Research on Signature Verification Method Based on Discrete Fréchet Distance

J L Fang, W Wu

Shen Yang JianZhu University Liaoning, China

E-mail: 1328767406@qq.com

Abstract. This paper proposes a multi-feature signature template based on discrete Fréchet distance, which breaks through the limitation of traditional signature authentication using a single signature feature. It solves the online handwritten signature authentication signature global feature template extraction calculation workload, signature feature selection unreasonable problem. In this experiment, the false recognition rate (FAR) and false rejection rate (FRR) of the statistical signature are calculated and the average equal error rate (AEER) is calculated. The feasibility of the combined template scheme is verified by comparing the average equal error rate of the combination template and the original template.

1.INTRODUCTION

With the rapid development of electronic transactions and the increasing demand for social security and identity authentication, biometrics-based identification technology has been rapidly developing. Among them, hand-written signature authentication is one of the most promising technologies in biometric identification technology. Handwritten signatures have different uniqueness between people and constant stability over a period of time, making signatures difficult to forge and fake. At present, the Hausdorff distance as a measure of distance is widely used to determine the similarity between two point sets, while the continuous Fréchet distance [1] is used to study the continuity of two consecutive curves. Zhang Zhigang[2], Qiu Yiming[3] have done to determine the similarity of the two curves of the study; HOLM et al.[4] applied the continuous Fréchet distance to the similarity of the protein structure arrangement; C. Bargetz.J[5] proposed the definition of discrete Fréchet distance on the basis of continuous Fréchet distance, and Jiang Minghui et al.[6] applied it to determine the structure of protein, and achieved some effect, but discrete Fréchet Distance in the signature certification has not been a comprehensive application; You Qingcheng et al.[7] considered the signature of the location, pressure and grip pen tilt, the use of HMM-SVM hybrid model for signature verification. Faundez-Zanuy[8] proposed a DTW algorithm based on feature victimization for online signature verification. Huang Chengjie et al [9] proposed an improved DTW online signature authentication method, the use of special points on the signature segmentation, and then use the sub-signature for DTW certification.

Based on the discrete Fréchet distance, this paper compares the template features with the signatures to be tested, and uses a reasonable threshold to sign the authentication within the range of tolerance. Finally, a multi-feature combination template is generated to break through the single signature authentication in traditional signature authentication limitation. In this paper, the SVC2004 signature database is used to ensure the reliability of the experimental results.

2.SIGNATURE AUTHENTICATION ALGORITHM

A. Abbreviations and Acronyms. Definition 1 ①Given a polygon chain with n highest points $P = < p_1, p_2, ..., p_n >$, a k-step along P, the highest point of the division P becomes k non-intersecting nonempty subsets $\{P_i\}_{i=1,...,}$,meet $P_i = < p_{n_{i-1}+1}, \ldots, p_{n_i} >$ and $1 = n_0 < n_1 < \ldots < n_k = n$.

②Given two polygon chains A and B, AB combination step consists of k steps $\{A_i\}_{i=1\ldots k}$ along A and k step $\{Bi\}_{i=1\ldots k}$ along B, Satisfy formula $1 \le i \le k$, and there are only one high point in A_i and B_i.

③The cost of a combination of $W = \{(Ai, Bi)\}$ along the chains A and B is:

$$d_F^w(A,B) = \max_i \max_{(a,b)\in A_i \times B_i} dist(a,b) \qquad (1)$$

Where $dist()$ is the Euclidean distance between A and B, The discrete Fréchet distance between chains A and B is:

$$d_F^w(A,B) = \min_W d_F^W(a,b) \qquad (2)$$

This combination is called the Fréchet arrangement of chains A and B. Since the discrete Fréchet distance is only able to examine the distance between the peak points, it is far from enough to judge the similarity of the curve. Therefore, this paper introduces the definition of the new judgment curve similarity.

Definition 2 Let $A = < a_1, \ldots, a_m >$ and $B = \langle b_1, \ldots, b_n \rangle$ be two curves of discrete points, $d_F^1(A,B)$ is the discrete Fr-échet distance between their highest points, and $d_F^2(A,B)$ is the Fréchet distance between their lowest points. Suppose there is ε, if $| d_F^1(A,B) - d_F^2(A,B) | \le \varepsilon$ then called A, B similar, otherwise they are not similar.

As shown in Fig. 1, a peak point and a trough point are taken as an example in which a and m are a peak point and a trough point of curve A, respectively, b and n are a peak point and a trough point of curve B, d1 is the distance between their crests a and b, and $d2$ is the distance between their troughs m and n. If | $d1$-$d2$ | <ε is true for a given ε, then curves A and B are similar, otherwise they are not similar.

