

Multistage degradation modeling for BLDC motor based on Wiener process

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Abstract. Brushless DC motors are widely used, and their working temperatures, regarding as degradation processes, are nonlinear and multistage. It is necessary to establish a nonlinear degradation model. In this research, our study was based on accelerated degradation data of motors, which are their working temperatures. A multistage Wiener model was established by using the transition function to modify linear model. The normal weighted average filter (Gauss filter) was used to improve the results of estimation for the model parameters. Then, to maximize likelihood function for parameter estimation, we used numerical optimization method- the simplex method for cycle calculation. Finally, the modeling results show that the degradation mechanism changes during the degradation of the motor with high speed. The effectiveness and rationality of model are verified by comparison of the life distribution with widely used nonlinear Wiener model, as well as a comparison of QQ plots for residual. Finally, predictions for motor life are gained by life distributions in different times calculated by multistage model.

1. Introduction

Brushless DC (BLDC) motors are widely used in a variety of drive and servo systems with good speed control and start characteristics. Wang Lingling[1] used the traditional zero-failure test method to verify the average life of the motors, and gave a test plan to evaluate whether the batch of motors was passed. Due to the long test time, she discussed the method for verifying the average life of an electric motor by accelerated life test. Wang Jian[2] made a useful exploration on the reliability demonstration by accelerated test. The results show that the scheme shortens the test time effectively. Due to the high reliability and long life characteristics of the motor, the life test often can hardly obtain enough failure life data. If the degradation data can be effectively utilized, it can make up the problem of insufficient information on the life expectancy of long life products. The modeling methods based on performance degradation data are effective complement to the traditional life prediction method, which have been successful in theoretical research and engineering application. Among them, Wiener process has a wide range of applications in the degradation modeling. Wang[3] described the degradation process of the product as linear drift Wiener process and proposed an adaptive method for predicting the remaining life of the product. The model was used to describe the degradation process of the bridge. Based on time scale transformation, the degradation processes of transistors and cables were studied by Whitmore[4]. Tseng and Peng established a nonlinear Wiener process model to describe the performance degradation of LED. Si established the nonlinear drift model by Wiener process, using standard Brown motion to describe the uncertainty of degradation on the time axis, then gave the approximate expression of the failure density function and the probability density function of the



residual life. Wang Xiaolin[5] summed up the theory of multiple nonlinear degradation modeling, and gave cases for analysis. However, in most cases, as the degradation of the motor continues, its dominant mechanism is changing, resulting in degradation appearing multi-stage, not a single model linear or non-linear with time-scale transformation.

2. Accelerated Degradation Test

Accelerated degradation test technique[6] is method combined of degradation test and accelerated test, by failure mechanism remains unchanged, to extrapolate and predict life under normal stress levels using product performance degradation data at high stress levels. Upon the manufacture's request, we carried out an accelerated test and life evaluation for a batch of motors. In this research, the accelerated tests for BLDC motors were loaded at high temperature (75 °C) and high load stress, and the rise-temperatures were considered the degradation data of the motor.

3. Wiener Process

3.1. Linear wiener process

From the practical point of view, randomness needs to be considered in the degradation modeling: the randomness in the sample performance and the randomness in the degradation on the time axis. The randomness of the samples depicts the samples differences, manifesting different degradation rates; the randomness on the time axis characterizes fluctuations in product degradation (control system fluctuations, product performance fluctuations, observation fluctuations). To this end, the models based on stochastic process are favored by many scholars. If $\{X(t) : t \geq 0\}$ satisfies:

- ① the increments in any time satisfies the normal distribution:

$$DX_i = X(t_{i+1}) - X(t_i), Dt_i = t_{i+1} - t_i, DX_i \sim N(uDt_i, \sigma^2 Dt_i)$$

Where, u for drift coefficient and σ for diffusion coefficient.

- ② The increments in any time are independent of each other.

Then the stochastic process[7] is called a linear Wiener process, which is used by Tseng and Tsai to describe the product degradation process. The model is:

$$X(t) = X(0) + ut + \sigma B(t) \quad (1)$$

Where, u for drift coefficient and σ for diffusion coefficient, $B(t)$ for Brownian motion.

We used chi-square test to verify that the temperature data of the accelerated degradation record were normally distributed and chose 50 data before sample starts steady degradation. Results as see Table 1.

Table 1. Data distributions.

temperature range	amount
$u+0.3 \sim u+0.4$	22
$u-0.2 \sim u-0.1, u+0.1 \sim u+0.2$	19
$u-0.3 \sim u-0.2, u+0.2 \sim u+0.3$	8
$u-0.4 \sim u-0.3, u+0.3 \sim u+0.4$	1
$<u-0.4, >u+0.4$	0

Its Chi square test is: $\chi^2 = 1.87 < 9.49 = \chi_{0.05}^2(4)$. The assumption that degradation data were normally distributed was accepted and the error caused by assumption was acceptable to do the theoretical calculation.

