

Mathematical Model of Stress-Strain State of Curved Tube of Non-Circular Cross-Section with Account of Technological Wall Thickness Variation

S P Pirogov¹, N N Ustinov² and N I Smolin²

¹Industrial University of Tyumen, 38 Volodarskogo Str, Tyumen, 625000, Russia

²Northern Trans-Ural State Agricultural University, 6 Respubliki Str, Tyumen, 625003, Russia

E-mail: piro-gow@yandex.ru, ustinovniknik@mail.ru, tlt_dir@mail.ru

Abstract. A mathematical model of the stress-strain state of a curved tube of a non-circular cross-section is presented, taking into account the technological wall thickness variation. On the basis of the semi-membrane shell theory, a system of linear differential equations describing the deformation of a tube under the effect of pressure is obtained. To solve the boundary value problem, the method of shooting is applied. The adequacy of the proposed mathematical model is verified by comparison with the experimental data and the results of the calculation of tubes by the energy method.

1. Introduction

Curved tubes of a non-circular cross-section are widely used as structural elements of pipelines, elastic sensitive elements of devices, power elements of machines in various industries [1-3]. As a result of tube bending during the manufacturing process, the fibers at the maximum bend radius are stretched, while at the minimum radius they are compressed. This leads to a change in the calculated wall thickness of the tube along the perimeter of the cross section [7-10].

2. Materials and methods

Let us consider the possibility of constructing a mathematical model of the stress-strain state of a curved tube of a non-circular cross-section with allowance for technological wall thickness variations based on the semi-membrane shell theory [4].

The stress-strain state of a curved tube of a non-circular cross-section (Fig. 1 a) is described by differential equations [3, 4]:

$$\begin{cases} \left(\frac{1}{t} \psi' \right)' - \mu_0 \vartheta \cos \alpha_0 = -m \sin \alpha_0 \\ \left(t^3 \vartheta' \right)' + \mu_0 \psi \cos \alpha_0 = -q f_0 \end{cases} \quad (1)$$

In these equations, the unknown functions are ψ and ϑ of dimensionless coordinate η :

$$\eta = s/r; \quad r = P/2\pi, \quad (2)$$

where s – the length of the meridian measured from the selected origin; r – the "reduced radius" of the section; Π – the perimeter of the cross section of the middle surface of the tube.

Function ϑ is the angle of rotation at the point of section η (Figure 1 b), that is, the change in the initial angle of inclination of tangent $\alpha_0(\eta)$:

