

Benefits of magnesium wheels for consumer cars

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Abstract. Advantages and disadvantages of magnesium wheels are considered based on a mechanical model of a car. Magnesium wheels are usually applied to racing cars as they provide slightly better strength/weight ratio than aluminum alloys. Do they provide notable benefits also for the everyday user when the car speeds do not exceed allowed speed limit? Distinct properties of magnesium rims are discussed. Apart from lighter weight of magnesium alloys, they are also good in dissipating the energy of vibrations. The role of energy dissipation in the rim of a wheel is estimated by a quarter car model. Improvements to safety by using the magnesium wheels are considered. Braking distance and responsiveness of the car is studied both with and without using an Anti Blocking System (ABS). Influence of rim weight on various handling parameters of the car is quantitatively tested.

1. Introduction

Magnesium wheels have been frequently applied to racing cars as every reduced gram of the vehicle improves its lap times. This results from slightly better strength/weight ratio of magnesium alloys [1] in comparison with aluminum alloys. The weight of rim can be reduced by about 1/3 with respect to aluminum rim. Even better strength/weight ratio can be achieved by such metal processing technique as forging instead of less costly casting. Strong and fire proof Mg alloys are obtained with addition of Si, Mn, Al, Zn, Zr and other elements. There are also more costly Mg alloys with rare earth elements [2] included to get even better mechanical properties. But novel aluminum alloys are not much behind as some of them have nearly similar strength/weight ratio. Rim constitutes only a part of the total suspension weight where tire, axle and brakes contribute, too. Magnesium wheels of racing cars are used not only on tarmac where frequent acceleration, deceleration and cornering takes place but also for non-paved roads where car grip with uneven surface is essential [3]. One more distinct feature of magnesium is a strong dissipation of vibrations [4]. However, most of vibrations of the car are absorbed by the suspension and tires [5, 6]. Notable dissipation of vibration in the rim can occur only for very high frequencies at > 1 kHz that are not in characteristic range of oscillations of consumer cars. To evaluate these vibrations quantitatively we will consider rim deformation in a quarter car model [7]. Another feature of magnesium rims is efficient heat transfer that reduces heat pressure on brake disks, brake pads and brake drums in comparison with steel rims. However, heat conductivity of aluminum rims is approximately similar. Magnesium rims still has advantage as it heats up less than aluminum rim while braking but serious consequences are possible if it overheats. Magnesium rims provides unusual aesthetics of the car standing out from the crowd. That is key point for customers looking for alternatives of aluminum rims. It is more significant for consumer cars to have extra safety rather than to increase maximal speed



and acceleration rate. Most important factors for safety is minimal braking distance and better handling capabilities of the car. It is clear intuitively that braking distance depends minimally on rim properties as wheels account for a small fraction of the car on every road with or without ABS. However, better handling of the car is essential in critical situations like in Moose test. There, a better responsiveness of the car is a must have. Thus, a lighter wheel may bring certain benefits which we would like to evaluate.

2. Mathematical model of the car

2.1. Quarter car model

Quarter car model [8] has been frequently applied to modeling of car dynamics for analyzing influence of tires, axle, disks, springs, road response, etc. Quarter car model will be used also in our case as it considers the main characteristics required for analyzing the role of rim properties during car movement. Vertical coordinate is defined as z , see figure 1. Displacement from unloaded state L_i of a component (deformation) is

$$\Delta z_i = z_{i+1} - z_i - L_i, \quad (1)$$

where index i denotes component: 0 is road, 1 - tire, 2 - rim, 3 - axle (unsprung weight without wheel), 4 - sprung weight, see table 1. The equation of momentum becomes

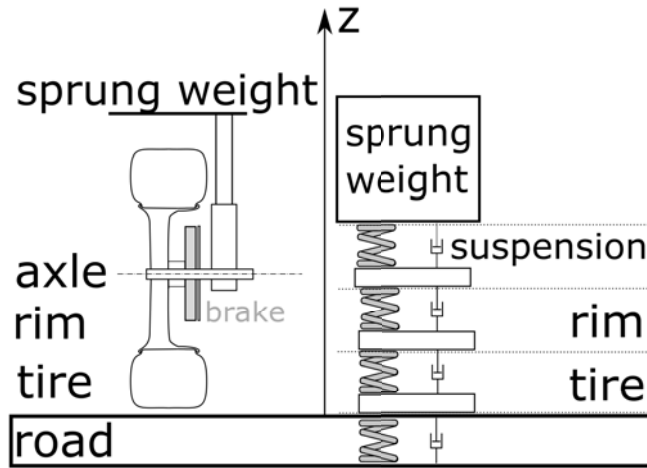


Figure 1. Scheme of the quarter car model.

