi. \tag{9}$$

Multiplying and dividing the right-hand side by r^2, we get after the transformations:

$$\iint_A f dA = 2A\int_0^1 g(\rho)\rho d\rho. \tag{10}$$

Completing the transformation of the integral of the biharmonic operator according to [84], we finally write:

$$I \approx \left(K_f{}^2\Phi_{g1} + K_f\Phi_{g2}\right)/\,A = K_f\left(K_f\Phi_{g1} + \Phi_{g2}\right)/\,A. \tag{11}$$

where

$$\begin{aligned}\Phi_{g1} &= \frac{1}{2}\int_0^1\left(g^{IV}\rho - 12g''' - 21g''\rho^{-1} - 3g'\rho^{-2}\right)d\rho,\\ \Phi_{g2} &= \frac{\pi}{2}\int_0^1\left(2g^{IV}\rho + 14g''' + 22g''\rho^{-1} + 3g'\rho^{-2}\right)d\rho,\end{aligned} \tag{12}$$

The sign of the approximate equality in (11) appeared under the transformation of integrals by means of the Bunyakovsky inequality. We substitute integrals (9) and (11) into expressions (6). After the necessary transformations, we get:

$$\begin{cases} W_0 \approx \dfrac{qA^2}{D}\dfrac{1}{K_f{}^2\Phi_{g1} + K_f\Phi_{g2}}, \\ \omega^2 \approx \dfrac{D}{2A^2 m}\dfrac{K_f{}^2\Phi_{g1} + K_f\Phi_{g2}}{\int_0^1 g\rho d\rho} \end{cases} \tag{13}$$

Since all the values of the definite integrals occurring in the expressions (13) are constant numbers depending on the accuracy of the choice of function g (ρ), they can be represented as the proportionality coefficients K_w, K_ω and B. Then

$$W_0 = K_w\frac{q}{D}\frac{A^2}{K_f{}^2 + BK_f}, \qquad \omega^2 = K_\omega\frac{D}{m}\frac{K_f{}^2 + BK_f}{A^2}, \tag{14}$$

where $$K_w = 1/\Phi_{g1}; \quad K_\omega = \frac{1}{2}\Phi_{g1}\Big/\int_0^1 g\rho d\rho; \quad B = \Phi_{g2}/\Phi_{g1}.$$

Strictly speaking, the signs of approximate equalities should be put in expressions (14), in view of (12) and the approximation of function g (ρ).

Let us multiply the expressions (14) to each other:

$$W_0\omega^2 = K_w K_\omega \frac{q}{m} = K\frac{q}{m}. \tag{15}$$

Taking into account that the coefficients K_w and K_ω depend on the shape of the plate, the following regularity can be obtained from the expression (15): for elastic isotropic plates of identical shapes with homogeneous boundary conditions, the product of the maximum deflection W_0 from the action of the uniformly distributed load q per square of their fundamental frequency of transverse oscillations in the unloaded state, ω^2 with accuracy up to the dimensional factor q/m is a constant. Thus, it is mathematically and rigorously proved that for the whole set of plates with homogeneous boundary conditions the product $W_0{\cdot}\omega^2$ will be represented by a single curve. An important feature of the formulated regularity is the fact that the product $W_0{\cdot}\omega^2$, which is considered in it, does not depend on the flexural rigidity and dimensions of constructions.

Forms of plates can be very diverse – from round to infinitely elongated. It is quite appropriate to expect that the boundary values of the curve $K = W_0{\cdot}\omega^2$ will correspond exactly to these plates.

2. Investigation of the maximum deflections and frequencies of the free transverse vibrations of composite plates

The calculate construction is a round plate consisting of two layers (Figure 1).