$$\vartheta(\eta) = \alpha_0(\eta) - \alpha(\eta). \quad (3)$$

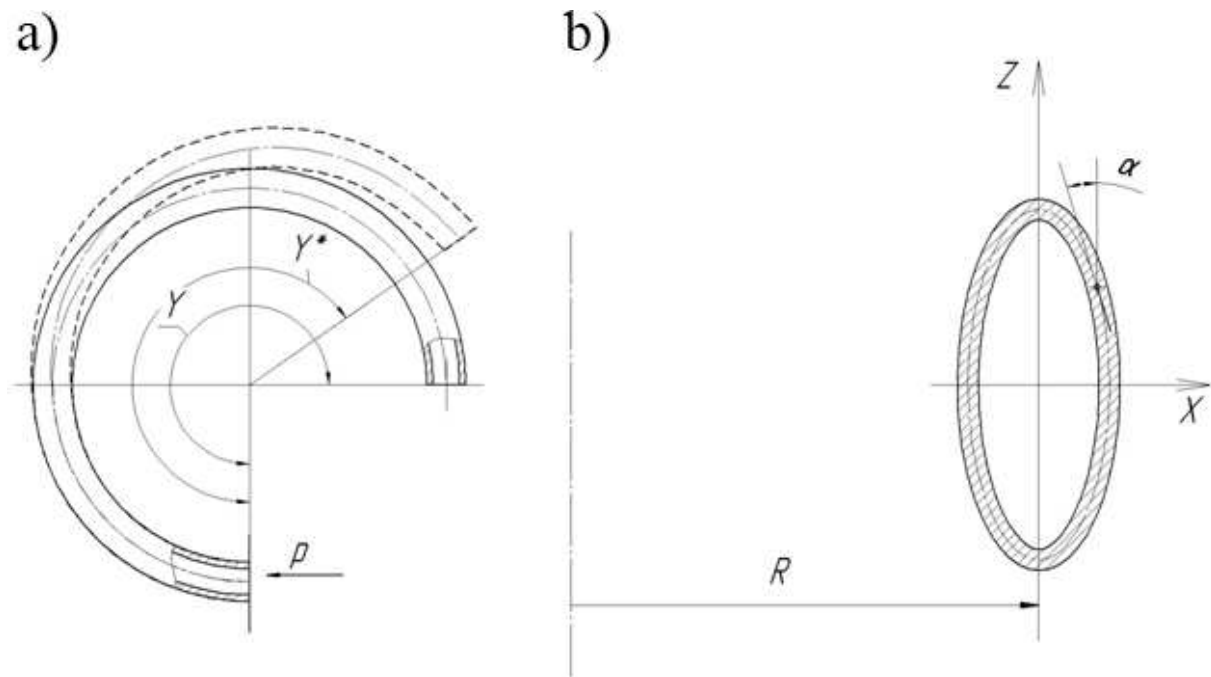


Figure 1. A curved tube of a non-circular cross-section
a – deformation under pressure; b – meridional section

Function $\psi(\eta)$ characterizes the stress state in the rod and has the form:

$$\psi(\eta) = \sqrt{12(1-\nu^2)} \frac{r}{Eh_m^2} \int_0^\eta T_1(\eta) d\eta, \quad (4)$$

where ν – Poisson's ratio; E – Young's modulus of the tube material; h_m – the thickness of the spring wall at the point with coordinate $\eta=\pi/2$; T_1 – the normal force per unit length of the cross-section. Values with index 0 refer to the state of the tube without deformation.

Function $t(\eta)$ depends on the law of variation of the wall thickness and is defined as follows:

$$t(\eta) = h(\eta)/h_m, \quad (5)$$

where $h(\eta)$ – the cross-section wall thickness.

Parameters q , μ and m take into account normal pressure p , the curvature of the axis and its variation:

$$q = 12(1-\nu^2) \frac{pr^3}{Eh_m^3}, \quad \mu = \sqrt{12(1-\nu^2)} \frac{r^2}{Rh_m}, \quad m = \mu - \mu_0, \quad (6)$$

where R – the radius of the central axis of the tube.

The shear force in the section with $r=1$ from the unit load is represented in (2) by the function:

$$f_0 = -\cos \alpha_0 \int_0^\eta \cos \alpha_0 d\eta - \sin \alpha_0 \int_\pi^\eta \sin \alpha_0 d\eta. \quad (7)$$

Equations (1) are valid if the material of the tube is elastic and homogeneous, the rotation angle is small compared to the unity, and dimensions of the cross section are small compared to R .

In equations (1), the shape of the midline of the cross-section is given by functions $\cos \alpha$ and $\sin \alpha$. Due to the symmetry of the mean line of the section with respect to the x and z axes (Fig. 2), it is sufficient to determine the given functions on interval $0 \leq \eta \leq \pi/2$, so expressing the values of angle $\alpha(\eta)$ through the geometric parameters of the "universal" section [1], one gets:

$$\alpha = \begin{cases} \alpha_1 = \arctg \left(\frac{B - B_1}{A - B_1} \right), & 0 \leq \eta < \eta_1; \\ \alpha_2 = \frac{r(\eta - \eta_1)}{B_1}, & \eta_1 \leq \eta < \frac{\pi}{2}. \end{cases} \quad (8)$$

In (8):

$$r = \frac{2}{\pi} \sqrt{A^2 + 2B_1^2 + B^2 - 2B_1(A+B) + B_1}, \quad \eta_1 = \frac{\sqrt{(B - B_1)^2 - (A - B_1)^2}}{r}.$$