$$m_i \frac{d^2 z_i}{dt^2} = m_i g + k_i \Delta z_i - k_{i-1} \Delta z_{i-1} - c_i \frac{d \Delta z_i}{dt} + c_{i-1} \frac{d \Delta z_{i-1}}{dt}, \quad (2)$$

where z_0 is road surface position. Variables k_i denote stiffness of the component and c_i - damping coefficients. Sprung weight is the last component of the system which is assumed to be rigid:

$$k_4 = c_4 = 0. \quad (3)$$

The roads are not always rigid but may deform under the weight of car, e.g. dirt roads. The quarter car model enables to add elasticity k_0 and damping factor c_0 for such roads. Parameters of the selected road are

$$k_0 = H(-\Delta z_0) k_{road}, \quad (4)$$

$$c_0 = H(-\Delta z_0) c_{road}, \quad (5)$$

Table 1. Properties of car components.

i	name	material	m , kg	L , m	k , N/m	ψ , -
1	tire		8.5	0.127	$2.25 \cdot 10^5$	0.2
2	rim	steel(Fe)	10	0.19	$1.28 \cdot 10^9$	0.005
2	rim	aluminum alloy(Al)	5.5	0.19	$4.42 \cdot 10^8$	0.02
2	rim	magnesium alloy(Mg)	4.5	0.19	$2.88 \cdot 10^8$	0.25
3	axle		25	0.35	$2.00 \cdot 10^4$	2.0
4	sprung mass		300	-	-	-

where $k_{road} = 10^7$ N/m, $c_{road} = 10^5$ N s/m and

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases} \quad (6)$$

is Heaviside step function. The Heaviside function takes into account that deformation of the road only occurs if wheel is in contact with the road. Characteristic parameters of a consumer car are given in the table 1. These data are derived from the best selling car model in the world, namely Toyota Corolla. Version of Corolla 2016 is used with base equipment and wheels. Damping coefficient of a component is derived from:

$$c_i = \frac{\psi_i}{2\pi} \sqrt{m_i k_i}, \quad (7)$$

where specific damping capacity ψ_i is ratio of energy lost and stored per cycle. Very high damping capacity is for suspension as it is accompanied by hydraulic or pneumatic shock absorbers improving ride quality. Damping of oscillations by the rim are usually disregarded but we will them include in order to estimate what this distinct feature of magnesium (table 1) can bring. Uneven road surface is assumed to be sinusoidal $z_0 = h_0 \sin(2\pi x/\lambda)$ with given wavelength λ and amplitude h_0 . The period of car oscillations in road bumps is $T = \lambda/v$, where v is horizontal velocity of the car.

2.2. Friction model

Behavior of the car on the road depends on the grip with road surface. If the car has a good grip with the road it becomes more responsive to the drivers commands and shortens braking distance. That is the car is not sliding horizontally on the surface neither in the direction of the car motion nor in the normal direction. Horizontal coordinate in the direction of car motion is assumed to be x . Friction force is given by

$$F_x = \mu F_z, \quad (8)$$

where μ is coefficient of friction and F_z is the load of the tire to the road surface. The coefficient of friction $\mu = \mu(s)$ (see figure 2) can be estimated by Magic formula [9, 10] that depends on wheel slip ratio s [11]

$$s = \frac{v - \Omega R}{v}, \quad (9)$$

where Ω is the wheels rotational velocity, and wheel radius is $R = L_1 + L_2$.

