IOP Conf. Series: Materials Science and Engineering **357** (2018) 012007 doi:10.1088/1757-899X/357/1/012007

Physical and mathematical modeling of process of frozen ground thawing under hot tank

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Abstract. A description of a new non-stationary thermophysical model in the "hot tank-frozen ground" system is given, taking into account mass transfer of pore moisture. The results of calculated and experimental data are presented, and the position of the thawing front is shown to be in good agreement with the convective heat transfer due to moisture migration in the thawed ground.

1. Introduction

Processes, united by the general influence of the cryogenic factor, are very diverse and complex in nature [1]. Freezing, frost heaving and thawing of moist soil are complex thermodynamic processes in freezing bases. In the process of thawing as the temperature field changes, moisture migration to the freezing front takes place [2].

In this regard, the study of the processes of heat and mass transfer in the ground and their interaction with engineering structures acquire particular urgency. It is important to solve the joint task "environment - construction - ground".

The experimental setup and the results of the experiments on thawing of frozen ground under a hot oil tank are presented in [4, 5].

It was experimentally discovered that when a frozen moist ground is heated from above, a vertical filtration flow of thawed water may occur, and, owing to this, a significant (approximately twofold) increase in the mean velocity of the thawing front movement.

This can be explained by the fact that since the volume of water is less than the volume of ice, pores appear in the thawed ground through which melt water moves down to the center of the melt front, convective heat transfer occurs, which increases the thawing rate [4].

The dependences of the temperature change of the gas space and the hot heat-transfer medium over time have also been obtained [5]. At the beginning of filling an empty tank, the temperature of oil fell as it was heating the walls of a cold metal tank. After reaching the thermal balance of the metal wall and oil, the temperature of the heat-transfer medium reached the value to which it was heated in the oil tank - 57 °C. In the stand-by mode, the temperature of the heat-transfer medium decreased exponentially (according to Shukhov's formula).

A change in the temperature of the gas space was slightly different: in the injection mode, the temperature increased sharply, but when the oil reached the maximum temperature, the temperature of the gas space continued to grow for some time. The increase in the temperature of the gas space is

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evidently due to evaporation of light fractions of oil, as well as a complicated nature of convective gas flows in the gas space inside the tank.

2. Analytical solution

The authors in [4] found an approximate analytical quasi-stationary solution of the two-dimensional Stefan problem in cylindrical coordinates *r*, *x* in the classical formulation, both taking into account the transport of pore moisture and without taking it into account.

The system of equations for the ground thawing problem (the two-dimensional Stefan problem in cylindrical coordinates *r*, *x* in the classical formulation without convection) has the form [4]:

$$\frac{\partial T_{1,2}}{\partial t} = a_{1,2}\left(\frac{\partial^2 T_{1,2}}{\partial r^2} + \frac{1}{r}\frac{\partial T_{1,2}}{\partial r} + \frac{\partial^2 T_{1,2}}{\partial x^2}\right), \tag{1}$$

where subscripts 1 and 2 refer to the thawed and frozen zones, respectively. These zones are separated by a moving surface (the melt front) $F(r,x,t) = 0$. The initial condition is the absence of a thawed zone and the equality of the temperature in the entire region to initial value T_0 (in this case T_0 = -9°C). The boundary conditions on the fixed boundaries have the form:

$$T_{1,2}\left(x=0, r>R\right) = T_0\,,\ -\lambda_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=0, r<R} = q = \frac{W}{\pi R^2}\,,\ \left.\frac{\partial T_{1,2}}{\partial r}\right|_{r=0} = 0\,,\ T_2(r\to\infty, x\to\infty)\to T_0\,,$$

where R - radius of the tank, W - heater wattage (in this model R = 12cm, W = 500W). At the melt front, the temperatures are equal $T_1 = T_2 = 0$°C, and the energy balance condition is given, often called the Stefan condition:

$$\left.\left(\lambda_1\left|gradT_1\right| - \lambda_2\left|gradT_2\right|\right)\right|_{F=0} = L\frac{\partial F/\partial t}{\left|gradF\right|}\,, \tag{2}$$

where $\lambda_{1,2}$ - thermal conductivity coefficients in the thawed and frozen zones, L - volumetric heat of melting (J/m^3).