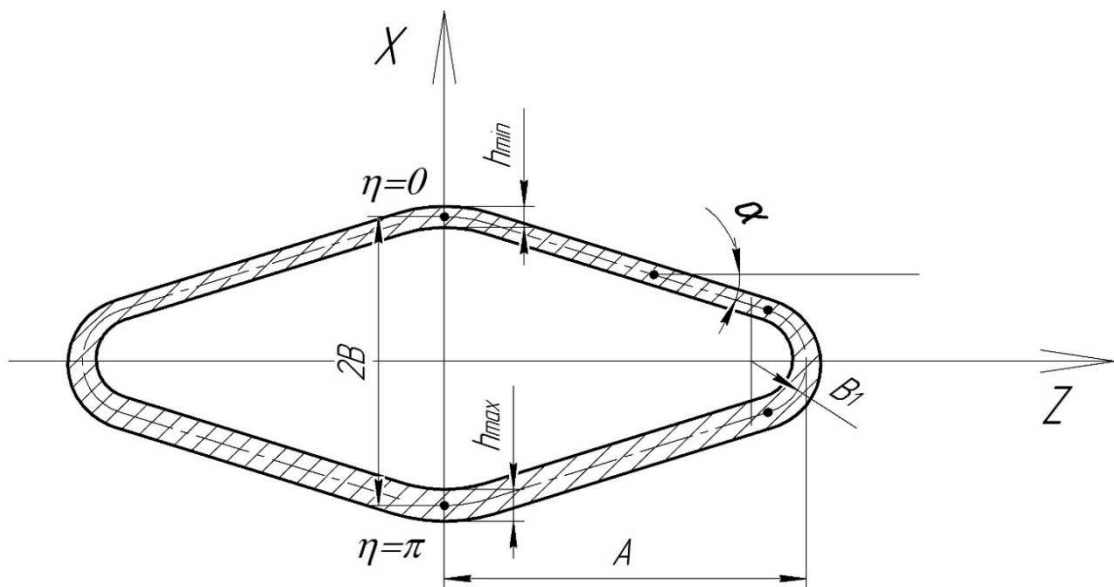


Figure 2. A model of the tube cross-section with regard to technological wall thickness variation

To model the technological wall thickness variation, the law of variation of the wall thickness along the perimeter of the section is represented in the form of a piecewise function:

$$h(\eta) = \begin{cases} h_{\max} & 0 \leq \eta \leq \eta_1 \\ h_{\min} + (h_{\max} - h_{\min})(\eta_2 - \eta)/(\eta_2 - \eta_1) & \eta_1 < \eta \leq \eta_2 \\ h_{\min} & \eta_2 < \eta \leq \eta_3 \\ h_{\min} + (h_{\max} - h_{\min})(\eta - \eta_3)/(\eta_4 - \eta_3) & \eta_3 < \eta \leq \eta_4 \\ h_{\max} & \eta_4 < \eta \leq \eta_5 \end{cases}, \quad (9)$$

where h_{\max} – maximum section wall thickness; h_{\min} – minimum section wall thickness; $\eta_1, \eta_2, \eta_3, \eta_4$ – coordinates that determine the law of variation of the wall thickness along the perimeter of the section (Fig. 1).

Let us introduce notations: $\frac{1}{t}\psi' = \varphi, t^3\vartheta' = u$. Then the system of equations (1) in a normal form can be written as:

$$\begin{cases} \varphi' = \mu_0 \cos \alpha_0 \vartheta - m \sin \alpha_0 \\ u' = -\mu_0 \cos \alpha_0 \psi - q f_0 \\ \psi' = t \varphi \\ \vartheta' = \frac{1}{t^3} u \end{cases}. \quad (10)$$

The system of equations is supplemented by boundary conditions:

$$\begin{cases} \psi(0) = 0 \\ \vartheta(0) = 0 \end{cases}, \begin{cases} \psi'(\pi) = 0 \\ \vartheta(\pi) = 0 \end{cases}. \quad (11)$$

In the linear approximation, the required functions can be represented as the sum of two parts proportional to the parameter of the change in curvature m and to the parameter of the normal pressure:

$$q: \psi = m\psi_m + q\psi_q, \vartheta = m\vartheta_m + q\vartheta_q. \quad (12)$$

As a result of substituting expressions (12) into system (1), let us obtain two systems:

$$\begin{cases} \left(\frac{1}{t}\psi_m' \right)' - \mu_0 \vartheta_m \cos \alpha_0 = -\sin \alpha_0 \\ \left(t^3 \vartheta_m' \right)' + \mu_0 \psi_m \cos \alpha_0 = 0 \end{cases}, \begin{cases} \left(\frac{1}{t}\psi_q' \right)' - \mu_0 \vartheta_q \cos \alpha_0 = 0 \\ \left(t^3 \vartheta_q' \right)' + \mu_0 \psi_q \cos \alpha_0 = -f_0 \end{cases}. \quad (13)$$

Using the algorithm for solving system (1), let us determine from systems (13) functions $\psi_m, \psi_q, \vartheta_m, \vartheta_q$.

