

An integrated production-inventory model for food products adopting a general raw material procurement policy

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Abstract. Studies on an integrated production-inventory model for deteriorating items have been done extensively. Most of the studies define deterioration as physical depletion of some inventories over time. This definition may not represent the deterioration characteristics of food products. The quality of food production decreases over time while the quantity remains the same. Further, in the existing models, the raw material is replenished several times (or at least once) within one production cycle. In food industries, however, a food company, for several reasons (e.g., the seasonal raw materials, discounted price, etc.) sometimes will get more benefit if it orders raw materials in a large quantity. Considering this fact, this research, therefore, is aimed at developing a more representative inventory model by (i) considering the quality losses in food and (ii) adopting a general raw material procurement policy. A mathematical model is established to represent the proposed policy in which the total profit of the system is the objective function. To evaluate the performance of the model, a numerical test was conducted. The numerical test indicates that the developed model has better performance, i.e., the total profit is 2.3% higher compared to the existing model.

1. Introduction

Inventory plays an important role in industry since 40% of companies' total asset is invested in the form of inventory [1]. Since EOQ was introduced by Harris in 1915, it has been developed extensively. Currently, business becomes more competitive. Thus, a conventional approach where a company only focuses on managing its own inventory is no longer sufficient. The joint economic lot size (JELS) model is firstly introduced by [2], where a company establishes corporations with its customers/retailers. The producer and retailer manage inventory collaboratively due to cost reduction of the joint system. Reference [3] introduced a joint economic-lot-size for purchaser and vendor based on a lot for lot basis. Later, [4] advanced it by developing one setup multiple delivery JELS that allowed the retailer to receive finished goods in a smaller batch during the production cycle. Unlike the lot for lot basis, the single setup multiple deliver strategy successfully reduced inventory cost in retailer. However, the study still disregarded the material procurement plan in inventory system. Reference [5] included raw materials procurement plan in the single vendor single buyer (SVSB) system while [6] generalized the model by applying a general procurement policy. Extensive reviews regarding the integrated production-inventory model can be seen in [7] and [8]

The aforementioned studies, however, do not pay sufficient attention to quality losses in food products. Multi-echelon models considering quality losses in raw materials and finished goods is proposed by [9] and [10]. In the models, the quality of raw materials follows a kinetic model function while the quality of finished goods is perceived by customers based on product's expiration dates. For



the latter, since the customers perceive the quality of aging products as declining, the customers' willingness to pay (WTP) for the product is also decreasing and influence the demand rate. To maintain the demand, retailers reduce the products' price and promote the discounted products [11]. Representing this phenomenon, [9] and [10] developed a shelf-life based price function where the price of finished goods is varied during the selling period. The inventory models, however, still limit the frequency of raw material replenishment. There must be at least one lot of raw materials is replenished during one production cycle. This restriction reduces the flexibility of the model in representing some real cases. In food industries, manufacturers sometimes deal with a situation where, for some reasons, they may need to purchase raw materials in a bigger quantity. Considering the possibility of ordering one lot of raw materials to fulfil more than one production cycle (i.e. implementing a more general procurement policy) will give companies more alternatives. Therefore, this study relaxes the procurement policy in the current research by applying a more general policy in a SVSB system dealing with food inventory issues.

The remaining part of this paper is organized as follows. Section 2 defines the objective problem as well as its assumptions and notations to develop the proposed model. The model development is presented in Section 3 while the numerical test and the result are discussed in Section 4. Finally, Section 5 summarizes this study.

2. Problem definition, assumptions, and notations

This study extends [9] by applying a more general raw material procurement policy into the SVSB system. The integrated system deals with problems of managing food inventories, i.e., the quality of raw material losses during storage time and the finished goods suffering from perceived value decrease. The raw material enters warehouse in maximum quality condition (Q_{\max}) and, by the time, its quality degrades until the minimum, but it is still acceptable to be used (Q_{\min}). Different from [9], the proposed model considers options either to order raw materials more frequent ($m \geq 1$) or less frequent ($m \leq 1$) during the production cycle (m denotes the number of ordering frequencies). However, regardless of which option is chosen, the ordering cycle should be shorter than the usable period of the raw materials. This is important to ensure that no shortage occurs due to spoilage/damage materials. Therefore, in this study, a constraint is added to prevent the ordering cycle of raw material longer than its usable period.

