

Statistical Bayesian method for reliability evaluation based on ADT data

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Abstract. Accelerated degradation testing (ADT) is frequently conducted in the laboratory to predict the products' reliability under normal operating conditions. Two kinds of methods, degradation path models and stochastic process models, are utilized to analyze degradation data and the latter one is the most popular method. However, some limitations like imprecise solution process and estimation result of degradation ratio still exist, which may affect the accuracy of the acceleration model and the extrapolation value. Moreover, the conducted solution of this problem, Bayesian method, lose key information when unifying the degradation data. In this paper, a new data processing and parameter inference method based on Bayesian method is proposed to handle degradation data and solve the problems above. First, Wiener process and acceleration model is chosen; Second, the initial values of degradation model and parameters of prior and posterior distribution under each level is calculated with updating and iteration of estimation values; Third, the lifetime and reliability values are estimated on the basis of the estimation parameters; Finally, a case study is provided to demonstrate the validity of the proposed method. The results illustrate that the proposed method is quite effective and accuracy in estimating the lifetime and reliability of a product.

1. Introduction

For today's highly reliable products, it is difficult to estimate their field reliabilities due to the extremely long test time under field conditions. Therefore, ADT is frequently conducted in the laboratory for predicting the products' reliability under working conditions. The main idea of ADT is to test products under harsh environmental stresses to accelerate the performance degradation on the condition of constant failure mechanism and obtain sufficient data for reliability analysis in a short time, lower costs and higher efficiency. So it has many applications, such as the reliability analyses of liquid-crystal display (LCD) [1], light-emitting diode (LED) [2] and batteries [3][4].

The core method of ADT-based reliability estimation is to find a suitable degradation model which is capable of incorporating the relationships between the applied stresses and the product's degradation [5]. The existing degradation analysis methods can be categorized into two broad classes [6], degradation path models and stochastic process models which are widely used in different combination and some extension.

Due to the stochastic process models can describe the temporal variation of the degradation process in a finite time interval, they have gained more attention from researchers, especially the typical ones like Wiener process, Gamma process [7][8]. Because Wiener process has a higher generalization and a strong ability to describe a nonlinear degradation process. It gets more intensive applications. Moreover, products in practice, their degradation increment in an infinitesimal time interval can be



viewed as an additive superposition of a large number of small external effects, while Wiener process with its normally distributed increment can perfectly satisfy this property [6].

However, there are still some limitations need to consider when adopting Wiener process in ADT. Usually, when combining Wiener process and acceleration model to estimate the degradation process, the parameters are hard to calculate. Traditional estimation method fit degradation ratio with least square method and maximum likelihood estimation algorithm. But the solution process and the estimation result of degradation ratio are imprecise which may affect the accuracy of the acceleration model and the extrapolation value. The Bayesian method is proposed to solve this problem for its great capability of data fusion and the ability of parameter inference. In literature, Wang [9] combine Bayesian inference Gamma process with ADT to predict lifetime. Xu [10] propose a fully Bayesian approach to deduce parameters in SSALT. The data processing in these literatures always follows the same pattern which is to unify the degradation data from different stress level into the unified data at the working stress level before estimation. The accuracy of degradation data and some key information may lose during the processing which may also affect the veracity.

In this paper, a new data processing and parameter inference method based on Bayesian method is proposed to handle degradation data. The main idea of this method is to utilize the estimation values and processed data of previous stress level to contribute the estimation under next stress level. It can maximize the utilization of test information to gain the best result through update and iteration. It can also eliminate the deviation effect caused by data processing and improve the estimation accuracy. This method can be a useful tool to apply in ADT design optimization.

The remainder of this paper is organized as follows. In Section 2, general degradation model based on Wiener process is introduced, and the reliability function is deduced. In Section 3, first, the initial value of degradation model is estimated, then the parameters of prior and posterior distribution under each stress level are obtained with data processing under every stress level. Finally, the reliability estimation value is calculated on the basis of the estimation values and parameters. In section 4, a case study is provided to demonstrate the validity of the proposed parameter estimation method as well as the reliability estimation method. Some conclusions were drawn in Section 5.

