

Probabilistic framework for product design optimization and risk management

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Abstract. Probabilistic methods have gradually gained ground within engineering practices but currently it is still the industry standard to use deterministic safety margin approaches to dimensioning components and qualitative methods to manage product risks. These methods are suitable for baseline design work but quantitative risk management and product reliability optimization require more advanced predictive approaches. Ample research has been published on how to predict failure probabilities for mechanical components and furthermore to optimize reliability through life cycle cost analysis. This paper reviews the literature for existing methods and tries to harness their best features and simplify the process to be applicable in practical engineering work. Recommended process applies Monte Carlo method on top of load-resistance models to estimate failure probabilities. Furthermore, it adds on existing literature by introducing a practical framework to use probabilistic models in quantitative risk management and product life cycle costs optimization. The main focus is on mechanical failure modes due to the well-developed methods used to predict these types of failures. However, the same framework can be applied on any type of failure mode as long as predictive models can be developed.

1. Introduction

In engineering it still appears to be an industry standard to handle product design with deterministic models and risk management with qualitative tools [1], [2], [3], [4]. This is rather surprising considering quantitative probabilistic methods are widely in use in many other fields such as finance and insurance [5]. Such methods have also been introduced to engineering applications more than 50 years ago [6]. In addition, little research exists to show the actual performance of traditional qualitative risk management methods [5], [7]. On the contrary, literature suggests that even simple quantitative models beat expert judgement at prediction and decision making within many practical applications [8] [9]. While the performance of qualitative risk management methods remains uncertain in the light of scientific research, there is evidence showing the superiority of quantitative methods [5], [10], [11]. Based on the trend seen in other fields and based on the literature on performance comparisons it appears reasonable to work towards implementing probabilistic quantitative methods in engineering practices as well. It is to be noted, however, that these methods should be simple enough to be practical.

The reliability level to which a certain product should be designed, is a common topic of debate. One party often prefers to drive manufacturing costs down, while the other argues for higher reliability. From a financial perspective, this debate boils down to a product life cycle costs optimization. In other words, there should be a reliability level that optimizes the trade-off between manufacturing and quality costs which realize in cases of failure. The idea of optimizing risk exposure in terms of costs within the design is not a new one. The literature introduces four general methods to analyse



engineering decisions: Life Cycle Cost Analysis (LCCA), Utility Theory (UT), Cumulative Prospect Theory (CPT) and Life Profitability Method (LPM) [12]. UT & CPT try to incorporate the risk preferences of the true decision maker into the objective function while LCCA is based on the financial value only [12], [13], [14]. LPM is a version of LCCA, capturing more financial information by covering positive cash flows as well [12]. We sympathize with the idea of trying to incorporate cognitive biases into decision analysis tool to have the optimizer to be in line with a true human decision maker. That makes perfect sense if the purpose of the model is to mimic a human decision maker. However, we concentrate on models which are financially rational, have the potential to differ from actual human opinion and aid in making better decisions. That leaves UT and CPT out of our interest. Another argument against UT, CPT and LPM is their added layer of complexity compared to LCCA. LCCA methods are simpler since they do not require decision maker specific parameters. Decision maker and application specific risk tolerances can be implemented simply as an optimization constraint while keeping the optimization process itself as a constant.

Several LCCA approaches have been suggested [15], [16], [17], [18], [19], [20]. Sørensen goes on to classify cost optimizing types of approaches as category IV reliability methods [21]. This paper introduces an alternative LCCA framework, which pursues a simpler and more practical approach to tackle the reliability optimization in everyday engineering work, while not sacrificing too much prediction accuracy. In order to define the process, we apply well-established engineering practices but also borrow from probability theory, financial models and decision analysis.

This paper provides a framework on which to define risk levels while striving to maximize product cost-effectiveness. This is achieved by applying optimization techniques on financial risk exposure, while considering design and legal requirements as well as environmental and safety risks as constraints. First, the process is performed on component level and then the component results are extended to the system level. The introduced probabilistic design process can be expressed as the steps shown in table 1. The order of chapters on this paper also follows the step numbers in table 1, each chapter delving deeper into these steps. In addition, chapter 7 applies the framework into practice presenting a practical example.

Table 1. Process steps for probabilistic design framework.

Step number	Description of a process step
1	Apply physical or statistical model of the system to predict the probability of failure for component design options
2	Calculate expected life cycle costs for component design options
3	Consider risk management and other constraints to define acceptable design space
4	Choose feasible design with acceptable level of risk and the lowest expected life cycle costs
5	Repeat the process for all the components of the system and finally verify risks on the system level

2. Model of the system

In order to estimate the failure probability of a component, a mathematical model is required. In principle, one can work with failure data to create a statistical failure probability model. However, in many practical applications product volume is too low to derive accurate failure probability estimations for the high reliability components within reasonable period of time. Another downside with the failure statistics is that they cannot predict the reliability of a new significantly different components. Therefore, a physics-based model capable of predicting the failure probability of any design is preferable. Material mechanical failure models are well-developed and quite accurate, therefore we concentrate on them within the scope of this paper. Moreover, if components are under static loading, models can be very simple. However, for many practical applications failure can also be

fatigue related. In principle, any fatigue model capable of estimating fatigue life will do. Nonetheless, the more sophisticated the model the better [22].