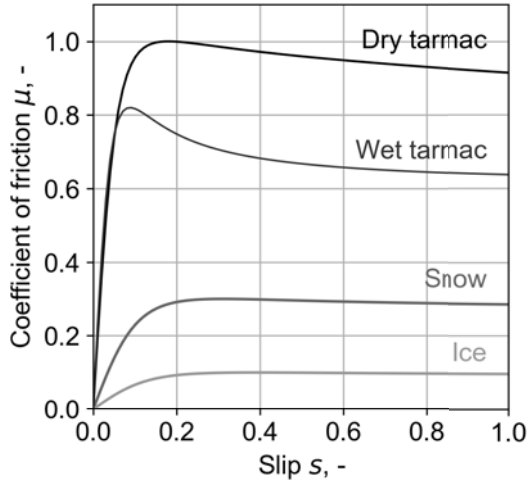


Figure 2. Tire longitudinal coefficient of friction for different road conditions [12].

Without wheels, the acceleration or deceleration rate of the car is given by

$$M \frac{d^2x}{dt^2} = -F_x, \quad (10)$$

where $M = \sum m_i$ is the quarter mass of the car. Wheel rotation must also be accounted, for, as wheels have non-zero inertial moment I :

$$I \frac{d\Omega}{dt} = rF_x - \tau, \quad (11)$$

where τ is braking torque, rF_x is road-tire friction torque with actual wheel radius $r = x_3 - x_1$. Moment of inertia I for the wheel with steel rim, aluminum rim and magnesium rim equal 1.215, 1.053 and 1.013 $kg \cdot m^2$, respectively. The difference is rather small because tire attributes have a dominating effect in moment of inertia. Tire load on the road is given by

$$F_z = c_0 \frac{d\Delta z_0}{dt} - k_0 \Delta z_0. \quad (12)$$

3. Frequency response of vibrating systems with quarter car model

Quarter car model is frequently applied to analyze car vibrations [8]. Oscillations are important for analyzing responsiveness of the car and driver comfort. Driver comfortably tolerates oscillations with periods of about one second. Therefore, the task of tires and springs is to reduce oscillations that have higher or lower frequencies as much as possible without notably compromising car responsiveness. Let us assume that harmonic oscillations occurs in the system with an angular frequency $\omega = \frac{2\pi}{T}$. Each component i of the car has an oscillations amplitude $|A_i|$ and phase $\arg(A_i)$:

$$z_i = A_i \exp(j\omega t) \quad (13)$$

where $j = \sqrt{-1}$. Let us apply the oscillating external force to the tire as it is in real case where tire is forced by uneven road $z_1 = \exp(j\omega t)$ or $A_1 = 1 + 0j$.

The frequency response follows from the solution of linear system of equations (section 2):

$$-\omega^2 m_i A_i = k_i (A_{i+1} - A_i) - k_{i-1} (A_i - A_{i-1}) - j\omega c_i (A_{i+1} - A_i) + j\omega c_{i-1} (A_i - A_{i-1}) \quad (14)$$

for each component $i = 2, 3, 4$, see table 1. It is clear that a lighter wheel will follow the external oscillations more easily, i.e, it will read the road better. That we can see for the axle

component in frequency diagram figure 4, where amplitudes of the quarter car components are shown excluding the tire which oscillates with a defined amplitude. Relative displacement of the rim almost coincides with the axle component for frequencies below 500 Hz, see figure 3. For higher frequencies, there are resonance peaks for rim oscillations. Due to the strong damping of magnesium the resonance peak of magnesium is much weaker than for steel and aluminum.