2. Degradation model based on wiener process

We assume that the degradation follows a Wiener process, then the degradation is modeled by the following

$$Y(t) = \sigma B(t) + d(s) \cdot t + y_0 \quad (1)$$

Where $Y(t)$ is the performance degradation process of product, $B(t)$ is the standard Brownian motion with the mean value of 0 and the variance of t , denoted as $B(t) \sim N(0, t)$, σ is the diffusion coefficient, which is a constant and does not change with stress or time, y_0 is a known initial value of product performance, and $d(s)$ is the drift coefficient, which represents the degradation rate of product and is a function of stress. It can be used to combine degradation model and acceleration model. Suppose the acceleration model of degradation rate is given by

$$d(s) = \exp[A + B\varphi(s)] \quad (2)$$

Where $\varphi(s)$ is a given functions of stress factor s . then $d(s)$ can be calculated by factor A and B and the relationship between stress and degradation data, the acceleration model, can also be built.

Known from the property of the Wiener process, the degradation increment ΔY during the unit time Δt follows a normal distribution with the mean of $d(s) \cdot \Delta t$ and the variance of $\sigma^2 \Delta t$ i.e.,

$$\Delta Y \sim N(d(s) \cdot \Delta t, \sigma^2 \Delta t) \quad (3)$$

Let k be the failure threshold and the product fails when $Y(t) - k < 0$, then the first passage time t to the threshold k has an inverse Gaussian distribution. The probability density function of the first passage time t is

$$f(t; y_0, k) = \frac{k - y_0}{\sigma \sqrt{2\pi t^3}} \exp\left\{-\frac{[(k - y_0) - d(s) \cdot t]^2}{2\sigma^2 t}\right\} \quad (4)$$

Then the reliability function of t is given by

$$R(t) = \Phi \left[\frac{l - y_0 - d(s)t}{\sigma\sqrt{t}} \right] - \exp \left(\frac{2d(s)(l - y_0)}{\sigma^2} \right) \Phi \left[\frac{l - y_0 + d(s)t}{\sigma\sqrt{t}} \right] \quad (5)$$

From the function above, we know that the key parameters for reliability prediction are $d(s)$ and σ^2 . After these parameters are obtained, the reliability prediction can be made.

3. Parameter and reliability estimation based on Bayesian method

3.1. Estimation of the initial value of A , B

We provide the method for estimating the unknown parameters in step stress accelerated degradation testing (SSADT). Suppose a set of stresses is $S = (s_1, s_2 \dots s_m)$ where m ($m > 2$) is the number of stress levels, while the number of the degradation increment data under the each stress level is $\Delta Y_i = (\Delta y_{i1}, \Delta y_{i2}, \dots \Delta y_{in_i})$, where $i = 1 \dots m$. Then the Linear Regression Equation can be easily obtained by (1) and (2).

$$E(y_i(t)) = \exp(A + B\varphi(s_i)) \cdot t + y_0 \quad (6)$$

$$\ln(E(\Delta y_i)) = B\varphi(s_i) + A + \ln(\Delta t) \quad (7)$$

By fitting the degradation increment data Δy_i under the each stress level s_i with (7), the initial value of A , B , which is A_0 and B_0 can be obtained. In addition, when the degradation data is hard to collect, A_0 and B_0 can be estimated through the degradation history or engineer experiences and then updated with real time test data in the following test process.

3.2. Data processing under s_1

The main idea of Bayesian method is to specify some prior probability, which is then updated to a posterior probability in the light of new, relevant data. That means the estimation under the next stress level can be contributed by the previous level. So, data under stress level s_1 should be handled first.

Known from the property of (3) and conjugate prior distribution theory that the product between the drift coefficient $d(s)$ and time interval Δt follows the normal distribution and the product between the diffusion coefficient σ^2 and time interval Δt follows the inverse gamma distribution. Which is

$$d(s) \cdot \Delta t \sim N(\mu, \tau^2) \quad (8)$$

$$\sigma^2 \cdot \Delta t \sim IG(b, a) \quad (9)$$

Where μ , τ are the mean value and the standard deviation of normal distribution, while a , b are the scale and shape parameter of gamma distribution.