Fatigue models are commonly applied in a deterministic manner [4]. That is, an adequate safety margin is required against failure. The safety margin is required because the model and its inputs are not perfectly accurate. Method is valid but, a difficult question arises: what is an adequate safety margin?

Some fatigue standards recommend a minimum allowed safety margin but in practice the accuracy of the inputs is case specific. For example, if the load is known precisely, a lower safety margin against material failure is adequate compared to a scenario where the load is highly uncertain. Deterministic models have no direct way to deal with uncertainty and the indirect way of compensating for uncertainty with a safety margin remains highly subjective.

It is actually the level of uncertainty and the magnitude of safety margin that in combination define the probability of failure. There is a simple solution to dealing with input uncertainty and quantifying failure probability. We can take any commonly applied mathematical material strength model and apply it stochastically instead of using a common deterministic engineering practice. Applying the Monte Carlo method is a practical way to implement the transition to stochastic modelling but requires the sources of uncertainty to be quantified. Table 2 lists the main sources of uncertainty for mechanical failure modes and categorizes them as defined by the Joint Committee of Structural Safety (JCSS) [15]. Based on table 2 many of the listed uncertainty sources are, at least to some extent, related to statistical or model uncertainty, which can be reduced by acquiring more measurement data [4]. Common designation for statistical and model uncertainties is epistemic uncertainty [4]. Epistemic uncertainties can be considered subjective in nature, since they are related to the amount of knowledge or data one possesses. Natural uncertainties, also called as aleatory uncertainties, are objective since they are inherent in nature [4].

Working with completely or partially subjective uncertainty estimates, however, requires one to lean towards a Bayesian view of probabilities. A frequentist can only handle uncertainties based on data, which of course should be the target anyway. In the absence of measurement data, engineering decisions still need to be made. A subjective quantitative estimate of parameter uncertainty remains a more effective option than facing the problem indirectly with deterministic safety margin methods. In the real world, in the presence of epistemic uncertainties, a failure probability estimate is always tied to the knowledge of the designer and the data available, rather than being a factual property of a certain design. We can only pursue objectivity by applying more accurate models, measuring input parameters more rigorously and constantly calibrating our estimates based on field data.

Table 2. Sources of uncertainty.

Description of uncertainty	Uncertainty category
Material strength	Natural, statistical
Geometry	Natural, statistical
Load magnitude and number of cycles	Natural, statistical
Failure point simulation	Model

Acknowledging these facts, however, does not diminish the practical applicability of the approach presented in this paper. From a decision analysis point of view, the source of uncertainty is irrelevant for decision making. All the uncertainties are treated mathematically in a similar manner in any case. [15], [22], [23]

Categorization of risk sources is, nonetheless, important since the higher the contribution of natural variability to total uncertainty, the closer to true failure frequency we can expect our failure probability prediction to land. In other words, if the contribution of model and statistical uncertainties is significant, model predictions are not necessarily in line with the measured failure frequency. This fact highlights the importance of the measurements in minimizing statistical uncertainties. Another

important step to tackle the issue is to apply a calibration loop between the model predictions and the number of experienced failures. Bayesian mathematics can be applied to gradually adjust the model output estimate of failure probability (prior) based on actual failure frequency to always have the best estimate (posterior) in the light of available knowledge. Whilst that is beyond the scope of this paper, see JCSS for an introduction into the topic [16].

It is common practice to measure material strength standard deviation by tensile testing. This is quite expensive and usually some level of statistical uncertainty remains on top of natural strength deviation inherent in all materials. Statistical measurement uncertainty should be taken into account while not relying completely on potentially inaccurate standard deviation provided by tests. While omitted here, it can be shown that uncertainty of uncertainty is mathematically nothing more than a larger uncertainty. In the light of this information, a lack of measurements does not prevent us from getting an estimate of the failure probability. In the absence of measurement data, input value uncertainties are naturally only higher. Probabilistic methods inherently account for parameter uncertainty, which is always case specific.

Geometry variations are defined by tolerances, therefore they can be directly controlled by design. Load uncertainty can be measured by the instrumentation of products or their operating environment. Different operating conditions and operators introduce natural load variability. In addition, naturally some statistical uncertainty remains due to a limited amount of measurement data available. Model uncertainty can be significant as well. Model uncertainty is a residual term that explains the remaining prediction inaccuracy in a component life when all the uncertainties of the model inputs are considered. Figure 1 presents the relationship between safety margin and probability of failure. Only material and load uncertainties are shown in figure 1 and they are represented by Gaussian distributions for the sake of simplicity. Safety margin is defined as a ratio between the most likely values of the material strength and the load. That is true with an assumption that the mathematical model is applied in such a manner that all the inputs are given as expected values and the model is unbiased and calibrated to give the best estimate of failure point of the material [4]. On these conditions, the failure probability can be defined as the area in which these two distributions overlap.