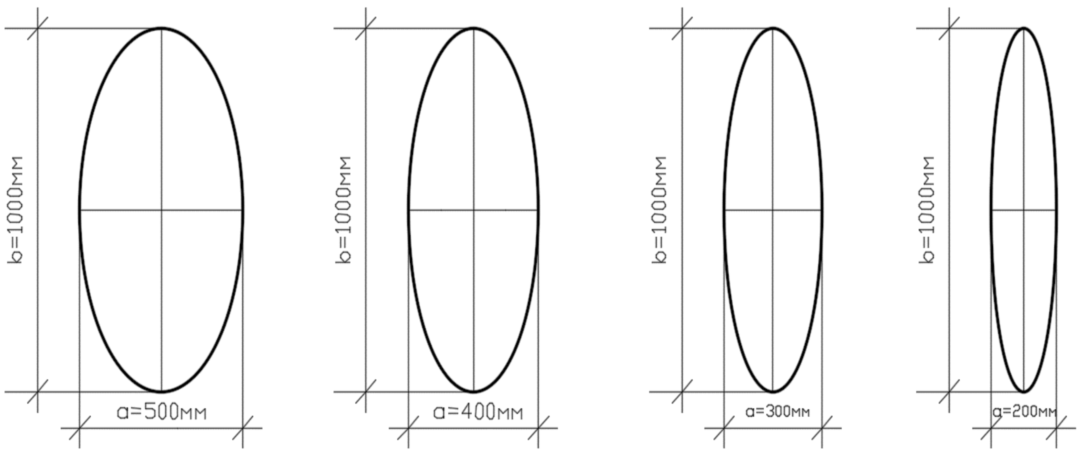

Figure 1. The Scheme of composite oval plates

Numerical studies of composite two-layer plates were carried out by the finite element method. When calculating the plates, two support schemes were investigated: rigid pinching on the contour and hinged support on the contour.

The distance between the layers was taken as the distance between the centers of gravity of the layers. Each layer was divided into 288 finite elements (Figure 2). Wood-chip boards with a volume weight of 740 kG/m^3 are accepted as plates. Modules of elasticity were adopted for wood chipboard E = 2600 MPa (according to the specifications for chipboards), for steel – E_{st} = 206000 MPa. All investigations were carried out under the assumption of elastic work of the material of the layers, vertical connections and shear connections.

A uniformly distributed load was taken q=1 kN / m^2 and was applied to the upper layer of the composite plate. To determine the free frequencies of the transverse vibrations of the plates, concentrated masses were applied to the structural units from the self-weight of the layers in accordance with the load area of the connections. Determination of vibration frequencies and deflections was carried out using the software complex “SCAD” [16]. The rigidity of transverse connections was taken as the appropriate steel dowel pin with a diameter of 2 mm and remained constant during the investigations. The stiffness of the EA_{cc} shear connections for all the plates varied from 100 to 105 kN with a step

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from 1 N to 104 N. The results of numerical studies of the plate are given in Tables 1-2.

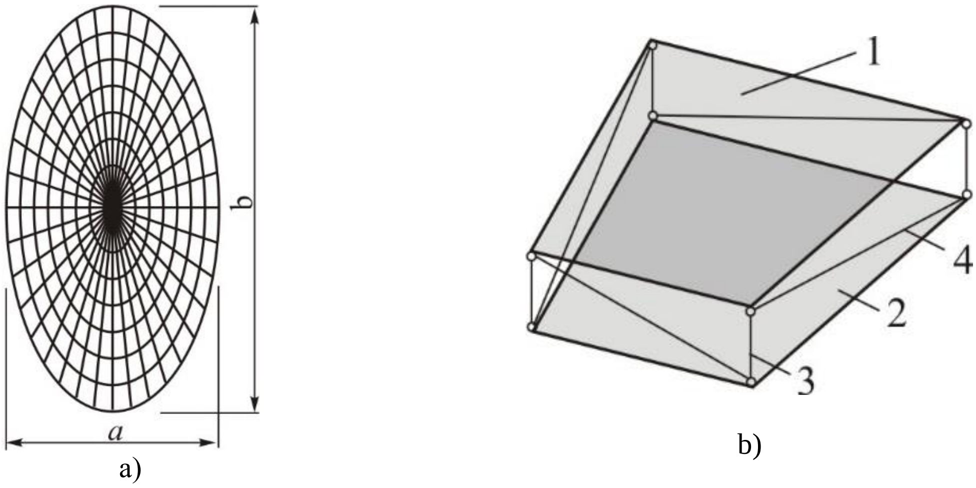

Figure 2. The finite-element calculation scheme of a composite plate (a) and elements of a composite plate with connections (b) (1 – the final element of the upper layer, 2 – the final element of the lower layer, 3 – transverse connections, and 4 – the shear connections)