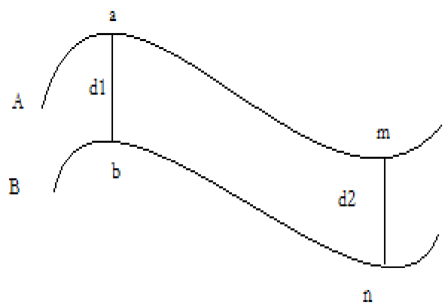


Fig.1 Two curves similar schematic diagram

The same is true for the two curves of the discrete points. When the discrete points are enough, find out all the peaks and valleys, and the number of extremes of the similar curves is not much different, and the Fréchet permutation is always the best arrangement of the arrangement, the arrangement of the calculated discrete Fréchet distance used as a judgment curve similarity can reduce the error.

B. Algorithm steps. ①*A*, *B* is a finite curve composed of two curves. First find the highest point and the lowest point of the curve, E.g. $A=\langle A_1,\ldots,A_m\rangle$, $a=\langle a_1,\ldots,a_n\rangle$, $B=\langle B_1,\ldots,B_m\rangle$ and $b=\langle b_1,\ldots,b_n\rangle$ where m ≤ n (A_1, ..., A_m and a_1, ..., a_n are *m* highest points and *n* lowest points of the chain A, B_1, ..., B_m and b_1, ..., b_n are *m* highest points and *n* lowest points of the chain *B*). If n - $m \geq 5$ (since the total number of signatures is between 200 and 300 points, the total of the high and low points is between 20 and 30, and the high and 20% Similarity, so the number of differences set here is limited to five), then they are not similar, the algorithm is over, otherwise the next step.

②Based on the chain *A* with a small peak point, the chain *B* with the peak point is divided into *m* steps using the definition of Fréchet permutation and the above constraint, and then find all the conditions to meet the division. Suppose there are *k* species ($k \in$ N), each division $W_j=\{(A_{i,}\ B_i)\}$, $1 \leq i \leq m$ and $1 \leq j \leq k$.

③In each division of Wj, first calculate the maximum distance between all the corresponding points in each step, and then find the maximum of the maximum distance in all the steps of this division. As follows:

$$d_F^W(A,B)=\max_i \max_{(a,b)\in A_i\times B_i} dist(a,b) \qquad (3)$$

④The best way to find a division is to find the minimum of the distance in all the division methods, which is the discrete Fréchet distance between chains *A* and *B*.

$$d_F(A,B)=\max_W d_F^W(A,B) \qquad (4)$$

⑤In this way, we can get the minimum discrete Fréchet distance $d_F^1(A,B)$ between the two peaks and the minimum discrete Fréchet distance $d_F^2(A,B)$ between the two troughs. Finally, with $d_F^1(A,B)-d_F^2(A,B)$, if the final result is less than a specific threshold ε, it is judged that the two curves are similar, otherwise they are not similar and the algorithm ends.

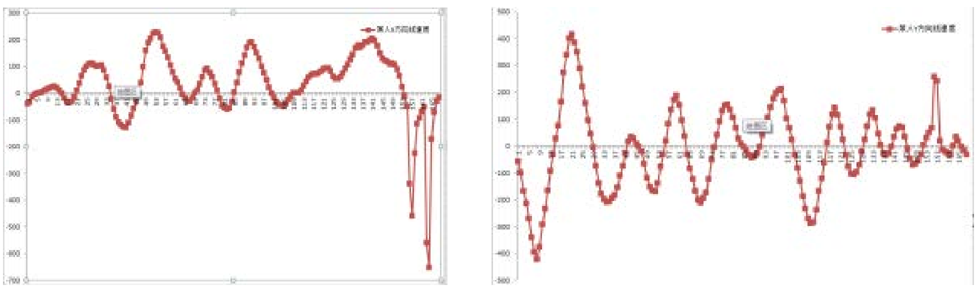
3.Parameter

A. Characteristic curve. Different signature features, parameter acquisition is different, Such as the signature of the horizontal, vertical, pressure signal signature characteristics can be directly through the handwriting device to collect data, and other signature features are calculated by the formula, the calculation formula as shown in Table 1.

TABLE 1 ONLINE SIGNATURE AUTHENTICATION FEATURE SET

Num	Feature	Description
1	X direction displacement	X(i)
2	Y direction displacement	Y(i)
3	Pressure	P(i)
4	XY Absolute displacement	$XY=sqrt(X(i)^2+Y(i)^2)$
5	X linear velocity	Vx(i)=(X(i+1)-X(i-1))/2
6	Y linear velocity	Vy(i)=(Y(i+1)-Y(i-1))/2
7	linear velocity	$V(i)= sqrt(Vx(i)^2+Vy(i)^2)$
8	X Acceleration	aX(i)=(Vx(i+1)-Vx(i-1))/2
9	Y Acceleration	aY(i)=(Vy(i+1)-Vy(i-1))/2
10	Centrifugal acceleration	a(i)=(Vx(i)aY-Vy(i)aX)/V(i)
11	Speed and X axis angle cosine value	cosV(i)=Vx(i)/V(i)

Taking the linear velocity as an example, the point *Vx*(*i*) in the X direction and the point *Vy* (*i*) in the Y direction are connected to the line by curve to obtain the linear velocity curve of *XY*. In combination with definition 1, the maximum value set *A* and the minimum value set *B* of the linear velocity curve are obtained. Calculate the wave troughs of discrete Fréchet distance $d_F^1(A,B)$ and $d_F^2(A,B)$. Figure 2 shows the true signed X-direction linear velocity curve and Y-direction linear velocity curve.