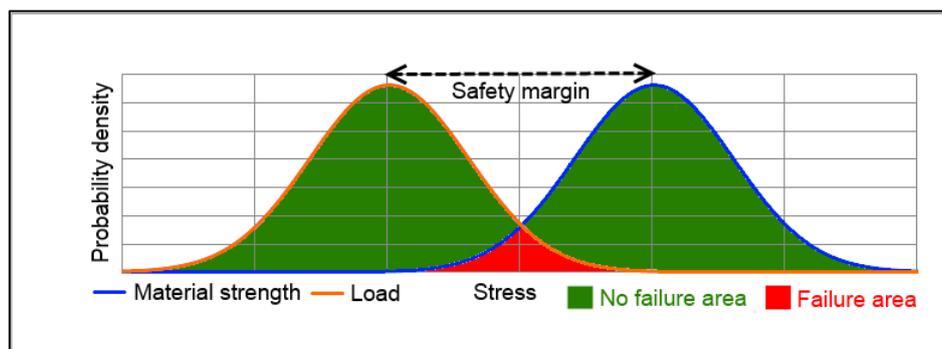


Figure 1. Safety margin and probability of failure.

There is a number of ways to solve this academic problem but we prefer a general method that allows us to define probability distributions freely to represent available measurement data as precisely as possible. We recommend to apply the Monte Carlo method on top of a fatigue model in order to estimate the probability of failure. Figure 2 shows the principle behind the Monte Carlo method simulating the failure probability. In its simplest form, we take all the internal safety margin away from the underlying mathematical model or introduce a correction term to have it estimate the expected failure point [4]. This of course requires deep understanding of the model and the simulated phenomena or a significant amount of data which to use in model calibration. The Monte Carlo process varies input parameters within their probability distributions randomly. The method gives an approximation of failure probability as a ratio between unfavorable outcomes to the total number of iterations. The code for the Monte Carlo method is simple to program and implement on top of any

mathematical strength model the analyst is familiar with. Therefore, it is a perfect fit for various practical engineering problems.

Engineers generally apply finite element method (FEM) to solve the stress of structures. The use of FEM is computationally demanding and therefore to reduce computation time it is preferable to seek for procedures where the Monte Carlo iteration loop is a post-processing step for FEM. Such a procedure can be easily implemented if linear FEM is applied as shown in figure 2. That is, the stress results can be scaled and the superposition principle can be applied to them. Fortunately, practical strength related problems can be often solved with reasonable accuracy by applying linear FEM.

We generally apply FEM to solve the stress distribution on the structure and post-processes the results with the Monte Carlo algorithm to solve the probability of failure. Findley criterion is generally a good choice as a multiaxial fatigue failure criterion for steel materials [24], [25]. However, if the Findley criterion is applied on a massive steel structure, the method inherently contains some internal safety margin against complete breaking of the structure. That is because the criterion does not differentiate between crack initiation and growth phases. Existence of the internal safety margin also depends on the definition of a failure: complete breakage versus the point where significant cracks first appear.

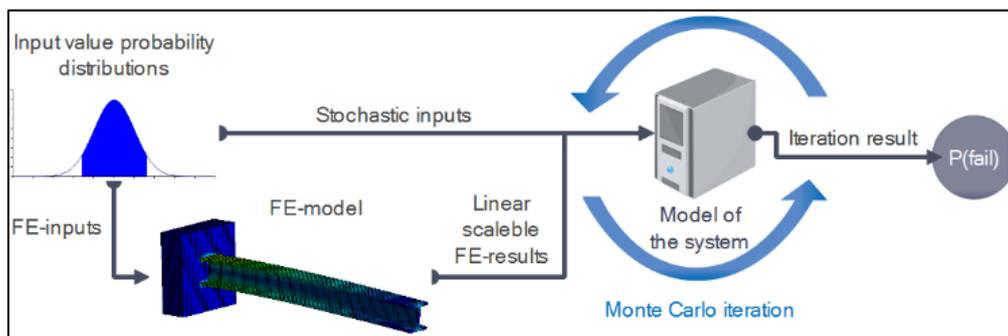


Figure 2. Process flow chart on solving failure probability.

3. Life cycle costs

Capability to solve the failure probability of a component has applications not only on risk management but also on minimizing product life cycle costs. We define life cycle costs as a sum of all the costs experienced during the product lifetime, including manufacturing costs. All the life cycle costs are also discounted to the same time period to be comparable. In efficient markets the product with lower life cycle costs will have a competitive edge against the product with higher life cycle costs, all else being equal. In practice, the competitive edge should realize as a higher sales margin or increased sales of an optimized product. Therefore, the exercise of minimizing life cycle costs can also be seen as a process of maximizing the customer value of a product having certain given features.