In terms of data $\Delta Y_1 = (\Delta y_{11}, \Delta y_{12}, \dots \Delta y_{1n_1})$ which is under stress level s_1 , the initial value of hyper parameters in (8) and (9) can be calculated. Known from the property of conjugate prior distribution, the calculation formula is given by

$$a_1 = \frac{1}{2} \sum_{j=1}^{n_1} (\Delta y_{1j} - \overline{\Delta Y_1})^2 \quad (10)$$

$$b_1 = \frac{n_1}{2} \quad (11)$$

$$\mu_1 = \frac{1}{n_1} \sum_{j=1}^{n_1} \Delta y_{1j} \quad (12)$$

$$\tau_1^2 = \frac{1}{n_1} \cdot \frac{1}{n_1 - 1} \sum_{j=1}^{n_1} (\Delta y_{1j} - \mu_1)^2 \quad (13)$$

Where $\overline{\Delta Y_1}$ the mean of data under is s_1 , n_1 is the number of the data, and a_1 , b_1 are the scale and shape parameter of gamma distribution (9) while μ_1 , τ_1^2 are the mean value and the standard deviation of normal distribution (8). Then known from the property of normal distribution the estimation value of $d(s_1) \cdot \Delta t$ under s_1 is

$$E(d(s_1) \cdot \Delta t) = u_1 \quad (14)$$

3.3. Prior and posterior distribution under s_2

Known from (2) and the initial value of A , B , which is A_0 and B_0 , for s_i and s_j the specific value of acceleration model is

$$p_{ij} = d(s_i)/d(s_j) = \exp(-B_0 \cdot (1/s_i - 1/s_j)) \quad (15)$$

In terms of (8) one already know that the product between the drift coefficient and time interval under s_j follows normal distribution, i.e.

$$d(s_j) \cdot \Delta t \sim N(\mu_j, \tau_j^2) \quad (16)$$

Then on the basis of normal distribution one can obtain

$$d(s_i) \cdot \Delta t = p_{ij} \cdot d(s_j) \cdot \Delta t \sim N(p_{ij} \cdot \mu_j, p_{ij}^2 \cdot \tau_j^2) \quad (17)$$

Based on the property of Wiener process that σ is the diffusion coefficient, which is a constant and does not change with stress or time, so the distribution of $\sigma^2 \Delta t$ does not change as well. Then after the specific value is conducted, the hyper parameters calculated under s_1 can convert to prior distribution of parameter under s_2 with

$$a_{20} = a_1 \quad (18)$$

$$b_{20} = b_1 \quad (19)$$

$$u_{20} = p_{21} \cdot u_1 \quad (20)$$

$$\tau_{20}^2 = p_{21}^2 \cdot \tau_1^2 \quad (21)$$

Where μ_{20} , τ_{20}^2 are the mean value and the standard deviation of the prior distribution under s_2 , which is a normal distribution, while a_{20} , b_{20} are the scale and shape parameter of the prior distribution under s_2 , which is a gamma distribution.

As discussed above, the product between the drift coefficient and time interval still follows the normal distribution and the product between the diffusion coefficient and time interval follows the inverse gamma distribution as well. So the hyper parameter of posterior distribution is given by

$$a_2 = a_{20} + \frac{n_2}{2} \left(\frac{1}{n_2} \sum_{j=1}^{n_2} (\Delta y_{2j} - \overline{\Delta Y_2})^2 \right) + \frac{n_2}{2} \frac{(\overline{\Delta Y_2} - p_{21} \cdot \overline{\Delta Y_1})^2}{n_2/n_1 + 1} \quad (22)$$

$$b_2 = b_{20} + \frac{n_2}{2} \quad (23)$$

Where a_2 , b_2 are the scale and shape parameter of the posterior distribution under s_2 , n_2 is the number of degradation data, and $\overline{\Delta Y_2}$ is its mean value.

Then the estimation value of $\sigma^2 \Delta t$ is

$$E(\sigma^2 \cdot \Delta t | \Delta y_2) = \frac{a_2}{b_2 - 1} \quad (24)$$

Next, the mean value and the standard deviation of the posterior distribution under s_2 , which is μ_2 , τ_2^2 can be expressed as

$$u_2 = \frac{\overline{\Delta y_2} / E(\sigma^2 \cdot \Delta t) + u_{20} \tau_{20}^{-2}}{1 / E(\sigma^2 \cdot \Delta t) + \tau_{20}^{-2}} \quad (25)$$

$$\tau_2^2 = \frac{1}{1 / E(\sigma^2 \cdot \Delta t) + \tau_{20}^{-2}} \quad (26)$$