Table 1. Results of numerical studies of a composite oval plate rigidly pinched on the contour

Plate size *a* (мм)	$lgEA_{cc}$	Circular frequency of the fundamental tone, ω (c^{-1})	Maximum deflection, $W_{o\ мах}$ (мм)	Circular frequency of the fundamental tone, calculated by the analytical method, ω (c-1)	Maximum deflection calculated by the analytical method, Wo мах (мм)	$K=W_o\,\omega^2/(q/m)$	$K=W_o\cdot\omega^2/(q/m)$ based on analytical W_0 и ω	Deviation K from K_{analyt} %
500	0	39.785	150.6985	39.127	150.557	1.624954	1.62486	-0.00576
	1	41.899	143.8618			1.638201		-0.82107
	2	45.389	131.2015			1.590229		2.131343
	3	58.926	95.238			1.572061		3.249425
	4	82,414	68.4372			1.585948		2.394808
	5	98.742	51.2315	97.548	50.657	1.611177		0.842096
400	0	47.021	144.8235	36.04	144.234	1.625039	1.625	-0.00242
	1	50,114	137.9868			1.646427		-1.31859
	2	63.412	125.3265			1.603825		1.30305
	3	67.875	85.1114			1.534732		5.55498
	4	95.411	62.5622			1.634143		-0.56266
	5	103.254	39.3565	93.488	44.957	1.64626		-1.30832
300	0	67.771	129.065	66.522	127.458	1.533186	1.627	5.766049
	1	69.885	122.2283			1.62742		-0.02579
	2	72.375	109.568			1.655604		-1.75806
	3	85.912	65.7659			1.712715		-5.26829

	4	103.4	49.2167			1.70241		-4.63491
	5	121.888	29.011	120.478	19.05	1.570312		3.484234
200	0	76.458	110.578	75.748	109.457	1.602798	1.628	1.548021
	1	78.572	103.7413			1.667748		-2.44152
	2	81.062	91.3468			1.72332		-5.85505
	3	94.599	53.2789			1.694106		-4.06056
	4	112.087	28.7297			1.600325		1.699958
	5	130.575	18.524	129.748	16.248	1.564619		3.893208

Table 2. The results of numerical studies of a composite oval plate hinged on the contour

Plate size *a* (мм)	$\lg EA_{cc}$	Circular frequency of the fundamental tone, ω (c^{-1})	Maximum deflection, $W_{o\ мах}$ (мм)	Circular frequency of the fundamental tone, calculated by the analytical method, ω (c-1)	Maximum deflection calculated by the analytical method, Wo мах (мм)	$K=W_o\,\omega^2/(q/m)$	$K=W_o{\cdot}\omega^2/(q/m)$ based on analytical W_0 и ω	Deviation K from K_{analyt} %
500	0	45.253	87.351	45.224	86.873	1.638076	1.578	-3.66747
	1	45.537	86.073			1.634433		-3.45277
	2	50.039	66.449			1.523621		3.56903
	3	64.949	40.088			1.54857		1.900491
	4	97.542	25.117			1.621876		-2.70529
	5	119.57	18.211	148.984	17.247	1.532364		2.978134
400	0	59.412	73.188	62.474	73.044	1.571213	1.577	0.368296
	1	62.696	71.91			1.557859		1.228703
	2	70.159	59.686			1.619192		-2.60574
	3	88.364	34.847			1.499599		5.161434
	4	115.479	22.595			1.660647		-5.03704
	5	122.507	15.573	167.405	14.347	1.63301		-3.42988
300	0	68.808	40.371	68.776	40.222	1.656309	1.575	-4.90907
	1	69.092	39.093			1.617146		-2.6062
	2	72.637	35.87			1.639991		-3.9629
	3	99.24	30.24			1.645375		-4.27716
	4	120.41	20.1			1.519775		3.633778
	5	141.78	13.87	114.897	13.765	1.611159		-2.24427
200	0	85.36	33.47	75.587	33.57	1.656775	1.5783	-4.9358
	1	90.257	27.009			1.495289		5.330837
	2	93.24	23.14			1.526128		3.202349
	3	118.111	17.44			1.523691		3.367447
	4	137.517	13.96			1.52832		3.054332

	5	158.368	4.248	122.968	4.157	1.512163		4.15544

3. Investigation of the stiffness coefficient of an oval composite plate

In the framework of the study, the problem of studying the stiffness coefficient of the seam ξ was also solved, depending on the stiffness of the EA_{cc} shear connections, which varies from 10-6 to 109 kN. The stiffness coefficient of the seam ξ is determined by the results of dynamic tests of composite plates, which greatly simplifies the evaluation of the rigidity of the structure in experimental studies.