3.1. Expected manufacturing costs

By manufacturing costs, we refer to all the costs such as material and work that are due to the manufacturing and design process of a component. Chapter 2 introduced a general methodology to solve failure probability. It is also a straightforward practice to estimate manufacturing costs for a component design based on historical data of manufacturing similar components. Estimation of the two quantities for several design options allows to curve fit a function for expected manufacturing costs $E[M(f)]$. In practice, the function can be discrete but for optimization purposes continuous functions generally behave better. Of course, it is possible to optimize with a continuous function and deal with the discretization at the later phase. Therefore, a simple curve fitting will suffice for all practical purposes. Figure 3 shows an example of such a function fitted by three design points.

When varying any single design detail, $E[M(f)]$ will in many practical cases have a monotonic downward sloping shape. That is, manufacturing costs decrease while the probability of failure increases. This behavior is obvious when considering design features such as the number of bolts on a joint, plate thickness, weld a-dimension, tighter geometry tolerances, etc. Obviously, there is no reason

to consider design details that increase reliability but have no adverse effect on costs. These are no-brainers and optimization techniques are not required. The method should be applied to aid on decision making in the design details that are trade-offs between reliability and manufacturing costs.

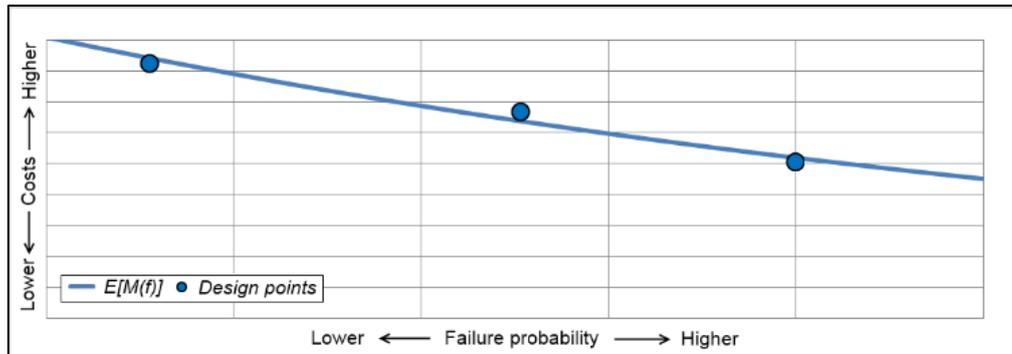


Figure 3. Estimating manufacturing costs function by curve fitting.

3.2. Expected quality costs

In this paper we define quality costs to include all the costs realizing in case of a component failure. These costs include, but are not necessarily limited to repair costs, downtime costs and costs due to brand loss. Gardoni et.al. suggest taking revaluation of an asset and optional insurance coverage into account in objective function. [12]. However, since the use of our method is limited to aiding design related engineering decisions we omit these parameters. On the other hand, Gardoni et.al. proposes a parameter to cover downtime or a revenue loss [12]. Here the optimal decision may depend on the point of view of a decision maker. The owner of the product is interested in the downtime costs while for the manufacturer the brand loss (decrease of future sales due to a bad customer experience) can be more important. In general the cost of repairing a failed component is a function of working time and spare parts. If the failure leads to downtime of a product or a brand loss for the manufacturer, resulting costs can be significant and need to be factored in. Quality costs in general are highly application specific but it should be possible to establish a reasonable estimate if some effort is put into it. Estimating quality costs is nothing more than a quantitative expression of the financial impact of failure, an exercise that is often performed qualitatively for risk matrix definition [26].

Quality costs Q_i only realize in case of failure, therefore we should be interested in expected quality costs which are a product of the probability of failure and the cost of failure. The probability of failure as a quantity needs to be tied to a time period. For many products, one annum is a natural choice for the length of the time period. Annual probability of failure f_i does not need to be constant for every period i , rather it may increase towards the end of product lifetime due to the cumulative nature of fatigue damage. For the sake of simplicity graphs in chapter 3 are plotted as if the annual probability of failure were a constant quantity.

If manufacturing costs occur at time t_0 then quality costs occur at time t_{0+i} , where i is an arbitrary time period number. Therefore, to compare apples to apples, we need to discount the expected quality costs from every failure period to the same time period as expected manufacturing costs. This is a standard practice in finance and can be carried out by applying the discounted cash flow formula [27]

$$E[Q(f)] = \sum_{i=1}^n \frac{f_i * Q_i}{(1+r)^i} \quad (1)$$

for summing the quality cost cash flows over the product lifetime n . From manufacturer's perspective, quantity n on the formula (1) can be considered as warranty time as well. The discount term r is usually an input from the financial department. In general discount term r can be defined as a cost of capital plus stock market real risk premium. For private companies that can be approximated by summing long term credit interest rate and earnings yield. The expected quality costs function $E[Q(f)]$

has an exponentially increasing shape as the probability of failure increases as shown in figure 4. This is in line with intuition, when the probability of failure approaches zero, quality costs should approach zero as well.