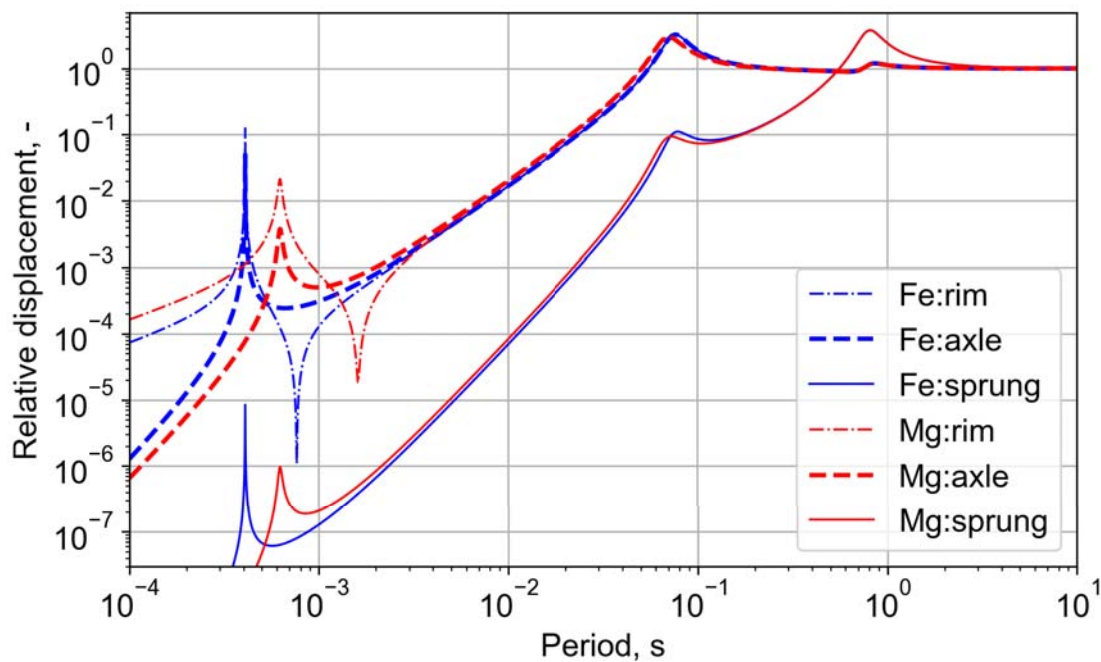


Figure 3. Relative amplitude of oscillating components $|A_i| / |A_1|$ on uneven road.

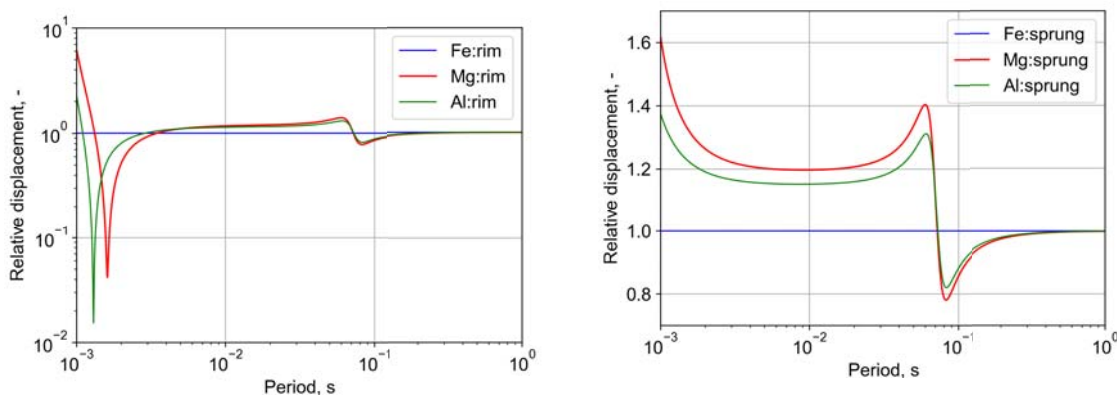


Figure 4. Relative amplitude of rim oscillations $|A_2| / |A_2^{Fe}|$ on uneven road.

Figure 5. Amplitude ratio of sprung mass with respect to the case with steel rim $|A_4| / |A_4^{Fe}|$.

Oscillations of the car body, i.e. sprung weight are much smaller than wheel oscillations. As we can see in figure 5, there are also counter effect of lighter wheels: slightly increased car body

vibrations, when driving on uneven surface with small wavelength $\lambda = vT$ of the bumps ($\lambda \approx 0.5$ m at speed $v > 50$ km/h), e.g. on gravel road, whereas it is opposite for longer vibrations ($\lambda > 1$ m at $v = 50$ km/h). It happens because tire of heavier wheel absorbs high frequency oscillations better. Nevertheless, the situation is opposite at longer bumps: car with steel rim will feel the bumps more as suspension with heavier wheel is closer to the resonance frequency of the spring. Lighter wheels help to increase the lifetime of tires as there are smaller tire oscillations for the same forcing by the road. Dampening factor of vibrations of magnesium rim is 10 times higher than of aluminum rim. However, figure 3 shows that notable damping by magnesium rim occurs at periods of less than a millisecond that are largely damped by tires on road with fine waviness. Therefore, damping of oscillations by the rim can be disregarded as amplitudes of axle and rim components almost coincide, see figure 4. Nevertheless, strong dampening increases durability of suspension brackets and brakes. The elastic and damping properties of springs have to be slightly adapted for given weight of the wheel in order to improve driver's comfort. This is, however, applied rarely for consumer cars.