The proposed model is developed based on the following assumptions:

- The production rate and demand rate are constant;
- One unit raw material is required to produce one finished unit goods;
- Once the customers' willingness to pay (WTP) decreases, the product's price declines linearly to maintain the demand;
- All delivering batches enter the retailer' warehouse before the customers' WTP decline;
- The ordering cycle of the raw material is shorter than its usable period;
- Lead time is neglected, and no shortage is allowed.

Further, the notation below is used to symbolize the model.

For the manufacturer

P	:	production rate (unit product/unit time)
D	:	demand rate from retailer (unit product /unit time)
k	:	deterioration rate (unit quality/ unit time)
Q_{\max}	:	maximum quality level of raw material (unit quality)
Q_{\min}	:	minimum quality level of raw material (unit quality)
$Q(t)$:	remaining quality level of raw material at time t (unit quality)
c_{raw}	:	purchasing cost of raw material (\$/unit product)
A_{raw}	:	delivering cost of raw material (\$/shipment)
q_{raw}	:	ordering size of raw material (unit product)

H_{raw}	:	holding cost of raw material (\$/unit product/ unit time)
c_{loss}	:	cost of quality loss (\$/unit quality/unit time)
$L(m,T)$:	total cost of quality loss (\$/unit time)
$TC_{\text{raw}}(m,T)$:	total cost of raw material system (\$/unit time)
c_{mfc}	:	manufacturing cost (\$/unit product)
A_{mfc}	:	production setup cost (\$/setup)
H_{mfc}	:	holding cost of finished goods (\$/unit product/unit time)
\bar{I}_{mfc}	:	average inventory of finished goods at the manufacturer (unit product)
$TC_{\text{mfc}}(m,T)$:	total cost of finished goods system (\$/unit time)
$TP_{\text{mfc}}(m,n,T)$:	total profit of the manufacturer (\$/unit time)

For the retailer

c_{ret}	:	product selling price from manufacturer to retailer (\$/unit product)
H_{ret}	:	holding the cost of finished goods at the retailer (\$/unit product/ unit time)
A_{ret}	:	transportation cost for delivering product to the retailer (\$/shipment)
τ_{sl}	:	product shelf-life (unit time)
τ_{start}	:	the time when customer's WTP starts decreasing (unit time)
p_{max}	:	maximum product price (\$/unit product)
$p(t)$:	price of the product at age t (\$/unit product)
E_i	:	age of batch i when entering retailer (unit time)
$TP_{\text{ret}}(n,T)$:	total profit of the retailer (\$/unit time)
$TP(m,n,T)$:	total profit of the integrated supply chain system (\$/unit time)

Decision variables

m	:	the number of raw material procurement cycles
n	:	the number of finished goods deliveries
T	:	the length of production cycle

3. Model development

The proposed model is developed in two phases. The first is developing the quality loss model of raw material which implements a general procurement policy.

3.1. Modelling the quality loss in raw material using a general procurement policy

Reference [9] has developed a mathematical model to represent the quality loss in raw material during the storage time as shown by equation (1) and (2). However, the model is restricted only for systems that adopt one procurement policy, i.e., $m \geq 1$. In the food industry, however, companies sometimes are benefitted from purchasing raw materials in huge lot size. Hence, one raw material replenishment may cover more than one production cycle. The illustration of such situation can be seen in figure 1 where $m=1/2$ or one replenishment to supply two production cycles. Also, in [9], there is no guaranty that the length of ordering cycle is always shorter than product's usable time. This is contradictive with the assumption that no shortage is allowed. As can be seen in figure 1, intuitively, the shortage will occur if product's usable period (t_Q) is shorter than the length of procurement cycle (t_{raw}). This study, therefore, solves this problem by adding a constraint ensuring that t_{raw} is always shorter than t_Q as shown by equation (3).