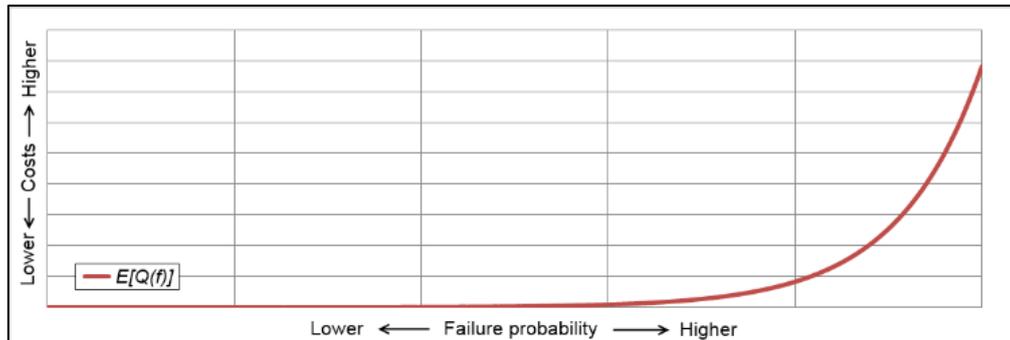


Figure 4. Quality costs function.

3.3. Expected life cycle costs

Following Gardoni et.al. we define the expected life cycle costs $E[L(f)]$ as a sum of expected manufacturing cost, discounted expected quality costs and discounted expected life maintenance costs $E[C]$ according to formula [12]

$$E[L(f)] = E[Q(f)] + E[M(f)] + E[C]. \quad (2)$$

Since $E[C]$ is not a function of failure probability f , it can be set to zero while having no impact on the optimization result in terms of f [20]. $E[L(f)]$ has a parabolic shape as shown in figure 5. The minimum of this parabola is an optimum reliability level, considering manufacturing and quality costs. On a free optimization case this is the optimum reliability level to target in the design.

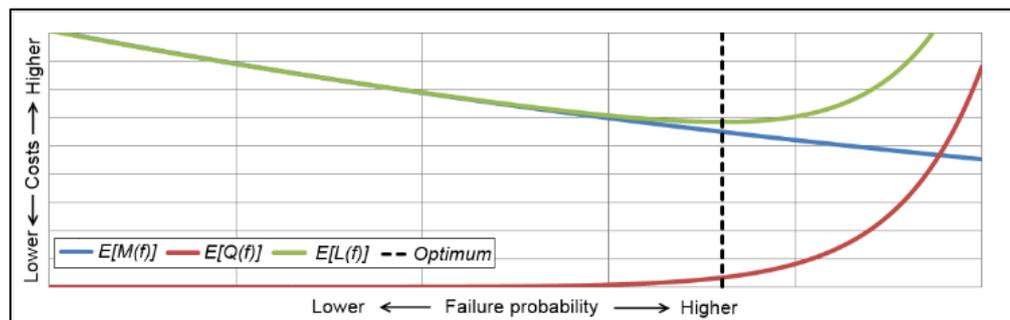


Figure 5. Life cycle costs function.

The optimum is shown in figure 5 as a vertical dashed line. Of course, in many practical engineering applications it is not possible to have a perfectly accurate estimate of $E[L(f)]$. Fortunately, practical experience shows that the optimal reliability level is not highly sensitive to the accuracy of the underlying parameters. That is, the method has been found to be a useful decision analysis tool in practice even in the presence of significant uncertainties. A recommended practice is to run a sensitivity analysis or the Monte Carlo simulation to quantify the confidence on reliability optimization as well. However, within the scope of this paper we treat reliability optimization in a deterministic manner.

4. Risk management and other constraints

As always in case of a practical optimization problem, constraints should be identified. Risk management can introduce a set of constraints to the design process. In addition, any rules and design

guidelines that are considered important, can be introduced as optimization constraints. This is true even if the constraints are not a function of failure probability or directly related to the objective function i.e. life cycle costs function [15]. Quite often, there are legal constraints such as standards that need to be followed or classification society rules that need to be fulfilled. Taking the constraints into account is straightforward. Just limit feasible design space by excluding the design options that are outside of the chosen constraints.

There are several approaches to deal with risk but we introduce only the four more common ones. The first and the simplest is a risk neutral approach. The risk neutral decision maker does not care about risk, but only tries to maximize return [12]. In our application that would be equal to minimizing life cycle costs. In theory this is a valid strategy but not commonly applied in practice due risk aversion typical for most decision makers [9], [12], [28]. The second possible strategy is to define an acceptable level of risk and optimize only within this constraint [19]. This is a common approach for risk averse decision makers. The third option is to form a risk corrected return measure which quantifies how many units of return is achieved per unit of risk [29]. The third approach is common for risk seekers and in general in the field of finance. The fourth option is to build risk preferences into objective function [13], [12]. In this paper we choose to use the second approach due to its simplicity and conservativeness.