4. Braking with and without ABS

One of the most important aspects of car safety is braking distance. The braking force can be well estimated with Magic formula, see sub-section 2.2 and (9). Do the lighter wheels improve braking distance? Let us test this on wet tarmac without ABS, i.e., when wheels can become blocked. Consider that initial velocity v_0 is 90 km/h, and the road is uneven with wavelength $\lambda = 1$ m and amplitude 1 cm. Such amplitude is insufficient to cause wheel of losing contact with the road surface. The set of equations of section 2 are solved numerically for initial conditions $x|_{t=0} = 0$, $\frac{dx}{dt}|_{t=0} = v_0$ and $\Omega|_{t=0} = \frac{v_0}{2\pi r}$ in the time interval $[0, t_b]$, $Mv_0 = \int_0^{t_b} F_x dt$. Value of braking torque 1400 N·m is applied characteristic for hard braking. Figure 6 shows that braking distance is almost independent of wheel weight. The car with steel rim decelerates more at one side of road bump and the opposite at the other side. The average load on the surface remains weakly dependent on wheel weight. Thus, the braking distance is about the same for all rim materials.

Now, consider the braking by ABS. ABS enables to reduce braking distance on solid road without losing the control of the car while braking. ABS is usually required in critical situations, when a sudden object appears on the road, that must be bypassed as it is in the moose test. Basically, ABS system detects wheel lockup, then commands to release brake pressure until the wheel rotation spins up [13]. Wheel lockup is usually defined as critical value of slip ratio s (9). Most effective and universal s is usually around 0.2 for all kinds of roads (see figure 2) when friction between road and tire is at maximum. Let the critical slip ratio be 0.3. The braking distance and time is clearly shorter then braking using ABS, see figure 6. Wheel rotation rate accelerates and decelerates multiple times during critical braking on wet tarmac. Inertial moment for light metal wheel is lower. Therefore, it has lower period between the cycles. On the other hand, lower period reduces time interval when the grip with road is reduced, i.e. when wheel has lowest rotational speed. The braking distance is few centimeters longer with magnesium rim and this difference is acquired in the final part of deceleration where road roughness comes into play, see figure 6. This difference can arise just from the phase of road roughness. Larger difference can only occur if the time interval between successive wheel lockups is comparable with duration required for braking. The speed of rotation with light metal wheel oscillates faster, that minimizes the time interval with reduced grip, see figure 7. Highest oscillations of deceleration rate are when the frequency of road at given speed is comparable with the resonance frequency of the tires. For the given road with wavelength $\lambda = 1$ m, the resonance occurs at velocities around 50 km/h. For higher velocities, road grip with heavier wheel oscillates even less because the road roughness is then absorbed more effectively. But the situation is opposite at lower car velocities with more steady deceleration rate of lighter wheel. It is clear that a

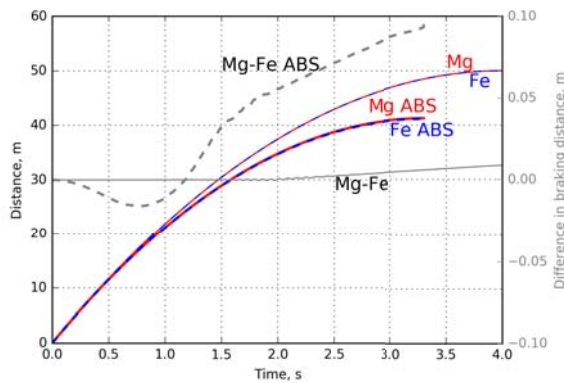


Figure 6. Braking distance on wet uneven tarmac with initial velocity 90 km/h for car with magnesium and steel rims both with and without ABS. Secondary axis shows difference between them (Mg-Fe).