An oval plate with dimensions b = 1000 mm, a = 500 mm was taken as a calculation construction. Two conditions for supporting the plates along the contour were considered: a hinged and rigid pinching. Supports along the contour of the plate were located at the connections of the finite elements of the layers, and their boundary conditions were the same.

The plates are connected by transverse connections in order to avoid removal or convergence of the layers with respect to each other, their rigidity is $EA_{cc} = 8^3$ kN, which corresponds to the rigidity of a steel dowel pin with a diameter d = 2mm. The distance between the layers was assumed to be 10 mm, which corresponds to the thickness of the layers. A chipboard was taken as a layer.

The calculation was carried out in the software package SCAD. As a result of the calculation, the fundamental frequency of the transverse vibrations and the value of the distributed moments were determined:

$$\sum_{i=1}^{n+1} M_x^i = M_x ; \sum_{i=1}^{n+1} M_y^i = M_y ; \sum_{i=1}^{n+1} M_{xy}^i = M_{xy} ; \sum_{i=1}^{n=1} q_i = q \tag{16}$$

$$M = \frac{M_x + M_y}{1 + \mu_{conv}}, \tag{17}$$

when $\mu_I = \mu$, $\mu_{conv} = \mu$.

Let us consider a particular case of a composite plate of two working layers. For this, we set the number of seams n = 1 for equations (16) and (17). We have the system of equations:

$$\begin{cases} \dfrac{\nabla^2 T}{\xi} = \delta \cdot T + \dfrac{N_1}{E_1^{nk} \cdot h_1} - \dfrac{N_2}{E_2^{nK}} - \dfrac{c \cdot M}{D_0}; \\ D_0 \nabla^2 W = -M + c \cdot T \end{cases} \tag{18}$$

where D_0 is the actual cylindrical stiffness equal to

$$D_0 = n \sum_{i=1} D_i ; D_i = \frac{E_i \cdot h_i^3}{12(1-\mu^2)}, (i = 1{,}2); \quad \delta = \frac{c^2}{D} + \frac{1}{E_1^* \cdot h_1} + \frac{1}{E_2^* \cdot h_2}; \quad E_i^* = \frac{E_i}{1-\mu_1^2} \tag{19}$$

where, E_i^* is the modulus of elasticity of the layers in the composition of the composite plate, while the indices of the seams are omitted, since the seam in our cases is one.

Eliminating the parameter T from the system of equations (18), we obtain:

$$\nabla^2 \nabla^2 W - \xi \cdot \delta \cdot \nabla^2 W = -\frac{\nabla^2 M}{D_0} + \xi \cdot M \cdot \delta \frac{\delta \cdot D_0 - c^2}{\delta \cdot D_0^2}, \tag{20}$$

And knowing that

$$\nabla^2 M = -q, \nabla^2 \nabla^2 W = -\frac{\nabla^2 M}{D_{conv}} = \frac{q}{D_{conv}}, \nabla^2 w = -\frac{M}{D_{conv}}. \tag{21}$$

where D_{conv} is cylindrical rigidity of some conventional continuous plate.

We get:

$$\frac{q}{D_{conv}} + \xi \cdot \delta \frac{M}{D_{conv}} = \frac{q}{D_0} + \xi \cdot \delta \cdot M \frac{\delta \cdot D_0 - c^2}{\delta \cdot D_0^2}. \tag{22}$$

For the plate:

$$D_M = \frac{\delta \cdot D_0^2}{(\delta \cdot D_0 - c^2)}. \tag{23}$$

On the basis of this identity, the equation can be written:

$$\frac{q}{D_{усл}} + \xi \cdot \delta \frac{M}{D_{усл}} = \frac{q}{D_0} + \xi \cdot \delta \frac{M}{D_M}. \tag{24}$$

We express the stiffness coefficient of the shear connection from this identity:

$$\xi = \frac{q \cdot (\frac{1}{D_0} - \frac{1}{D_{conv}})}{q \cdot M (\frac{1}{D_{conv}} - \frac{1}{D_M})}. \tag{25}$$

where D_0 is the actual cylindrical rigidity, D_{conv} is the cylindrical stiffness of a conventional solid plate, D_m is the cylindrical rigidity of the monolithic plate with the longitudinal modulus of elasticity in the seam area, M is the maximum moment.