So the estimation value of $d(s_2) \cdot \Delta t$ can be given by

$$E(d(s_2) \cdot \Delta t) = u_2 \quad (27)$$

3.4. Prior and posterior distribution under s_i

The Prior and posterior distribution of s_i can be calculated the same as s_2 , so the solving process is established. Suppose the hyper parameters of posterior distribution under s_{i-1} are a_{i-1} , b_{i-1} , μ_{i-1} , τ_{i-1}^2 respectively. Then A_0 and B_0 can be updated by (7), and by further calculating with (15)(18)(19)(20)(21) the scale and shape parameter of gamma distribution, α_{i0} and b_{i0} , as well as the mean value and the standard deviation of normal distribution, μ_{i0} and τ_{i0}^2 , can be obtained.

Finally, the hyper parameters of posterior distribution can be given on the basis of the hyper parameters of prior distribution, which are

$$a_i = \alpha_{i0} + \frac{n_i}{2} \left(\frac{1}{n_i} \sum_{j=1}^{n_i} (\Delta y_{ij} - \overline{\Delta Y}_i)^2 \right) + \frac{n_i (\overline{\Delta Y}_i - p_{i-1} \cdot \overline{\Delta Y}_{i-1})^2}{2 n_i / n_{i-1} + 1} \quad (28)$$

$$b_i = b_{i0} + \frac{n_i}{2} \quad (29)$$

$$E(\sigma^2 \cdot \Delta t | \Delta y_i) = \frac{a_i}{b_i - 1} \quad (30)$$

$$u_i = \frac{\overline{\Delta y}_i / E(\sigma^2 \cdot \Delta t | \Delta y_i) + u_{i0} \tau_{i0}^{-2}}{1 / E(\sigma^2 \cdot \Delta t | \Delta y_i) + \tau_{i0}^{-2}} \quad (31)$$

$$\tau_i^2 = \frac{1}{1 / E(\sigma^2 \cdot \Delta t | \Delta y_i) + \tau_{i0}^{-2}} \quad (32)$$

$$E(d(s_i) \cdot \Delta t) = u_i \quad (33)$$

Where $\overline{\Delta Y}_{i-1}$, $\overline{\Delta Y}_i$ are the mean value under s_{i-1} , s_i , respectively while n_{i-1} , n_i are the number of degradation data. μ_i , τ_i^2 are the mean value and the standard deviation of the posterior distribution under s_i , while a_i , b_i are the scale and shape parameter of the posterior distribution under s_i . So, with the calculation above, the hyper parameters of posterior distribution under s_i and other parameters are fully obtained.

3.5. Reliability estimation

According to the parameter estimation method above, m estimation values of product between the drift coefficient and time interval under m stress levels are gained. It can be written as $U = (E(d(s_1) \cdot \Delta t), E(d(s_2) \cdot \Delta t), \dots, E(d(s_m) \cdot \Delta t))$. Then (2) can be transformed into

$$E(d(s_i) \cdot \Delta t) = \exp[A + B\varphi(s_i)] \cdot \Delta t \quad (34)$$

Next, from (34) the final value of A , B which is \hat{A} and \hat{B} , can be obtained by fitting with the least-squares method, next the drift coefficient can be expressed by plugging working stress s_0 in (5) and the estimation value of σ^2 under the last stress level can be marked as final estimation value $\hat{\sigma}^2$. Finally, the reliability estimation value of product can be obtained.

4. Case study

In this section, we demonstrate the proposed model and the inferential method based on a numerical example. This example is used to demonstrate the validity and superiority of the proposed parameter estimation method. Suppose there is a temperature step-stress ADT applied to a product, where there are 4 test units and 4 temperature levels (60 °C, 80 °C, 100 °C, and 120 °C). The time of each level is 1250, 750, 500 and 500 h, and the inspection time interval is 5 h.

4.1. Degradation and acceleration model

As the stress factor is temperature, the Arrhenius acceleration function is applicable, so

$$d(T) = \exp[A + B/T] \quad (35)$$

Where T is the absolute temperature, A and B are the parameters. Suppose the degradation follows Winner process, then known from the discussion of section 2, the degradation increment ΔY during the unit time Δt is subject to a normal distribution with the mean of $d(s) \cdot \Delta t$ and the variance of $\sigma^2 \cdot \Delta t$,

which is (3) and the reliability model is (5), Let the initial value of y_0 be 100 and the failure threshold l be 40.