There are also three main categories of risks to consider when designing a product: financial, environmental and safety. Following recommendations given by JCSS, tangible and intangible risks are clearly separated [15]. Tangible i.e. financial risks are part of the reliability level optimization introduced in chapter 3. Therefore, depending on the risk profile, one may not want to add additional risk management criteria for financial risks. It is possible, at least qualitatively, to estimate consequences of failure to environment and safety of the people involved. Yet it is suggested to quantify intangible risks too whenever possible, since that allows the exploitation of generally accepted risk levels [30], [31]. Estimation of consequences of failure enables the definition of tolerable component failure probabilities. These tolerable risk management limits work as a constraint on the life cycle costs optimization.

As a practical note, we consider design requirements such as manufacturability, stiffness, vibrations, stability, noise, flow characteristics and classification society rules as deterministic constraints for the optimization problem. Risk management is naturally another layer of constraints.

5. Optimal component design

Chapter 2 gave a framework to predict failure probabilities for design options, chapter 3 showed how to estimate life cycle costs function and finally chapter 4 introduced risk management constraints for the design space. Figure 6 sums it all up in one graph. The feasible design space is shown on the white background after considering shaded area of constraints.

From this point on it is a simple exercise to minimize the $E[L(f)]$ function to find a global optimum for the reliability level $(1-f)$. In figure 6 the vertical dashed line, which represents the optimum reliability level, is now shifted towards left due to the chosen constraints. We also chose to run the optimization with a continuous function but the actual design options are often discrete. In that case, naturally the closest feasible design option to the optimum should be chosen.

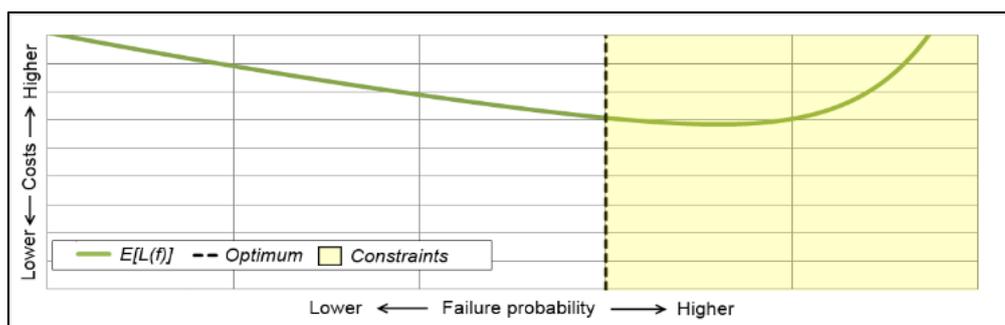


Figure 6. Life cycle costs function with constraints.

6. System level

Chapters 3-5 introduced a framework to optimize on a component level. Component level life cycle cost optimization should be adequate for the product as a whole as well. This is not necessarily so from the risk perspective. If the failure probabilities of individual components are less than perfectly correlated, the failure probability of a system will always be higher than the highest individual component failure probability. Therefore, for risk management reasons it is important to consider the system level failure probability as well.

If we let all the component level failure probabilities f_k be independent, the system level probability of failure f_s boils down to a simple sum product equation [32]

$$f_s = 1 - \prod_{k=1}^x (1 - f_k), \quad (3)$$

x being the number of components. Equation (3) can also be applied if one prefers a simple and conservative approach to risk management. In practice, component level failure probabilities are usually correlated. In the case of mechanical failure modes, the whole system often experiences the same load. If different components are dimensioned with the same failure criteria, model uncertainty may bias component failure probabilities in a mutual direction. On the other hand, the natural part of the material strength uncertainty is fundamentally uncorrelated. That is because material strength variation is mainly a local phenomenon due to impurities and micro cracks. Therefore, case specific component level correlation coefficients can be estimated with a reasonable accuracy based on quantified sources of uncertainty.

On the other end of the spectrum, in a perfectly correlated system, the product level failure probability equals to the highest component failure probability. The reality is usually between the two extremes, even negative correlation coefficients are conceivable. Thoft-Christensen et.al. present an analytical equation to solve the system level probability of failure for any correlation coefficient [33]. In practical engineering applications, the system level probability of failure for less than perfectly correlated components, can also be solved with the Monte Carlo algorithm.

7. Application example

This chapter applies the introduced framework into a simple example. Let us consider a high strength steel rod with a solid circular cross-section under fully reversed constant amplitude uniaxial uniform stress. The rod is a part of system that has the design life of 10 years. The goal is to optimize the rod diameter in terms of life cycle costs within the risk management constraints. The structure is assumed to be completely fatigue strength limited having no other limiting failure modes such as buckling or vibrations. To avoid separating crack initiation and growth phases, we define the failure as a point where significant cracks first appear. For this simplified example we also neglect fatigue strength correction factors such as size and surface quality. With these assumptions we can directly compare stress amplitude to a SN-curve derived from fatigue tensile tests. Moreover, the failure criteria is $\sigma_S > \sigma_A$, where σ_S is the fatigue strength taken from the SN-curve and σ_A is the stress amplitude in the plate. Therefore, the safety factor is defined simply as $S = \frac{\sigma_A}{\sigma_S}$. Chosen failure model is unbiased but we assume some model uncertainty in terms of failure point estimation. We apply Matlab based Monte Carlo code with one million iterations following the process shown in figure 2 to solve the failure probability estimates. The stress amplitude and the material fatigue strength are assumed to be uncertain but constant over the design life. The annual number of cycles is uncertain and independent for every year over the design life. FEM is not required in this case since the stress amplitude is simply defined by formula