$$L(m,T) = c_{\text{loss}} \frac{mP}{T} \int_0^{\frac{DT}{mP}} \Delta Q(t) dt \quad (1)$$

Subject to

$$m \geq 1 \quad (2)$$

$$t_Q > \frac{DT}{mP} \quad (3)$$

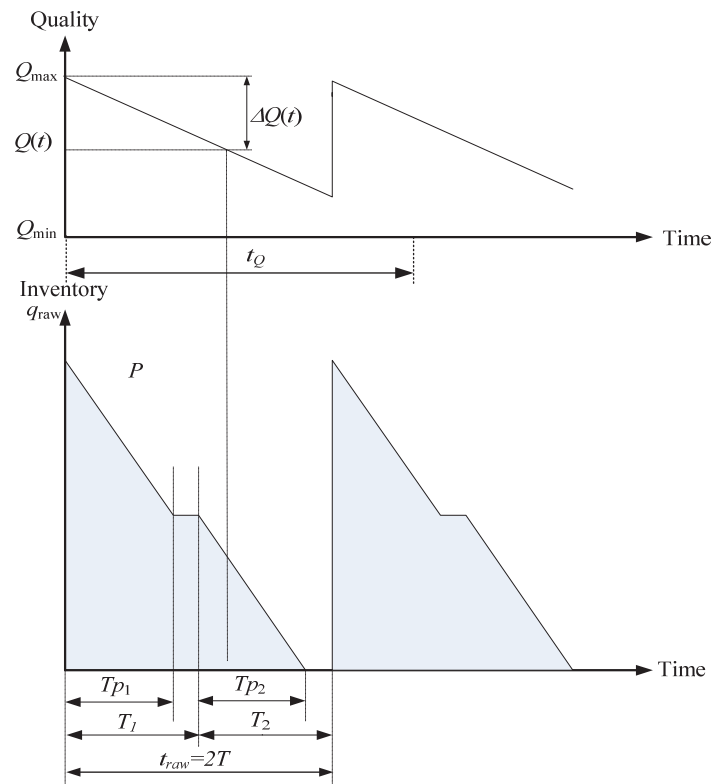


Figure 1. Quality loss in inventory using a general raw material procurement $m = \frac{1}{2}$ or $t_{\text{raw}} = 2T$

As mentioned previously, in this study, a general policy will be implemented in the model. Instead of restricting the number of ordering cycle ($m \geq 1$) such as in [9], the model considers the possibility of ordering raw material for covering more than one production cycle ($m < 1$). As can be seen in figure 1, one replenishment cycle of raw material is used to fulfill the demand for raw material in two production cycles. Hence, the value of m is equal to $\frac{1}{2}$. During the first production run length (T_{p1}), the quantity of raw material is depleted by raw material demand d_{raw} which is equal to the production rate or P and remains the same during the off-production period ($T_i - T_{p1}$). Further, at the next cycle, it is continuously depleted until the end of T_{p2} . Following the steps in [9], the total relevant cost per unit time due to the quality loss in the raw material is represented by equation (4)-(6). This formula complements [9] for the case m less than one.

$$L(m, T) = c_{\text{loss}} \frac{mP}{T} \sum_{i=1}^{1/m} \int_{(i-1)T}^{(i-1)T + DT/P} \Delta Q(t) dt \quad (4)$$

Subject to

$$m < 1 \quad (5)$$

$$t_Q > \left(\frac{1}{m} - 1 \right) T + \frac{DT}{P} \quad (6)$$

3.2. Modelling the average inventory in the integrated production-inventory system

The next phase of this section is to find a formula to calculate the average inventory in the integrated system. Figure 2 represents the inventory status of either raw materials or finished goods in a single manufacturer (vendor) single retailer (buyer) environment (or usually called SVSB).