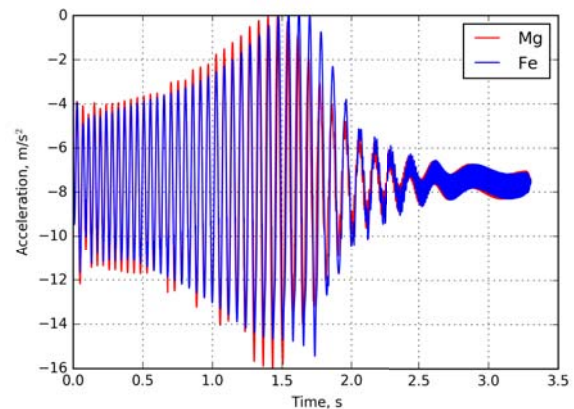


Figure 7. Deceleration rate of the car with ABS on wet uneven tarmac with initial velocity 90 km/h for car with magnesium and steel rims.

car with different rim weight requires different ABS settings [11]. Heavier wheel needs lower threshold s (9) as it is more difficult to accelerate it while braking and regain grip.

5. Weight advantages of magnesium wheels

Magnesium wheels have a lower weight than steel or aluminum wheels. This property leads to multiple advantages [8]:

- (i) Better grip on uneven surface, as lighter wheels read the road better. As a result:
 - It is easier to handle the car. Car motions are more predictable.
 - Reduced risk of car sliding.
 - Magnesium wheels allows higher speed for the same road without compromising grip with the road.
- (ii) Increased durability of tires as amplitude of oscillations reduces in tire (see section above).
- (iii) Easier handling of the car during emergency braking.
- (iv) Lighter wheels improves acceleration and deceleration of the car on road with good grip, as the mass of the car and moment of inertia of the wheels is reduced. Considering that most of the rim weight is distributed at its exterior, it makes up to 2.2 % improvement for the car, see table 1, with magnesium rims in comparison with steel ones.
- (v) Reduced mass of the car.

One drawback of lighter wheels is that the center of gravity of the car becomes slightly higher which affects car stability. Let us consider these effects quantitatively, assuming that an uneven road surface is sinusoidal with a given wavelength λ and amplitude.

5.1. Tire grip on road

Load of the tire on the road remains constant when driving on even surface and constant speed. Driving on an uneven road, the tire load on the surface is alternating but the average is the same as for even road. However, by increasing the vehicle speed, it is possible that the tire lose contact with the road surface and the car becomes irresponsible. Lighter wheels allow driving the vehicle with the same speed on more uneven road without compromising the grip of the car. Let us apply the quarter car model of section 2 to analyze responsiveness of the

car. Quantitatively, good car grip means high and continuous load on the surface which leads to better response to drivers input, see figure 8 where minimal tire load on the surface is shown over a wavy road with amplitude of 1 cm. By increasing the speed of the car or decreasing the wavelength of the road surface it is possible to make the wheel lose contact with the road surface. Then it is not possible for the driver to control the car. For various wheels critical roughness for the road when this happens is shown in figure 9. The difference between magnesium and aluminum rims is relatively small but the car with steel rims needs almost twice smoother road. Two minimums of the critical roughness are associated with resonance frequencies of tires and suspension. If we move to higher frequencies above 12 Hz, then tire of the heavier wheel can absorb the road oscillations better. But small period of oscillations makes these vibrations less essential for handling of the car, as the driver cannot lose road contact in such a small interval of time. The car is easily handleable if the grip is almost continuous. Therefore, it is better

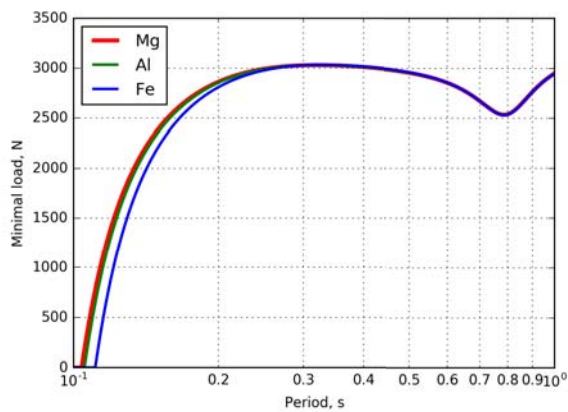


Figure 8. Minimal tire load vs. period $T = \lambda/v$ on uneven surface if the roughness amplitude is $h_0 = 1$ cm.