There is no exact analytical solution to this problem, but approximate quasi-stationary solutions of a number of related problems are known, published in [11, 12]. Following the procedure outlined in these works, let us seek the solution of equation (1) in form $T(F)$, where $F = const$ - isothermal surface equation; in particular, $F = 0$ - zero isotherm equation. Let us calculate function F in the form:

$$F = x - f(r) - \int_0^t g(t)dt\,, \tag{3}$$

where $f(r)$ and $g(t)$ - unknown functions in advance. Differentiating $T(F)$ by *r*, *x*, *t*, one will find:

$$\frac{\partial T}{\partial r} = -T'\cdot f'\,,\ \frac{\partial^2 T}{\partial r^2} = f'^2\cdot T'' - T'\cdot f''\,,\ \frac{\partial^2 T}{\partial x^2} = T''\,,\ \frac{\partial T}{\partial t} = -g\cdot T'\,, \tag{4}$$

where $T' = \frac{dT}{dF}$, $T'' = \frac{d^2T}{dF^2}$, $f' = \frac{df}{dr}$, $f'' = \frac{d^2 f}{dr^2}$. Substituting (4) into (1) and neglecting the difference between a_1 and a_2 (that is, let us assume $a_1 \approx a_2 = a$), let us obtain an ordinary differential equation with respect to function $f(r)$:

$$P\left(1+f'^2\right) - f'' - \frac{1}{r}f' + \frac{g}{a} = 0\,, \tag{5}$$

where

$$P = T''/T'\,. \tag{6}$$

For (5), an analytical solution can be obtained if one assumes that g and P are constants. However, after the temperature field is found, when it is substituted into the Stefan's condition (2), let us assume that g and P are functions, which allows us to find the velocity of the front motion. Physically, this means that the melt front moves so slowly that one can consider the temperature field to differ little from the stationary one (the quasi-stationary approximation). Introducing notations $P = B^2$, $B^2 + g/a = A^2$, $y = ABr$, $f' = -w'(y)/(B^2 w(y))$, let us transform (5) to the form:

$$w'' + \frac{1}{y} w' + w = 0, \tag{7}$$

the solution of which, as we know, is the Bessel function of the first kind of zero order $J_0(y)$. Returning to previous notations, let us obtain a solution for function $f(r)$:

$$f(r) = -\frac{1}{B^2}\left\{\ln\frac{273 + T_1(r)}{273} + \ln\left[J_0\left(ABr\right)\right]\right\}, \tag{8}$$

where $T_1(r)$ – ground surface temperature. Obviously, the solution obtained makes sense if:

$$ABr < 2.4 \tag{9}$$

where 2.4 - the first zero of the Bessel function J_0. Hence, assuming ABR = 2.4, there is:

$$g = a\left(\frac{2.4}{BR}\right)^2 - aB^2. \tag{10}$$

On the other hand, integrating (6), let us find:

$$T' = PT_1 e^{PF}, \quad T = T_1 e^{PF} + T_0. \tag{11}$$

Substituting in (2), one obtains the second relation between g and $P = B^2$:

$$g = \frac{B^2 T_1 |\lambda_1 - \lambda_2|}{L}. \tag{12}$$

Excluding B^2 from (10) and (12), let us obtain the relation between g and T_1:

$$g = \frac{2.4}{R}\sqrt{\frac{a\left(T_1|\lambda_1 - \lambda_2|\right)^2}{L\left(T_1|\lambda_1 - \lambda_2| + aL\right)}}. \tag{13}$$

The experimental dependence of the temperature of the ground surface at point r = 0 on time is shown in Figure 1 (curve 1 approximates the experimental points). Fig. 1 shows calculated curve 2 obtained as a result of numerical integration of function $g(T_1(t))$. As can be seen from the figure, the coincidence with the experimental points (denoted by the symbols +) is observed only for the beginning of warm-up, when the transfer of heat by melt water is negligible.

Let us write the equation of thermal conductivity taking into account the convective heat transfer. Here let us assume that only the convective transfer along the x axis is essential, and the transfer along the r axis is negligible:

$$\frac{\partial T}{\partial t} + v(r,t)\frac{\partial T}{\partial x} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right), \tag{14}$$

where $v(r,t)$ - average projection of the melt water rate on vertical axis x.

