

Dependence of energy characteristics of ascending swirling air flow on velocity of vertical blowing

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Abstract. In the model of a compressible continuous medium, for the complete Navier-Stokes system of equations, an initial boundary problem is proposed that corresponds to the conducted and planned experiments and describes complex three-dimensional flows of a viscous compressible heat-conducting gas in ascending swirling flows that are initiated by a vertical cold blowing. Using parallelization methods, three-dimensional nonstationary flows of a polytropic viscous compressible heat-conducting gas are constructed numerically in different scaled ascending swirling flows under the condition when gravity and Coriolis forces act. With the help of explicit difference schemes and the proposed initial boundary conditions, approximate solutions of the complete system of Navier-Stokes equations are constructed as well as the velocity and energy characteristics of three-dimensional nonstationary gas flows in ascending swirling flows are determined.

1. Introduction

The proposed gas model, as a moving compressible continuous medium possessing dissipative properties of viscosity and thermal conductivity [1], in numerical simulation of complex swirling air flows arising during cold vertical blowing [2], gives the main gas dynamic characteristics that coincide with the results of field experiments [3]. Comparison of calculated numerical values of geometric characteristics and kinetic energies [4] led to the conclusion that the energy of rotational motion obtained from the energy of the Earth's rotation around its axis will exceed the half of the kinetic energy of the entire flow only for the diameter of the vertical part of the flow of not less than 5 meters and blowing velocity not less than 15 m/s. In this case, the main contribution of 97% to the total kinetic energy of the ascending swirling flow is given by the kinetic energy of rotational motion. It was this part of the energy that was obtained by an ascending swirling flow from the energy of the Earth's rotation.

In this paper, the developed method for parallelizing the numerical algorithm for solving the complete system of Navier-Stokes equations is used to numerically study the dependences of the energy characteristics of an ascending swirling air flow on the velocity of a vertical blowing. In particular, the aim of the paper is to establish by the numerical experiments the character of the change in the velocity and energy characteristics of the formed swirling air flow at a sudden step-like decrease in the velocity of the vertical blowing.



2. Materials and methods

To describe complex nonstationary three-dimensional flows of a compressible continuous medium with dissipative properties of viscosity and thermal conductivity, the paper uses the complete system of Navier-Stokes equations, which, being written in dimensionless variables taking into account the action of gravity and Coriolis forces in vector form, has the following form [4-10]:

$$\begin{cases} \rho_t + \vec{V} \cdot \nabla \rho + \rho \operatorname{div} \vec{V} = 0, \\ \vec{V}_t + (\vec{V} \cdot \nabla) \vec{V} + \frac{T}{\gamma \rho} \nabla \rho + \frac{1}{\gamma} \nabla T = \vec{g} - 2\vec{\Omega} \times \vec{V} + \frac{\mu_0}{\rho} \left[\frac{1}{4} \nabla (\operatorname{div} \vec{V}) + \frac{3}{4} \Delta \vec{V} \right], \\ T_t + \vec{V} \cdot \nabla T + (\gamma - 1) T \operatorname{div} \vec{V} = \frac{\kappa_0}{\rho} \Delta T + \frac{\mu_0 \gamma (\gamma - 1)}{2\rho} \left\{ [(u_x - v_y)^2 + \right. \\ \left. + (u_x - w_z)^2 + (v_y - w_z)^2] + \frac{3}{2} [(u_y + v_x)^2 + (u_z + w_x)^2 + (v_z + w_y)^2] \right\}, \end{cases} \quad (1)$$

where the values of dimensionless coefficients of viscosity and thermal conductivity are as follows: $\mu_0 = 0.001$, $\kappa_0 \approx 1.46\mu_0$.

In system (1): t is time; x, y, z are Cartesian coordinates; ρ is gas density; $\vec{V} = (u, v, w)$ is gas velocity vector with projections onto the corresponding Cartesian axes; T is gas temperature; $\vec{g} = (0, 0, -g)$ is acceleration vector of force of gravity, $g = \text{const} > 0$; $-2\vec{\Omega} \times \vec{V} = (av - bw, -au, bu)$ is Coriolis forces' acceleration vector, $a = 2\Omega \sin \psi$, $b = 2\Omega \cos \psi$, $\Omega = |\vec{\Omega}|$; $\vec{\Omega}$ is angular velocity vector of Earth's rotation; ψ is latitude of point; O is the beginning of the Cartesian coordinate system rotating together with the Earth.