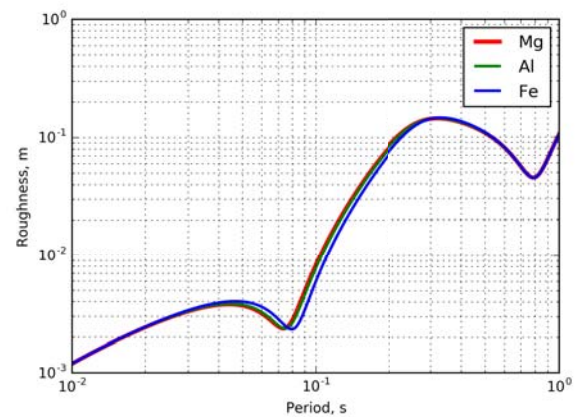


Figure 9. Critical roughness of the road vs. period $T = \lambda/v$ when wheel loses contact with the road.

that the amplitude of load oscillations is minimal. For example, driver is steering the wheels and the car responds weakly when the car load on the road is minimal. However, when the load oscillates to a higher value the driver's input can appear too large. Therefore, let us plot ratio of minimal vs. maximal load of tire on the surface, see figure 10. Again it is clear that magnesium and aluminum rims provide essential advantage over more heavy steel rims. More important is the duration of low grip of the car. Lighter wheel leads to shorter time interval with reduced grip at periods above 0.1 s, see figure 11. Reduced grip on the road can last about 0.01 s more for wheels with steel rims as compared to light metal rims.

6. Conclusions

Modeling results show that better dampening factor of magnesium wheels has minor influence in car handling. However, the lighter wheels substantially improves car responsiveness in many aspects. Also the tire lifetime is increased due to smaller oscillations in them. Braking distance is almost the same for any rim either with or without ABS but car responsiveness during critical braking is substantially improved by using combination of ABS and lighter wheels. One minor drawback of light metal wheels is higher car vibrations on road with fine waviness when slightly larger part of oscillations are transmitted to the sprung mass. Accounting the price of magnesium wheels and their durability, there are no significant advantages over aluminum wheels for every day user at velocities below 100 km/h. Most significant factor is that critical velocity is increased by a few percent on bumpy road when car still responds to driver. However, better look and

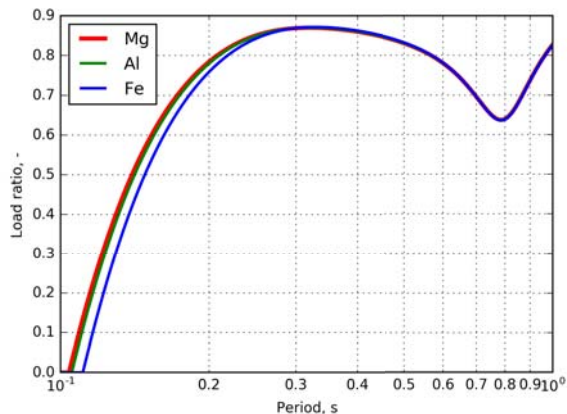


Figure 10. Ratio between minimal and maximal load on uneven surface vs. period $T = \lambda/v$ if the roughness amplitude is $h_0 = 1$ cm.

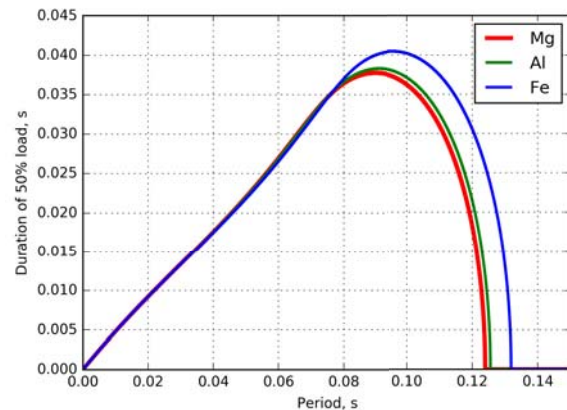


Figure 11. Time interval when the tire pressure on road is less than 50 % from normal vs. period $T = \lambda/v$ on road with roughness amplitude $h_0 = 1$ cm.

minimally better driving responsiveness will attract car enthusiasts especially for premium cars. Moreover, car acceleration rate increases by about 2 %. Most important benefits are for cars driving on an uneven surface especially with low profile tires. However, these wheels usually require larger rims where magnesium ones may appear to costly over aluminum ones if the lifetime of rims is included.

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