The results of the calculation of a rigidly pinched and hinged plate are given in Tables 3 and 4, respectively. According to the data presented in these tables, the graphs of dependence on the stiffness coefficient of the seam, on the stiffness of shear connections were constructed (Figure 3).

Table 3. Numerical studies of a plate rigidly pinched along the contour with a change in the stiffness of the shear connections

№ №	$lgEA_{cc}$	Circular frequency of the fundamental tone, ω (c^{-1})	Distributed moment, M_x (M_y) (N×м/м))	Maximum moment, according to formula (15), M (N×м/м)	Stiffness coefficient of the seam, $\xi \times 10^6$ (N/$м^3$)
1	-6	189.8418	34.30233	22.67527	115.3556
2	-5	189.8418	34.30233	22.67527	115.3556
3	0	189.8534	34.30233	22.67527	115.3556
4	1	189.958	34.28271	22.6451	115.5139
5	2	190.9941	34.09639	22.35844	124.0458
6	3	210.5056	32.60578	20.0652	131.4197
7	4	252.9856	27.99665	12.97423	229.1314
8	5	325.6355	26.04513	9.971889	356.9954
9	6	347.5162	25.80977	9.609798	384.0339

Table 4. Numerical studies of a plate hinged on the contour with a change in the stiffness of the shear connections

№ №	$lgEA_{cc}$	Circular frequency of the fundamental tone, ω (c^{-1})	Distributed moment, M_x (M_y) (N×м/м))	Maximum moment, according to formula (15), M (N×м/м)	Stiffness coefficient of the seam, $\xi \times 10^6$ (N/$м^3$)
1	-6	121.4964	48.69819	42.75722	85.2154
2	-5	132.1142	48.69819	42.75722	90.2231
3	0	134.4782	48.68839	42.74213	92.7145
4	1	141.6124	48.60013	42.60635	111.2254

5	2	160.0014	47.73714	41.27868	120.1471
6	3	170.2011	42.04928	32.52813	132.0017
7	4	189.6918	32.28186	17.50132	152.5628
8	5	261.6205	29.47716	13.1864	236.9935
9	6	285.026	29.0947	12.598	257.0889

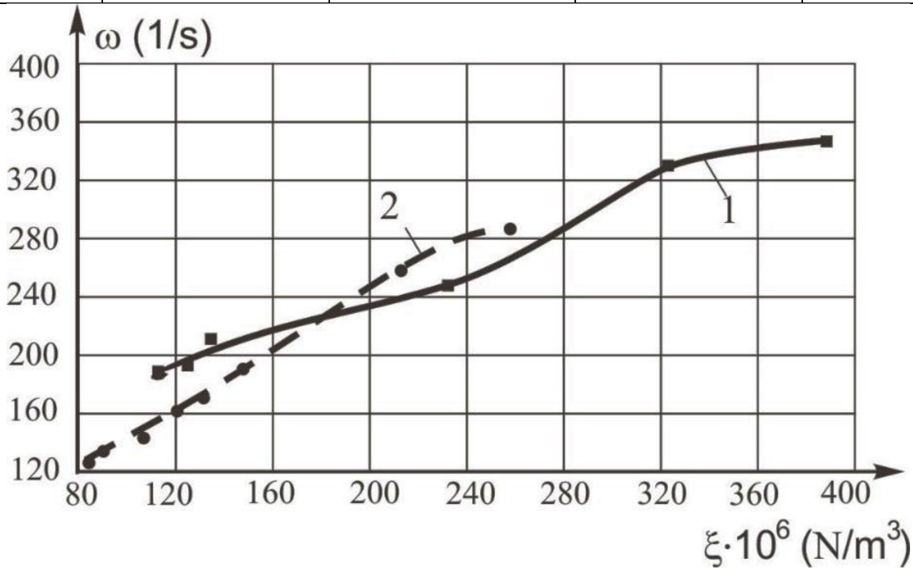

Figure 3. Dependence of the frequency of free vibrations (ω) on the stiffness coefficient of the seam (ξ) of the composite plate: (1 – "ω – ξ " dependence for rigid pinching along the contour; 2 – "ω – ξ " dependence for hinged support along the contour)