As it was mentioned above, it is necessary to seek a solution in form $T(F)$ and function F in the form:

$$F = x - f(r) - h(r)\int_0^t g(t)dt = x - f(r) - h(r)\cdot z(t), \; v(r,t) = g(t)\cdot h(r), \tag{15}$$

where function $f(r)$ is assumed to be known from the previous solution for a stationary medium, and function $h(r)$ - unknown. Differentiating $T(F)$ by r, x, t, and substituting in (14), one obtains a differential equation with respect to functions $f(r)$ and $h(r)$:

$$P\left[1 + \left(f' + h'z\right)^2\right] - \left(f'' + h''z\right) - \frac{1}{r}\left(f' + h'z\right) = 0, \tag{16}$$

where P, as it was mentioned above, is determined by (6). Replacing $y = P\cdot r$:

$$f' + h'z = -u'/(Pu), \tag{17}$$

let us obtain an equation for function *u*:

$$\frac{d^2u}{dy^2} + \frac{1}{y}\frac{du}{dy} + u = 0, \tag{18}$$

coinciding in the form with equation (7). The solution of this equation is the Bessel function $u = J_0(y) = J_0(Pr)$, and $u' = -P \cdot J_1(P \cdot r)$, where J_1 - the Bessel function of the first kind of the first order. From (17) let us find:

$$h'(r) = -\frac{1}{z}\left(\frac{u'}{Pu} + f'\right) = \frac{1}{z}\left(\frac{J_1(P \cdot r)}{J_0(P \cdot r)} - f'\right). \tag{19}$$

Using the experimental values of function $z(t) = \int_0^t g(t)dt$, one can numerically integrate (19). The result of the calculations is shown in Fig. 1, 2 as function $F(r,x,t) = 0$ at different times *t*, as well as in Fig. 1 as the dependence of the coordinate of the center of the front on time (curve 3). It can be seen that, in contrast to the solution in a stationary medium, it is possible to obtain a satisfactory agreement with the experimental data.

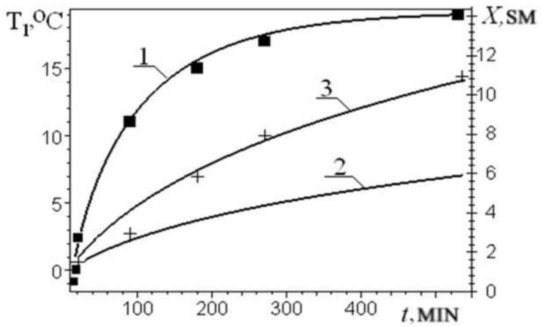

Figure 1. The temperature of the ground surface; 2 - the coordinate of the thawing front center without taking into account convective heat transfer; 3 - the same taking into account convective heat transfer; + - experimental points.

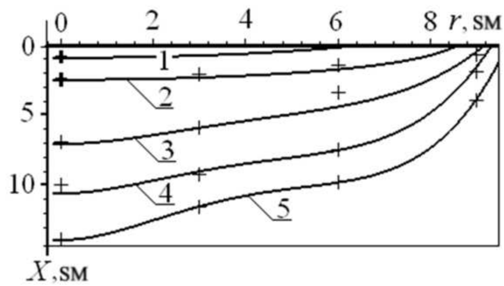

Figure 2. Position and shape of the thawing front taking into account convective heat transfer:
1 - t = 10 min; 2 - t = 90 min; 3 - t = 180 min; 4 - t = 270 min; 5 - t = 540 min.

3. Numerical modeling

Computer modeling of this problem was considered by several authors [6, 7, 8] who confirm the conclusion obtained experimentally by the authors of the article that the movement of the liquid in the ground creates an additional thermal flux that must be taken into account when solving the thermal conductivity equation. In low-water soils, this mechanism can be neglected, which does not apply to soils in the north of Russia, through which most of the existing and under construction pipelines and facilities of the fuel and energy complex pass [7].

Physical and mathematical modeling is based on a numerical solution of a system of equations consisting of thermal balance equations for the gas and oil phases, the heat-transfer medium flow equation, the thermal conductivity equation in a two-dimensional setting, taking into account phase transformations and migration of pore moisture.

The non-stationary thermophysical model can be represented in the following form [5]:

$$\begin{cases} Q_g = -Q_{g1} + Q_{go1} + Q_{go2} - Q_l \\ Q_l = Q_{l1} + Q_{lo} + Q_{ls} + Q_{\lg} \\ \dfrac{\partial T_s}{\partial t} = a_s \cdot \left(\dfrac{1}{r} \cdot \dfrac{\partial}{\partial r} \cdot \left(r \cdot \dfrac{\partial T_s}{\partial r} \right) + \dfrac{\partial^2 T_s}{\partial z^2} \right) + \vartheta_r \cdot \dfrac{\partial T_s}{\partial r} + \vartheta_z \left(\dfrac{\partial T_s}{\partial z} \right), \\ \vartheta_r = -K \cdot \dfrac{\partial w}{\partial r}, \vartheta_z = -K \cdot \dfrac{\partial w}{\partial z} \\ G = \pi R^2 \cdot \dfrac{dh}{dt} \end{cases} \tag{20}$$