The failure distribution corresponding to this degenerate model is the inverse Gaussian distribution, and the failure probability density function is:

$$f(t) = \frac{D}{\sqrt{2\pi\sigma^2 t^3}} e^{-\frac{(D-ut)^2}{2\sigma^2 t}} \quad (2)$$

Where D is the threshold level of $X(t)$. Coefficients[8,9] are gained by maximum likelihood estimation:

$$\hat{u} = \frac{\sum_{i=1}^n \Delta X_i}{\sum_{i=1}^n t_i}, \quad \hat{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^n \frac{(\Delta X_i - \hat{u} t_i)^2}{t_i}}$$

Where, $\Delta X_i = X(t_{i+1}) - X(t_i)$

3.2. Multistage wiener process

In this research, a transition function[10] was used:

$$G(\gamma, c; s_i) = (1 + \exp(-\gamma(s_i - c)))^{-1} \quad (3)$$

Where, s_i is time-related variables, and $s_i = t$ here; c is location parameter of transition; γ is speed parameter of transition.

The multistage Wiener process is expressed as:

$$X(t) = X(0) + (1 - G(\gamma, c; t))u_1 t + G(\gamma, c; t)(u_2 t + x_0) + \sigma B(t) \quad (4)$$

At this point, when degradation process hits the threshold in a certain time, the probability that the process has crossed the threshold level before this time is considered negligible, which is referred by [11]. In fact, the diffusion process - Brown motion may hit threshold many times. With assumption that the probability of this case is 0, the probability density function of the Wiener can be expressed as follows:

$$f(t) \approx \frac{1}{\sqrt{2\pi t}} \left(\frac{S(t)}{t} + \frac{\lambda(t, \theta)}{\sigma} \right) \exp\left(-\frac{S^2(t)}{2t}\right) \quad (5)$$

Where, $S(t) = \frac{1}{\sigma}(D - \lambda(t, \theta))$, $\lambda(t, \theta) = (1 - G(\gamma, c; t))u_1 t + G(\gamma, c; t)(u_2 t + x_0)$, $\theta = (u_1, u_2, \gamma, c, x_0)$ is the

parameter vector for λ .

So, its Likelihood function is:

$$L(\theta, \sigma) = \prod_i \frac{1}{\sqrt{2\pi t_i}} \left(\frac{S(DX_i, t_i)}{t_i} + \frac{\lambda(t_i, \theta)}{\sigma} \right) \exp\left(-\frac{S^2(DX_i, t_i)}{2t_i}\right) \quad (6)$$

Where, $S(DX_i, t_i) = \frac{1}{\sigma}(DX_i - \lambda(t_i, \theta))$.

The process of estimating the parameters is the process of solving $\arg \max L(\theta, \sigma)$.

4. Cases Studies

4.1. Computing method

We used simplex method[12, 13] to estimate the parameters in the model, which is as follows:

- Establish seven vertices of the regular simplex in the parameter space $\{u_1, u_2, \gamma, c, x_0, \sigma\}$, which is $\{d_1, d_2, d_3, d_4, d_5, d_6, d_7\}$, where $d_i = (u_{1i}, u_{2i}, \gamma_i, c_i, x_{0i}, \sigma_i)$;
- Calculate the logarithmic likelihood of the regular simplex vertices, record the results as $\{l_1, l_2, l_3, l_4, l_5, l_6, l_7\}$, select the smallest l_k , prepare corresponding d_k for deletion;
- Search the backup point : $d_{new} = \frac{1}{n} \sum_{j=1 \& j \neq k}^7 d_j + (\frac{1}{n} \sum_{j=1 \& j \neq k}^7 d_j - d_k)$;
- Compare the backup point with the worst point of the log likelihood function value $\{l_{new}, l_k\}$, if better then update $d_k = d_{new}$, or shorten the step size, or stop if meet the accuracy required to calculate.