As initial conditions for describing the corresponding flows of a compressible viscous heat-conducting gas in the case of constant values of the coefficients of viscosity and thermal conductivity, functions determining the exact solution of [11] of the system (1) are taken as follows:

$$u = 0, \quad v = 0, \quad w = 0, \quad T_0(z) = 1 - kz, \quad \rho_0(z) = (1 - kz)^{\gamma-1}, \quad (2)$$

where

$$k = \frac{l x_{00}}{T_{00}}, \quad l = 0.0065 \text{ K/m}, \quad x_{00} = 50 \text{ m}, \quad T_{00} = 288^\circ \text{K}, \quad \nu = \frac{\gamma g}{k} = \text{const} > 0. \quad (3)$$

Numerical experiments were carried out in the computational domain in the form of a rectangular parallelepiped (Fig. 1), which side lengths in a dimensionless form are $1 \times 1 \times 0.04$ (in a dimension form they are $50 \text{ m} \times 50 \text{ m} \times 2 \text{ m}$). Blowing the gas through a vertical pipe is modeled by setting vertical gas flow rate w as a time function in the form:

$$w(t) = M \cdot [1 - \exp(-10t)], \quad (4)$$

through a square hole of size 0.1×0.1 (in dimensional form $5 \text{ m} \times 5 \text{ m}$) in the center of the upper bound of the computational domain; M is the maximum blowing velocity.

For the density on all six bounds of a parallelepiped, there is the condition of the flow continuity [12]: $x = 0, \quad x = x^0, \quad y = 0, \quad y = y^0, \quad z = 0, \quad z = z^0$. The boundary conditions for the components of the gas velocity vector correspond to the non-flow conditions for the normal component of the velocity vector, and the symmetry conditions for the other two components of the velocity vector. For the temperature on all six faces, the thermal insulation conditions are set [12].

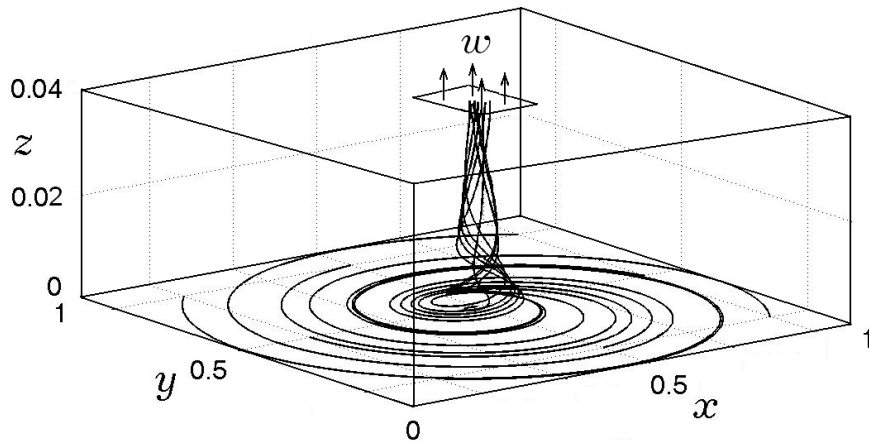


Figure 1. Computational domain

Calculations were carried out with the following input parameters: scale size values of density, velocity, distance and time were equal, respectively

$$\rho_{00} = 1.29 \text{ kg/m}^3, \quad u_{00} = 333 \text{ m/s}, \quad x_{00} = 50 \text{ m}, \quad t_{00} = x_{00} / u_{00} = 0.15 \text{ s}.$$

Difference steps in three spatial variables are $\Delta x = \Delta y = 0.005$ (dimensional value 0.25 m), $\Delta z = 0.004$ (dimensional value 0.2 m), and the time step was $\Delta t = 0.001$ (dimensional value 0.00015 s).

3. Results of calculations

The essence of the numerical experiments carried out is reduced to the following. Each separate calculation was started with the simulation of the gradual acceleration of the emerging ascending swirling flow with vertical air blowing through the upper opening at constant velocity $w = 20 \text{ m/s}$.