$$Q_g = C_g \cdot \rho_g \cdot \pi R^2 \cdot (H - h) \cdot \frac{d(T_g - T_0)}{dt}; \qquad Q_l = C_l \cdot \rho_l \cdot \pi R^2 \cdot (h) \cdot \frac{d(T_l - T_0)}{dt};$$

$$Q_{g1} = C_g \cdot \rho_g \cdot \pi R^2 \cdot (T_g - T_0) \cdot \frac{d(H - h)}{dt}; \qquad Q_{l1} = C_l \cdot \rho_l \cdot \pi R^2 \cdot (T_{lo} - T_l) \cdot \frac{d(h)}{dt}$$

$$Q_{go1} = k_{go1} \cdot (T_g - T_0) \cdot \pi R^2; \qquad Q_{lo} = k_{lo} \cdot (T_l - T_0) \cdot 2\pi R h$$

$$Q_{go2} = k_{go2} \cdot (T_g - T_0) \cdot 2\pi R \cdot (H - h); \qquad Q_{ls} = k_{ls} \cdot (T_l - T_s) \cdot \pi R^2$$

$$Q_l = \alpha_{gl} \cdot (T_g - T_l) \cdot \pi R^2 \qquad Q_{\lg} = \alpha_{\lg} \cdot (T_l - T_g) \cdot \pi R^2$$

where T_0 - ambient temperature, K; T_{lo} - temperature of the heat-transfer medium at the inlet to the tank at a constant flow rate G, K; k_{go1} - coefficient of heat transfer through the roof of the tank, $W/(m^2 \cdot K)$; k_{go2} - coefficient of heat transfer through the side wall of the tank in the gas space region, $W/(m^2 \cdot K)$; α_{gl} and $\alpha_{\lg}$ - coefficient of heat exchange from the gas-air mixture to the "hot" heat-transfer medium and back, $W/(m^2 \cdot K)$; k_{lo} - coefficient of heat transfer through the side wall of the tank in the liquid space region, $W/(m^2 \cdot K)$; k_{ls} - coefficient of heat transfer through the tank bottom from the "hot" heat-transfer medium to the foundation of the tank, $W/(m^2 \cdot K)$; $C_{g,l,s}$ and $\rho_{g,l,s}$ - heat capacity and density of the gas-air mixture, heat-transfer medium and ground, respectively, $J/(kg \cdot K)$, m; R, H - radius and height of the tank, m; h - height of the heat-transfer medium filling, m; G - flow rate of the heat-transfer medium, m^3/s, a_s - coefficient of thermal diffusivity of ground, m/s^2, ϑ_r and ϑ_z - filtration rate of pore moisture, $K -$ isothermal coefficient of moisture conductivity, m/s^2, w - moisture content by weight (subscripts g, l, s denote, respectively, gas, liquid and ground components of the system).

The technique for finding coefficients of heat transfer is described in [9, 10, 11]. System (1) is solved using an implicit scheme [12] with the following initial and boundary conditions:

Initial conditions:

at $t = 0$ $h = 0, T_g = T_0, T_l = T_{lo}$,

$$J = k_{ls} \cdot (T_l - T_s) \cdot \pi R^2 = 0$$

at $z = 0$ $T_s = T_0$

Boundary conditions:

$$z > 0, T_s = T_s(z)$$

$$z = L, T_s = T(L) = const$$

$$J = k_{ls} \cdot \pi R^2 \cdot (T_l - T_s) = \lambda \left. \frac{\partial T_s}{\partial z} \right|_{z=0}$$

4. Discussion of the numerical solution

The physical and mathematical modeling of the proposed non-stationary model in the "hot tank-frozen ground" system taking into account the effect of the migration of pore moisture gives a fairly accurate agreement with the experimental data, unlike the variant without taking it into account [4].

5. Conclusion

A significant influence of pore moisture migration on the shape and velocity of the thawing front during thermal interaction in the "hot tank-frozen ground" system has been established experimentally.

A non-stationary thermophysical model is developed that takes into account the migration of pore moisture, the numerical solution gives a fairly satisfactory agreement with the experimental data.

The solution of the non-stationary thermophysical system can be adapted to predict the halo of thawing when storing oil and oil products on frozen soils.

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