(a) X linear velocity (b) Y linear velocity

Fig.2 True signature X, Y axis direction linear velocity

B. Extreme Point Judgment Threshold Analysis

Use the data in the SVC2004 database Task1 user1, user2, user3, user4, user5 five true signatures, and skilled forged signature test, through experiments to determine their threshold. In the signature sample library to take each person's real signature sample 10, skilled forged signature samples to match the 10, so as to observe the changes in parameters on the rate of error and the impact of the rate of rejection.

The initial matching threshold of the extreme points of the coordinate curve is analyzed by taking the X axis displacement signed by user1 as an example. By experimenting with the signatures of real signatures and others forged in the signature sample library, we define the discrete Ferchet distances $d_F^1(A,B)$ and $d_F^2(A,B)$ of true and false signatures in conjunction with definition 1. Then, in conjunction with definition 2, the value of $d_F^1(A,B)$ - $d_F^2(A,B)$ is obtained and all discrete Ferchet distance differences are obtained as shown in Table 2.

TABLE 2 TRUE AND FALSE SIGNATURE DISCRETE FRÉCHET DISTANCE

User1	1	2	3	…	8	9	10
True	980	84	138	…	302	454	480
False	1306	780	975	…	1959	663	1006

The matching result is observed by setting the gradient value of ε to 250, 370, 490, 610, 730, and 850. So as to get the best threshold X displacement ε = 490, the matching results shown in Figure 3:

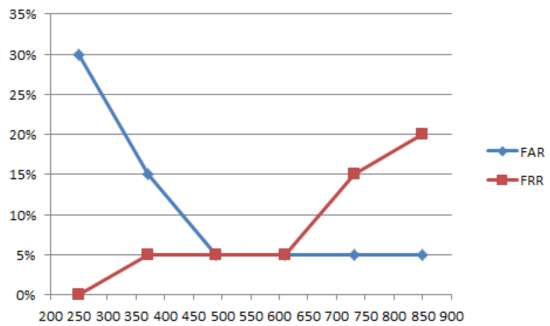


Fig.3 FAR and FRR curves with different values of ε

It can be quickly seen from Figure 3 the optimum threshold values ε of displacement X is 490, the calculated threshold value ε Similarly this experiment, the other characteristic parameters 10, as shown in Table3 (in mm) :

TABLE 3 OTHER PARAMETER THRESHOLDS Ε

Feature	*Y*(*i*)	*P*(*i*)	*XY*(*i*)	*Vx*(*i*)	*Vy*(*i*)
ε	200	600	800	300	300
Feature	*V*(*i*)	*aX*(*i*)	*aY*(*i*)	*a*(*i*)	*cosV*(*i*)
ε	400	200	110	520	1.3

4.SIGNATURE TEMPLATE SELECTION

A. Orthogonal test design. In order to reduce the number of experiments, this paper introduces the experimental scheme of orthogonal experiment design. The essence of orthogonal design is to use the method of orthogonal experiment to arrange the experiment and analyze the data. The survey index of this experiment is the average error rate (AEER) of the signature authentication. There are 11 factors, and the number of levels is 2, which means that the signature data entered by the user is selected twice. From this analysis can be found, this test to meet the orthogonal table L12(112)of the design, the orthogonal table as shown in Table 4, with the number 1 and 2 represent the number of horizontal, A1-A11 on behalf of the test factors, 1-12 On behalf of the experimental serial number.

TABLE 4 SIGNATURE VERIFICATION ORTHOGRAPHIC FORM

	A1	A2	A3	…	A9	A10	A11
1	1	1	1	…	1	1	1
2	1	1	1	…	2	2	2
3	1	1	2	…	2	2	2
4	1	2	1	…	1	1	2
5	1	2	2	…	1	2	1
6	1	2	2	…	2	1	1
7	2	1	2	…	1	2	1
8	2	1	2	…	1	1	2
9	2	1	1	…	2	1	1
10	2	2	2	…	2	1	2
11	2	2	1	…	1	2	2
12	2	2	1	…	2	2	1

As can be seen from Table 4, the orthogonal table gives a combination of the number of experiments and the number of factors, this feature coincides with the signature designation of the signature template and signature number of the combination of design. Therefore, we can use the orthogonal design table design signature template.