4.2. Failure probability distribution

The failure probability distribution is obtained by integrating the failure probability density function[14]:

$$F(t) = \int_0^t f(t) dt = \int_0^t \frac{1}{\sqrt{2\pi t}} \left(\frac{S(t)}{t} + \frac{\lambda(t, \theta)}{\sigma} \right) \exp\left(-\frac{S^2(t)}{2t}\right) dt \quad (7)$$

We predicted the distribution of life in the future, that is, calculated probability density function at different times:

$$f'(t|t_0) = \frac{1}{1-F(t_0)} f(t), t \geq t_0 \quad (8)$$

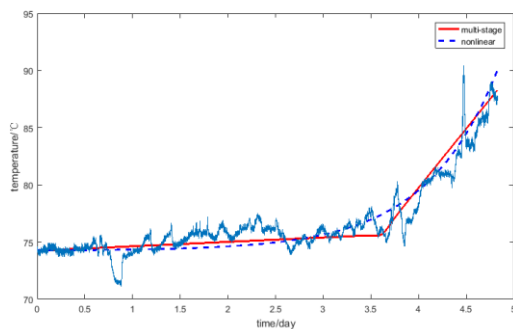


Figure 1. Comparison of Drift Process.

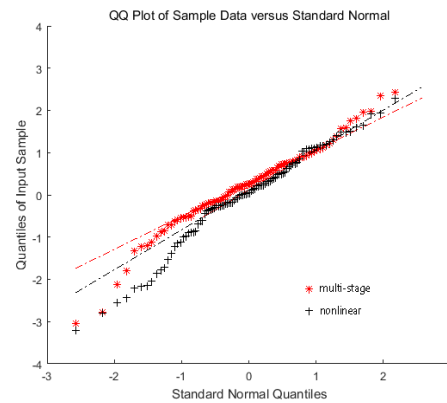


Figure 2. Residual QQ Plot.

4.3. Results

Using the method of this paper, we got the drift process as shown in Fig 1, with parameters: $u_1 = 0.3860$, $u_2 = 10.3378$, $c = 3.6259$, $r = 25.9924$, $x_0 = -35.8467$, $\sigma = 3.7550$. This shows the fact that the mechanism of degradation changes in 3.6 days, and the rate of change is pretty fast. The drift process obtained by the nonlinear method used in [4, 5] is also shown in fig1 as follows. Their Residual QQ Plot[15] are as Figure 2

The comparison of the failure distributions (Figure 3) shows that using nonlinear models to predict the life distribution of the product is earlier than practical. In the final stage, the degradation speed of the nonlinear model is faster than the actual degradation trend, resulting in a poor description for the degradation rate of the later stage of the motor. The failure distribution of the multi-stage degradation process is more optimized, and the description in whole period is also more appropriate. The life prediction of the motor under this stress carried out by using the multistage Wiener process model are shown as Figure 4.

5. Conclusions

In this research, aiming at the degradation characteristics of BLDC motor –nonlinear and multistage, we adapted the Wiener model, which was different from the individual nonlinear process, to research on modeling methods. This method was applied to solve the multi - stage Wiener process modeling problem, which is of great significance to the engineering application for the life prediction of the motor.

- We established the multi-stage Wiener process model to describe BLDC motor degradation. According to the multi-stage model theory proposed in this paper, the mechanism of motor in the process of degradation has changed, and the conversion speed is pretty fast.
- In the process of accelerated life test, attention should be paid to whether the mechanism of degradation changes; when the mechanism changes, the accelerated life test should be carried out with caution.

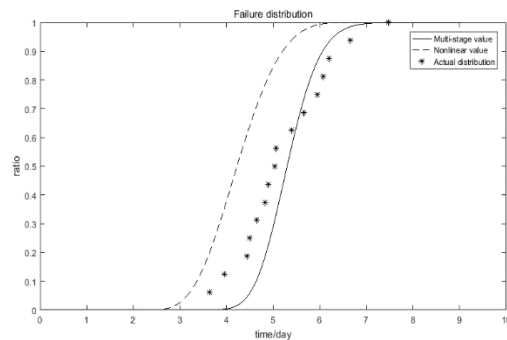


Figure 3. Comparison of Failure Distributions.

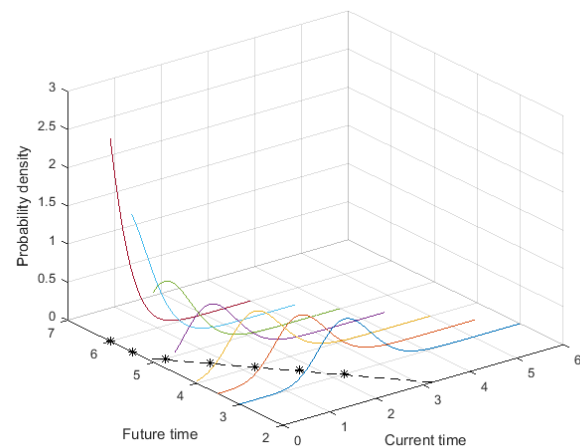


Figure 4. Life Prediction.

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