4.2. Estimation of the initial value of A, B

Based on the degradation data gathered from each temperature level (60 °C, 80 °C, 100 °C, and 120 °C), the degradation increment data $\Delta Y_{60}, \Delta Y_{80}, \Delta Y_{100}, \Delta Y_{120}$ are obtained, respectively. Then (7) is applied to fit the degradation increment data. So the initial value of A, B , which is A_0 and B_0 is calculated.

4.3. Prior and posterior distribution under each temperature level

4 temperature levels are applied to the product, so let 60 °C be the initial stress, the calculation sequence is 60 °C, 80 °C, 100 °C and 120 °C.

4.3.1. Data processing under 60 °C

Known from the property of conjugate prior distribution, (8) and (9) are chosen to be the form of the prior and posterior distribution of the product between the drift coefficient and time interval and the product between the diffusion coefficient and time interval.

Then the hyper parameters calculated with (10)~(13) are shown in Table 1.

Table 1. Hyper Parameters of Distribution under 60 °C

a_1	b_1	μ_1	τ_1^2
0.3284	100	0.0239	1.6501e-5

4.3.2. Prior and posterior distribution under 80 °C

The specific value of the drift coefficient under 80 °C and 60 °C can be known from(15), which is $p_{2,1}=2.2361$. Then by combining (18)~(21) the hyper parameters of prior distribution under 80 °C are calculated and shown in Table 2.

Table 2. The Hyper Parameters of Prior Distribution under 80 °C

a_{20}	b_{20}	μ_{20}	τ_{20}^2
0.3284	100	0.0534	8.2502e-5

Then based on degradation increment data, the hyper parameters of prior distribution and (22)(23)(25) (26), the hyper parameters of posterior distribution under 80 °C are gained and shown in Table 3.

Table 3. The Hyper Parameters of Posterior Distribution under 80 °C

a_2	b_2	μ_2	τ_2^2
0.4918	160	0.0535	8.0359e-5

4.3.3. Prior and posterior distribution under 100 °C and 120 °C

As the discussion in section 3, the Prior and posterior distribution under 100 °C and 120 °C as well as the parameter estimation values under every temperature levels are known from (15), (18)~(21), (28)~(33). The estimation values are shown in Table 4.

Table 4. The Parameter Estimation Values under Every Temperature Levels

	60°C	80°C	100°C	120°C
$d(T) \cdot \Delta t$	0.0239	0.0535	0.1098	0.2112
$\sigma^2 \cdot \Delta t$	0.0033	0.0031	0.0033	0.0032

4.3.4. Reliability estimation

From the estimation values in Table 4 and (34) the final value of A, B which is \hat{A} and \hat{B} are obtained, that is $\hat{A} = 4824.1$ and $\hat{B} = 92274.2$. Next, the drift coefficient under 25 °C is extrapolated with \hat{A} and

\hat{B} , that is $d(60\text{ }^\circ\text{C})= 8.7164\text{e-}4$. The estimation value of σ can also get from the estimation value of $\sigma^2 \Delta t$ under $120\text{ }^\circ\text{C}$. Finally, the reliability estimation value of product is obtained by plugging $d(60\text{ }^\circ\text{C})$ and σ in(5), which is shown in Table 5.

Table 5. The Estimation Value of Reliability and Lifetime

	3 year	5 year	7 year	10 year
R(T)	1	1	0.8384	0.0127

5. Conclusion

- 1) This paper applies Bayesian method to handle degradation data obtained from ADT with a new data processing and parameter inference method.
- 2) The estimation values and processed data of previous stress level can contribute the estimation under next stress level with updating and iteration to maximize the utilization of test information to gain the best result.
- 3) The method proposed can eliminate the deviation effect caused by data processing and improve the estimation accuracy and be a useful tool to apply in ADT design optimization.
- 4) A case study is provided to demonstrate the validity and superiority of the proposed parameter estimation method as well as the reliability estimation method.
- 5) This paper only considers product which subject single stress, products follow the multiple failure mode would be studied in future.

6. References

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