B. Signature template generation. According to the orthogonal Table 4, 11 signature parameters are selected according to the order of factors: $X(i)$、$Y(i)$、$P(i)$、$XY(i)$、$Vx(i)$、$Vy(i)$、$V(i)$、$aX(i)$、$aY(i)$、$a(i)$、$cosV(i)$, corresponding to $A1$-$A11$ in Table 4. Select the user two signature information as the experimental level, with the number 1, 2, which can be designed to generate the signature experiment of the 12 templates (m_1, m_2 ... m_{12} corresponding to Table 4, 1-12) program combination table. Where number 1 indicates that all the signature parameters of the user's first signature are selected in the database and number 2 is the characteristic parameter of the second signature of the user in the selected database.

C. *Experimental data and analysis.* For example, take user1's third signature as a template, written with u3.Select 10 real signatures and 10 false signatures to test u3, and the whole experiment is divided into the following.

IOP Conf. Series: Materials Science and Engineering **359** (2018) 012003 doi:10.1088/1757-899X/359/1/012003

The first step, by definition 3 for template and 20 signatures of 11 sets of stable characteristic curve of the minimum set $d_F^1(A,B)$ and maximum set $d_F^2(A,B)$, calculating the discrete Fréchet distance $d_F^1(A,B)$ - $d_F^2(A,B)$, finally get the difference of Fréchet distance set parameters.

The second step, according to the threshold epsilon of each characteristic value in Table 3, statistics FAR and FRR, and obtains the values of FAR and FRR of the u3 template.

The third step, according to the second step results, can be summed up u3 template average FAR value of 7.72% and the average FRR value of 8.18%. (Average FAR and FRR value is all the characteristics of FAR and FRR after the sum of the value).

And so on, you can calculate the average of the remaining twelve FAR templates and average FRR value. Through the above calculation, the average FAR and FRR all template values, the end of the experiment, the original signature template will mean FAR and FRR 12 template obtained respectively and untreated (user with a signature machine the average FAR and FRR) for comparison, ultimately determine the feasibility of combination of the template. In this experiment, orthogonal design experiment using the user first and second signature data, therefore also need to calculate the second signatures as the average FAR and FRR template (U2 table). Through the calculation of second times the average signature as FAR template the value is 12.27%, the average FRR value is 11.82%.

The average error rate (AEER) after averaging FAR and FRR is shown in Table 5 below**:**

TABLE 5 THE TEMPLATE OF THE AEER

template	m1	m2	m3	m4	m5
AEER	6.82%	7.5%	10.23%	8.41%	9.55%
template	m6	m7	m8	m9	m10
AEER	11.36%	12.5%	7.95%	9.55%	11.14%
template	m11	m12	u2	u3	
AEER	10%	8.18%	12.05%	7.95%	

5.Conclusion.

As it can be seen from table 5, the initial template and with second and third overall signatures as signature authentication templates, the average error rate is 12.05% and 7.95%, while the highest in the composite template is 6.82%.According to table 4, you can quickly get the combination template generation method : take the first signature *Y*(i) *XY*(*i*), *aX*(*i*), *aY*(*i*), *a*(*i*)and second signature of the *X*(*i*), *P*(*i*), *Vx*(*i*), *Vy*(*i*), *V*(*i*).Therefore, combination of *m*1 template is the user1 template signature.

And so on, it can be analyzed that the signature authentication template combination of 4 other users. Through the experiment, the signature of the user2 template using the third combinations in table 4, the average error rate of template AEER reached 7.95%, while the original template average error rate of AEER is 9.55%;The signature template of user3 uses eleventh combinations in Table 4, which makes the average error rate of AEER equal to 9.55%, while the average error rate of the original template AEER is 10%;the signature template of user4 uses eighth combinations in Table 4, which makes the average error rate of

AEER equal to 11.14%, while the average error rate of the original template AEER is 12.05%;the signature template of user5 uses fourth combinations in Table 4, which makes the average error rate of AEER equal to 10%, while the average error rate of the original template AEER is 11.14%.

6.SUMMARY

This paper mainly analyzes the characteristics of the Fréchet discrete distance of curves, combined with online signature verification, and presented a signature template combination of multiple features the signature, and gives the analysis of the experimental data and the corresponding experimental data. The acquisitions of signature characteristic curve, signature feature data calculation, extraction of curve extreme value are introduced. The progress of signature verification in the current environment is also pointed out. In the SVC2004 data set, the experimental part of this paper will be combined with multi signature template features traditional template to compare the results found using the traditional signature information the average error rate of AEER compared with the signature composition template average error rate of AEER is higher, indicating signature template combination is proposed to optimize the feasible.

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