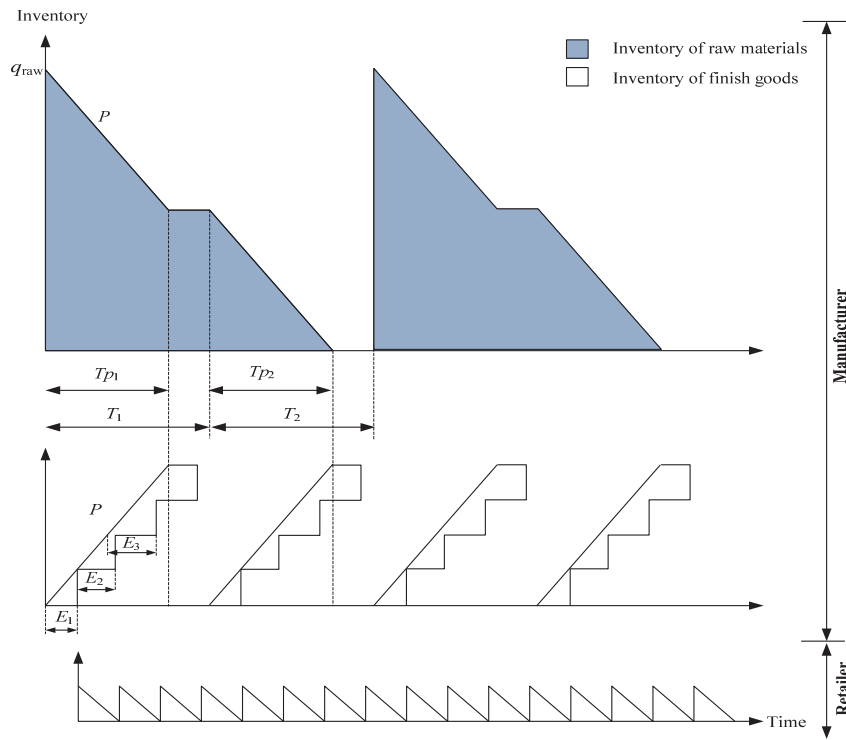


Figure 2. The integrated system using a general raw material procurement policy ($m=1/2$)

As figure 2 shows, this model relaxes the restricted rule in [9] by allowing $m=1/2$ (one replenishment of raw material to fulfill two production cycles, i.e., $t_{\text{raw}}=2T$). The relaxing influences the formula for calculating the average inventory of raw material. Equation (7) is the equation used in [9], complemented by equation (8) as a result of the general procurement policy.

$$I_{\text{raw}} = \frac{D^2 T}{2mP}, \text{ for } m \geq 1 \quad (7)$$

$$I_{\text{raw}} = mT \left[\frac{D^2}{2mP} + \sum_{i=1}^{1/m} (i-1) \frac{D^2 T}{P} \right], \text{ for } m < 1 \quad (8)$$

The inventory cost of carrying raw materials in the manufacturer's system is obtained by multiplying the average inventory in equation (7) or (8) with raw material inventory cost (H_{raw}). Hence, the total cost of procuring and holding raw materials in the manufacturer's system is presented by equation (9).

$$TC_{\text{raw}}(m, T) = C_{\text{raw}} D + A_{\text{raw}} \frac{m}{T} + H_{\text{raw}} I_{\text{raw}} + c_{\text{loss}} L(m, T) \quad (9)$$

Further, following [4], the equation for calculating the average of finished goods is represented by equation (10).

$$I_{\text{mfc}} = \frac{DT}{2n} \left[\frac{D}{P} (2-n) + (n-1) \right] \quad (10)$$

Similar to the previous step, the inventory cost of carrying finished goods is obtained by multiplying equation 10 with the cost for holding the finished goods. Hence, the total cost of processing and holding finished goods in the manufacturer's system is presented by equation (11) while equation (12) represents the total profit obtained from using the general raw material procurement policy.

$$TC_{\text{mfc}}(n, T) = C_{\text{mfg}}D + \frac{A_{\text{mfc}}}{T} + H_{\text{mfc}}I_{\text{mfc}} \quad (11)$$

$$TP_{\text{mfc}} = c_{\text{ret}}D - TC_{\text{mfg}}(m, n, T) - TC_{\text{ret}}(n, T) \quad (12)$$

The last stage is to formulate the mathematical model that accommodates the shelf-life based pricing. The formula for calculating the age of each batch (E_i) as well as the formula for calculating the total profit in retailer or $TP_{\text{ret}}(n, T)$ is similar to [9]. However, since it is assumed in this study that all batches should enter the retailers' warehouse before the customers' WTP declines, $E_i < \tau_{\text{start}}$ is added as a constraint to ensure that the age of all receiving batches is younger than τ_{start} . Equations (13)-(16) represent the total profit of the integrated system.