With the passage of time, the airflow velocity in computational domain D , and, consequently, its total kinetic energy:

$$W = \frac{1}{2} \iiint_D \rho (u^2 + v^2 + w^2) dx dy dz \approx \frac{1}{2} \sum_i \sum_j \sum_k \rho_{i,j,k} (u_{i,j,k}^2 + v_{i,j,k}^2 + w_{i,j,k}^2) \Delta x \Delta y \Delta z. \quad (5)$$

At the same time, the velocities of the circular (rotational) motion of the swirling flow also grows, as a result of which the kinetic energy of rotational motion increases

$$W_\varphi = \frac{1}{2} \iiint_D \rho v_\varphi^2 r dr d\varphi dz \approx \frac{1}{2} \sum_i \sum_j \sum_k \rho_{i,j,k} v_{\varphi i,j,k}^2 r \Delta r \Delta \varphi \Delta z. \quad (6)$$

The increase in the velocities and energies of the ascending swirling flow occurs until it reaches the steady mode of its operation, in which all thermodynamic, velocity and energy characteristics stop to change. Since the parameters of all ascending swirling air flows, for which this series of computational experiments were carried out, were the same, the time to enter the steady mode was the same for them.

After reaching the steady-state regime, a different continuation of the numerical experiment was chosen for each variant of the calculation. Namely at the same time, the value of the blowing velocity suddenly decreased to a lower value, in comparison with the initial one. A further description of the computational experiments is devoted to the results of the changes in the functioning of the ascending swirling flow after such jump in the blowing velocity in each of the calculation options.

Fig. 2 shows the dependency plots of the total kinetic energy of the ascending swirling airflow versus time.

On the left-hand side of the figure, the plots of changing the total kinetic energy of the flow coinciding with each other, right up to reaching the steady mode (point \bar{A} on the plots) at a dimensionless vertical blowing velocity of 0.06 (dimensional value $w = 20 \text{ m/s}$). In the steady mode

(point \dot{A}), the maximum value of the total kinetic energy is 0.001417 (dimensional value 25392640 J = 25.39 MJ), and the time to enter the steady mode is 17.5 minutes.