$$\sigma_A = \frac{4 \cdot F_d}{\pi \cdot D^2}. \quad (4)$$

Table 3 lists the required input parameters for this probabilistic product design optimization problem. We use two types of probability distributions for input parameters; Gaussian distribution and constant probability distribution. Applied SN-curve is a linear interpolation between σ_{S1e2} and σ_{S1e7} in the log-

log scale. We assume that component manufacturing costs consists completely of material costs. Cost of failure is also known and constant over time. The expected life cycle cost function $E[L(f)]$ is calculated separately for low and high uncertainty environments (HUE and LUE). These two environments can be seen as scenarios of different level of available measurement data. The expected life maintenance costs $E[C]$ are neglected in a similar manner as explained in chapter 3.

Table 3. Input parameters for the application example.

Quantity	Symbol	Expected value	HUE deviation	LUE deviation
Fatigue strength 1e7	σ_{S1e7}	500MPa	SD 50MPa	SD 25MPa
Fatigue strength 1e2	σ_{S1e2}	1 000MPa	SD 50MPa	SD 25MPa
Load amplitude	F_a	2 000kN	SD 500kN	SD 100kN
Annual number of cycles	N	5e5	$\pm 4e5$	$\pm 1e5$
Model uncertainty	M	-	± 50 MPa	± 10 MPa
Discount term	r	7%	0	0
Material density	ρ	7850kg/m ²	0	0
Rod diameter	D	75-110mm	0	0
Rod length	L	1000mm	0	0
Material cost	C	10€/kg	0	0
Cost of failure	Q	1 0000€	0	0
P(f) constraint	$P(f)a$	1%	0	0

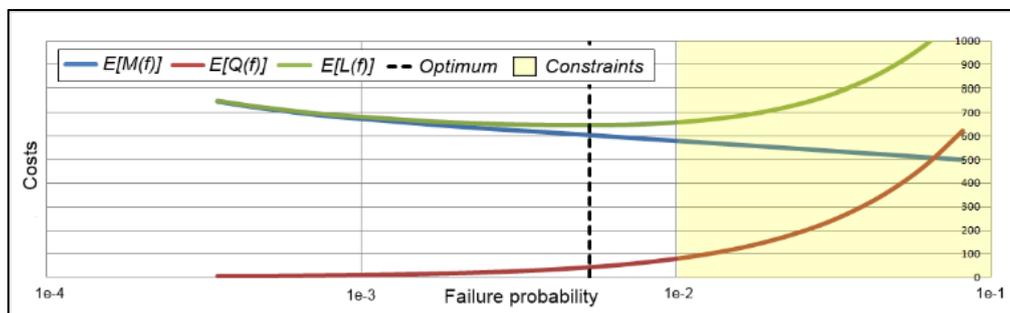


Figure 7. Life cycle costs function for HUE.

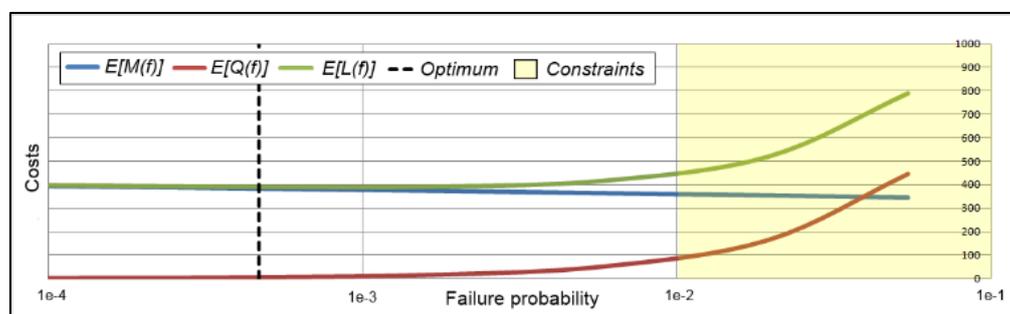


Figure 8. Life cycle costs function for LUE.

Figures 7 and 8 show optimization space for HUE and LUE. As expected $E[M(f)]$ has a downward slope, $E[Q(f)]$ has an upward slope while $E[L(f)]$ as a sum of the two has a parabolic shape. The Matlab code to calculate presented results is shown in the appendix. The quality costs are discounted to present value according to equation (1) with an annual time step.