Maximise:

$$TP(m, n, T) = TP_{\text{ret}}(n, T) + TP_{\text{mfc}}(m, n, T) \quad (13)$$

Subject to:

$$P \geq D; \quad (14)$$

$$E_i < \tau_{\text{start}}; \text{ for } i = 1, 2, \dots, n \quad (15)$$

$$T > 0; \quad (16)$$

4. Numerical example

In this numerical example, model in [9] is used as the benchmark model. Hence, equation (10) in [9] is compared to equation (13) with parameters shown by table 1.

Table 1. Parameters in numerical test

Manufacturer	P	D	k	c_{raw}	A_{raw}	H_{raw}	c_{loss}	c_{mfc}	A_{mfc}	H_{mfc}
Value	19,200	4,800	0.01	5	1000	1	10	5	600	6
Retailer	c_{ret}	H_{ret}	A_{ret}	O_{ret}	τ_{sl}	τ_{start}	p_{max}			
Value	35	7	25	50	0.208	0.167	50			

To determine the controlling variables (m, n, T), a genetic algorithm tool in Matlab 2009 is utilized. The result can be seen in table 2.

Table 2. Total Cost, Revenue and Total Profit of the SVSB System

	Decision Variables		
	m	n	T
Benchmark Model	1	4	0.1921
	Manufacturer	Retailer	Total
Total Cost	130,060	146,050	276,110
Revenue	144,000	239,970	383,970
Total Profit	13,940	93,920	107,860
	Decision Variables		
	m	n	T
Proposed Model	0.2500	3	0.1825
	Manufacturer	Retailer	Total
Total Cost	127,610	145,980	273,590
Revenue	144,000	240,000	384,000
Total Profit	16,390	94,020	110,410

As can be seen from table 2, decision variables obtained from the benchmark model are $m=1$, $n=4$ and $T=0.1921$. It means that during the period of 0.1921, the manufacturer procures raw material once and deliver the finished goods four times to retailers. This method leads to \$107,860 of the total profit.

On the other hand, the proposed model yields \$110,410 of the total profit, in which general raw material procurement is applied. The manufacturer of the latter strategy is allowed to order raw material in a larger quantity to fulfill the demand of four production cycles. In this case, the proposed strategy performs better regarding the total profit (2.3% higher). Further, still from table 2, the total cost in the manufacturer system is reduced from \$130,060 to \$127,610 leading to 17.6% increase in the total profit of the manufacturer applying the proposed strategy. Meanwhile, in the retailer system, the total cost declines slightly leading to 0.1 % increase in the total profit of retailers. Although in overall applying the general procurement strategy is advantageous for all players in the SVSB system, the manufacturer seems benefitted most.

5. Conclusion

In this study, an integrated inventory model for food product applying general raw material procurement has been developed. Unlike the previous model, the procurement policy in this model is more flexible. The manufacturer has more alternatives/strategies, i.e., establishing one, two or more raw material replenishments during one production cycle or ordering one raw material replenishment to support more than one production cycle. Using the given parameter, the developed model has a better performance, i.e., the total profit of the integrated system is higher than that of the current model. The numerical test shows that reducing the ordering frequencies and the length of production cycle potentially diminish the manufacturing and retailer costs. Applying the proposed strategy increases the total profit of either the manufacturer or retailers. However, the numerical test result also reveals that the manufacturer obtains more benefits from this strategy. Therefore, to distribute the advantages among players, a profit-sharing scenario between the manufacturer and retailers is worth to investigate in future research.

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