To solve the boundary value problem, the shooting method was used [4], also a program was written in the MATLAB environment, the integration of the Cauchy problems was carried out using the Runge-Kutta method of the 4th order of accuracy using the standard solver ode45.

In the case of fixing one of the ends of the tube, its free end under the action of normal pressure is able to move; the system of resolving equations is supplemented by the condition that the moment in the cross-section be equal to zero:

$$M = \frac{Eh_m^2 r}{\sqrt{12(1-\nu^2)}} \oint \psi \sin \alpha_0 d\eta = 0. \quad (14)$$

From (12), taking into account (14), let us determine the relative opening angle of the tube:

$$\frac{\Delta Y}{Yp} = \frac{12(1-\nu^2)}{\mu_0 E} \left(\frac{r}{h_m} \right)^3 \frac{m}{q}, \text{ where } -\frac{m}{q} = \frac{\oint \psi_q \sin \alpha_0 d\eta}{\oint \psi_m \sin \alpha_0 d\eta}. \quad (15)$$

3. Results and Discussion

It is of interest to assess the adequacy of the proposed model by comparing the calculation results with the experimental data and the results of calculating the tubes by the energy method. Table 1 shows the parameters of a number of manometric springs, borrowed from the work of M.P. Shumsky [6].

Table 1. Parameters of manometric springs of flat-oval cross-section with technological wall thickness variation

No. of sample	Measured pressure, [Mpa]	Semimajor axis of section A, [mm]	Seminor axis of section B, [mm]	Radius of rounding at the ends of the major axis of section B1, [mm]	Maximum wall thickness h_{\max} , [mm]	Minimum wall thickness h_{\min} , [mm]	Radius of the central axis of the spring R, [mm]	Modulus of elasticity of material E, [Mpa]
1	0.4	10.20	2.00	1.72	0.65	0.63	52.5	$1.2 \cdot 10^6$
2	0.4	10.21	1.82	1.18	0.64	0.62	54.0	$1.2 \cdot 10^6$
3	0.6	10.14	1.70	1.11	0.83	0.76	54.5	$1.2 \cdot 10^6$
4	1	9.85	2.28	1.59	0.93	0.85	54.0	$1.2 \cdot 10^6$

The study revealed a deviation in the value of the wall thickness from the calculated along the contour of the cross-section within wide limits.

Comparison of the results of calculating the relative opening angle with experimental and calculated data by M.P. Shumsky [6] is given in Table 2. In view of the lack of information on the law of variation in the wall thickness along the perimeter of the cross-section, the wall thickness at the outer and inner sections of the cross-section was assumed to be constant, equal to the minimum and

maximum values, respectively, and in the rounding sections a linear law of variation in wall thickness was taken.

Table 2. Comparison of results of calculating the sensitivity of springs taking into account technological wall thickness variation

No. of sample	Relative opening angle $\Delta Y/(Y) \cdot 10^2, [1/\text{MPa}]$				
	Experiment	Calculation by the Ritz method in the second approximation	Error $\Delta, [\%]$	Calculation by the proposed method	Error $\Delta, [\%]$
1	0.152	0.142	6.57	0.143	5.92
2	0.165	0.144	12.72	0.209	-26.66
3	0.155	0.172	-10.96	0.186	-20.0
4	0.149	0.150	-0.67	0.162	-8.72

As can be seen from Table 2, deviations of the calculation results using the proposed method and the Ritz method in the second approximation, with allowance for deviations of the real geometry of the tube from the theoretical one, should be considered satisfactory.

4. Conclusion

Thus, it can be concluded that the proposed model with sufficient accuracy allows us to evaluate the stress-strain state of a curved tube with allowance for technological wall thickness variations and can be used in solving problems of calculation and design.

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