The failure probability on x-axis represents the cumulative probability over 10 year design life. The reliability constraint does not limit the optimization problem in either of the cases. In case of LUE it is obviously more cost effective to design for the higher reliability level compared to HUE. LUE also gives significantly lower expected life cycle costs in the optimum reliability level compared to HUE case. This difference is an indication of the financial value of having more data in the design phase. The optimal rod diameters are 99 mm and 79 mm for HUE and LUE respectively. Furthermore, the optimal design safety factor for HUE is 2.0, while it is only 1.3 for LUE. Clearly, the conventional method of dimensioning against the safety factor does not properly capture the uncertainty, the risk level nor the cost aspects of the design. On the other hand, the probabilistic framework requires a bit more engineering effort. In the cases of expensive structures or high product volumes the cost of additional engineering work is clearly exceeded by the cost benefits of the optimal design.

Many structural engineering problems are of course significantly more complex. FEM is required to solve the stress level due to complicated geometry. There are usually many load cases and the failure criterion needs to account for multiaxial stress state. Also, significant effort is often required to estimate the expected quality costs. Although the example above is highly simplified, the approach itself is quite general and can be applied to the assessment of more complicated engineering problems as well.

8. Conclusion

Based on our literature review, quantitative probabilistic decision analysis methods appear to outperform standard deterministic and qualitative methods. We introduced a new quantitative probabilistic framework for designing safe and more cost-effective products. The presented approach is a significant upgrade on the commonly applied safety margin methods. The method is at its strongest as a decision making tool applied on design details that are trade-offs between reliability and manufacturing costs. There are other approaches that pursue the same goal, but the one presented here is shaped towards simplicity and practicality on actual engineering work. The framework has also practical applications beyond the ones presented in this paper. One of these applications is a process of quantifying added financial value of measurements. Another interesting use case is the exercise of estimating the value of extended warranty time. These applications were not considered here but are reserved for the future work.

9. Appendix

%MATLAB CODE TO SOLVE FAILURE PROBABILITY AND LIFE CYCLE COSTS FOR HUE

```
%INPUTS
Ni = 1e3; %number of iterations
Ny = 10; %number of years
C = 10; %material costs €/kg
rho = 7850; %density kg/m^3
Qf = 10000; %cost of failure kg/m^3
r = 0.07; %discount term
Lr = 1000; %length [mm]
Dmin = 90; %diameter min[mm]
Dmax = 110; %diameter max[mm]
Dstep = 1; %diameter step[mm]
Favg = 2000000; %AVG load [N]
Fsd = 500000; %SD load [N]
Navg = 5e5; %AVG annual cycles
Ndev = 4e5; %+/- DEV annual cycles constant distribution deviation
FATsd = 50; %SD fatigue strength [MPa]
Mdev = 50; %model deviation %+/- constant distribution deviation [MPa]
Nsn = [1e2 1e7]; %SN-curve cycles
Ssn = [1000 500]; % [MPa] SN-curve strength

%cycles distribution
Nmin = Navg-Ndev;
Nmax = Navg+Ndev;

%SN curve function
SN = @(x) 10 ^ ((log10(Ssn(1)) - log10(Ssn(2))) / (log10(Nsn(1)) - log10(Nsn(2))) * (log10(x) - log10(Nsn(2))) + log10(Ssn(2)));

%iterate over all diameters
Dnum = 0;
for D = Dmin:Dstep:Dmax
    Dnum = Dnum + 1;
    F(Dnum, 1) = D;
    F(Dnum, 2:end) = 0;
    A = pi*(D/2)^2; %stress area
    M(Dnum) = C * rho * A/1e6 * Lr/1000; %material costs
```

```

%interpolate fatigue strength based on SN-curve
FAT = SN(Navg*Ny);
SfDEt(Dnum) = FAT / (Favg / A); %safety factor

%initial values of F
for year = 1:1:Ny
F(Dnum, year+1) = 0;
end

%monte carlo iteration
for MC = 1:1:Ni
Ntot = 0;
Merr = (Mdev * 2) * rand(1) - Mdev;

%Stress
Savg = Favg / A;
Ssd = Fsd / A;
S = randn(1) * Ssd + Savg;

%random component of FAT strength
FATrand = randn(1) * FATsd;

for year = 1:1:Ny

%FAT strength for the year
Na = (Nmax - Nmin) * rand(1) + Nmin;
Ntot = Ntot + Na;
FAT = SN(Ntot) + FATrand + Merr;

%count the number of failures
Sf = FAT / S;
if Sf < 1
F(Dnum, year+1) = F(Dnum, year+1) + 1;
for year2 = year+1:1:Ny
F(Dnum, year2+1) = F(Dnum, year2+1) + 1;
end
year = Ny;
end

%calculate quality costs
if year > 1
Q(Dnum) = Q(Dnum) + (Qf * (F(Dnum, year)/Ni - F(Dnum, year-1)/Ni)) / (1+r)^year;
else
Q(Dnum) = Qf * F(Dnum, year)/Ni / (1+r)^year;
end

end
L(Dnum) = M(Dnum) + Q(Dnum); %life cycle costs
end

%failure probability function
F(Dnum, 2:end) = F(Dnum, 2:end) / Ni;
Pf(Dnum) = F(Dnum, Ny);
end

semilogx(Pf,M,Pf,Q,Pf,L) %plot functions

```

10. References

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