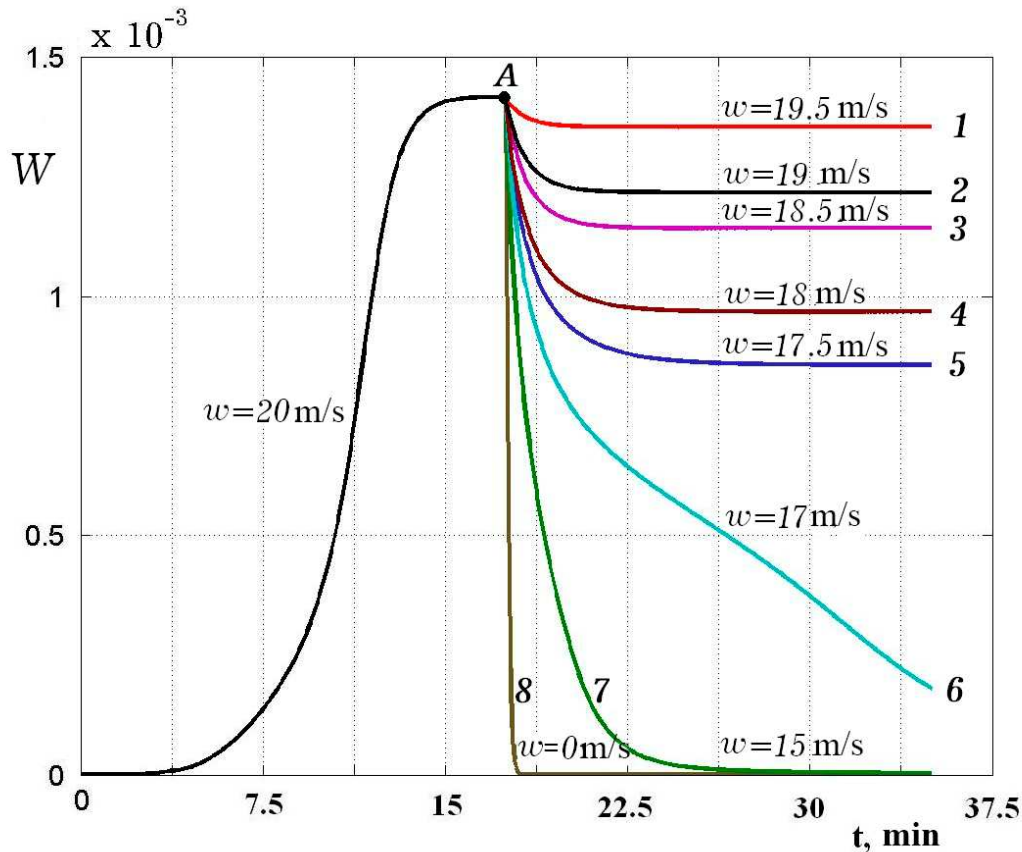


Figure 2. Dependences of the kinetic energy of a swirling flow on time

The right-hand side of the figure shows the dependency plots of the total kinetic energy on time after an step-like decrease in the velocity of the vertical blowing at point \dot{A} up to velocities: $w_1 = 19.5$ m/s – plot 1; $w_2 = 19$ m/s – plot 2; $w_3 = 18.5$ m/s – plot 3; $w_4 = 18$ m/s – plot 4; $w_5 = 17.5$ m/s – plot 5; $w_6 = 17$ m/s – plot 6; $w_7 = 15$ m/s – plot 7; $w_8 = 0$ m/s – plot 8.

Plot 1 shows a gradual decrease in the kinetic energy of the flow and its entering the steady mode up to value 0.001354 (dimensional 24.26 MJ) during the time of $t_1 = 4.37$ minutes.

Plot 2 reflects a gradual decrease in the kinetic energy of the flow and its entering the steady mode to value 0.001217 (dimensional value 21.81 MJ) during the time of $t_2 = 6.25$ minutes.

Plot 3 represents the decrease in the kinetic energy of the flow and its entering the steady mode to value 0.001142 (dimensional value 20.46 MJ) during the time of $t_3 = 8.51$ minutes.

Plot 4 shows a gradual decrease in the kinetic energy of the flow and its entering the steady mode to 0.0009686 (dimensional value 17.36 MJ) during the time of $t_4 = 11.25$ minutes.

Plot 5 reflects a gradual decrease in the kinetic energy of the flow and its entering the steady mode to value 0.0008578 (dimensional value 15.37 MJ) during the time of $t_5 = 13.75$ minutes.

Plot 6 represents a gradual decrease in the kinetic energy of the flow without its entering the stationary mode. Extrapolation of the dependence of the total kinetic energy on time shown by the plot makes it possible to estimate the time until the complete flow stop, which is $t_6 = 35$ minutes for a given blowing velocity.

Graph 7 visualizes the process of reducing the total kinetic energy of the flow to zero value, that is, the process of a complete flow stop occurs during time $t_7 = 12.5$ minute.

And, finally, plot 8 shows the process of sufficiently rapid reduction of the flow kinetic energy to zero value, or the flow moving stop during the time $t_8 = 3.5$ minute.

Further analysis of the calculation results presented in the figure can be carried out for two different ranges for reducing the blowing velocity. With a step-like decrease in the blowing velocity in the range from 20 m/s to 17.5 m/s, the swirling air flow does not collapse, but only smoothly enters the corresponding steady mode with a reduced value of the velocity characteristics, and, therefore, with a smaller margin of total kinetic energy. Moreover, with a decrease in the blowing velocity in this range, the time to reach the corresponding stationary mode increases almost linearly from 4.37 minutes to 13.75 minutes.

With regard to the step-like decrease in the blowing velocity in the range from 17 m/s to 0 m/s, it follows from the calculations that with such changes in the blowing velocity, the swirling air flow does not enter the steady mode, but gradually stops. At the same time, the velocity, and, consequently, the energy characteristics are smoothly reduced to zero values. The decay time of the flow reduces with a decrease in the blowing velocity from 35 minutes to 3.5 minutes.

A special case is a sharp decrease in the blowing velocity from 20 m/s to 0 m/s (plot 8). In fact, such numerical experiment corresponds to a sharp termination of the blowing velocity. The boundary conditions set in the numerical experiment do not allow air to escape from the computational domain. Therefore, the swirling flow quickly stops to exist. This numerical experiment simulates the destruction of a tornado by stopping the vertical gas flow.

4. Conclusion

Detailed three-dimensional nonstationary calculations simulated the flows arising in experiments on blowing air in the pipe from the bottom up. Numerical calculations of these flows set the boundary for the velocity, geometric and energy characteristics of the flow, when the Earth rotation begins to introduce an essential additive into the kinetic energy of the flow. Calculations of nonstationary three-dimensional flows made it possible to determine the time for these flows to enter the steady mode, taking into account energy characteristics. This justifies the obvious idea from the physics point of view: energy in tornadoes and tropical cyclones does not arise immediately and from nowhere. It accumulates over time from an understandable and well-grounded source – the kinetic energy of the Earth rotation around its axis.

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