As a result of numerical studies, the maximum deflections and fundamental vibration frequencies for oval-shaped plates were determined under different support conditions. The maximum error for hard-clamped plates was 5.76%, and the minimum error was 5.85%; in case of the hinged support, the maximum error was 5.30%, and the minimum error was 5.03%.

From the obtained graphs, it can be concluded that the stiffness of the seam depends on the fundamental vibration frequency of a composite plate.

References

[1] Rzhanitsyn A 1976 Calculation of composite plates with absolutely tight cross couplings. *Researches on the theory of constructions, issue* **XXII** (Moscow: Stroyizdat) pp. 120-133.
[2] Rzhanitsyn A 1986 Composite rods and plates. (Moscow: Stroyizdat), 316 p.
[3] Kalmanok A 1950 Construction mechanics of plates (Moscow: Mashstroyizdat), 303 p.
[4] Shaposhnikov N 1968 Calculation of plates on a bend according to the finite-element method (Works of MITE) **250**, pp. 134 – 144.
[5] Korobko V I 1989 About one "remarkable" regularity in the theory of elastic plates (*The news of higher education institutions. Construction and architecture*) **11** pp. 32-36.
[6] Korobko A V, Chernyaev A A and Shlyakhov S V 2016 Application of the technique of the method of interpolation according to the shape coefficient for the calculation of triangular and quadrangular plates using widely known geometric parameters *Construction and Reconstruction* **4** pp. 19-29.
[7] Korobko A V, Chernyaev A A and Shlyakhov S V 2017 The relationship between the problems of transverse bending and free vibrations of elastic plates in the form of polygons circumscribed around a circle, with boundary conditions of rigid pinching and hinged support along the contour *Construction and Reconstruction* **1** pp. 39-49.

[8] Korobko A V and Savin S Y 2011 Bending of rectangular orthotropic plates with homogeneous and combined boundary conditions *Construction and Reconstruction* **5** pp. 29-34.

[9] Korobko A V and Savin S Y 2013 Free oscillations of rectangular orthotropic plates with homogeneous and combined boundary conditions *Construction and Reconstruction* **1** pp. 13-18.

[10] Turkov A V and Marfin K V 2012 Interrelation of maximum deflections and frequencies of natural vibrations of composite square isotropic plates depending on the number of symmetrically located shear connections *Construction and Reconstruction* **2** pp. 44-48.

[11] Turkov A V and Marfin K V 2012 Determination of the stiffness coefficient of the seam of a square compound isotropic plate based on its fundamental frequency *Construction mechanics and calculation of constructions* **5** pp. 38-44.

[12] Turkov A V and Marfin K V 2013 Determination of the stiffness coefficient of the seam of a circular composite isotropic plate at its fundamental frequency *Construction mechanics and calculation of constructions* **4** pp. 58-62.

[13] Turkov A V and Marfin K V 2013. Determination of the stiffness coefficient of the seam of a circular composite isotropic plate at its fundamental vibration frequency. *XVI International scientific and practical conference of students, undergraduates, graduate students and young scientists “Construction – the formation of the environment of life”* (Moscow). Pp. 187-190.

[14] Turkov A V and Karpova E V 2015 The study of a seam stiffness coefficient for a triangular composite isotropic plate depending on its basic frequency of oscillations in case of different rigidness of shift bindings *Construction mechanics and calculation of constructions* **2** pp. 66-69.

[15] Karpova E V 2015 Investigation of the stiffness coefficient of the sea of composite triangular isotropic plates, depending on the number of symmetrically located shear connections. *XVIII International Interuniversity Scientific and Practical Conference* (Moscow, Moscow State University of Civil Engineering). Pp. 238-242.

[16] Semenov A A and Gabitov A I 2005 The SCAD project computer system in educational process (Moscow: ASV publishing house) 152 p.