Effect of Capillary Tube's Shape on Capillary Rising Regime for Viscos Fluids

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Abstract. When properties of the displacing fluid are considered, the rising profile of the penetrating fluid in a capillary tube deviates from its classical Lucas-Washburn profile. Also, shape of capillary tube can affect the rising profile in different aspects. In this article, effect of capillary tube's shape on the vertical capillary motion in presence of gravity is investigated by considering the properties of the displacing fluid. According to the fact that the differential equation of the capillary rising for a non-simple wall type is very difficult to solve analytically, a finite element simulation model is used for this study. After validation of the simulation model with an experiment that has been done with a simple capillary tube, shape of the capillary tube's wall is changed in order to understand its effects on the capillary rising and different motion regimes that may appear according to different geometries. The main focus of this article is on the sinusoidal wall shapes and comparing them with a simple wall.

1. Introduction

Although capillary rising is a phenomenon that is known from many years ago [1], but according to the fact that its new applications like manufacturing new detectors and measurement instruments are appearing more and more nowadays [2-4], it seems that the need for further researches about the effect of the different parameters on the capillary fluid motion in order to take the numerous advantages of this phenomenon and increasing our efficiency in using capillary-based instruments is increasing. Three main parameters which are less-known about vertical capillary rising, are effects of gravity, tube shape and viscosity of the displacing fluid. Some authors provided models for spontaneous imbibition [5], while some others investigated effects variably shaped apertures [6]. Recently, the effects of viscosity and gravity for simple wall capillary tubes are investigated [7]. In this study, after validation of our finite element model with an experiment, we are going to investigate the effects of changing tube's geometry on the vertical capillary rising, while considering both gravity and viscosity effects of fluids, which were neglected partially or completely in the past studies. It will be shown that the new geometry will affect the regime of motion in an especial way, and according to various applications of capillary tubes, this new phenomenon can be used in the future for different applications.

2. Governing equations

Equations that govern the physics of this study are the continuity equation (equation (1)) along with the Navier-Stoke's momentum equations for incompressible fluid (equation (2) to equation (4)) in all three dimensions.
Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

With u, v, w as the velocity magnitude in x, y and z direction, respectively.

Navier-Stokes equations:

$$x - component: \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + F_x \quad (2)$$

$$y - component: \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + F_y \quad (3)$$

$$z - component: \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + F_z + \rho g_z \quad (4)$$

Where , μ and are average density and viscosity , respectively. Also, F_x , F_y and F_z are magnitude of the surface tension force in Cartesian coordinate system.

3. Simulation

3.1. Finite element modelling

In order to model the physics, we used a finite element modelling program. According to the fact that the tube has a circular cross section, we used the axisymmetric solving, so the symmetry that exists in this problem can be used and calculation time reduces. An image of a simple capillary tube's mesh is shown in Figure 1. For the fluid flow, wetted wall boundary condition is applied for walls. In the outlet, the atmospheric pressure is set, and for the inlet, the pressure calculated and set according to hydrostatic fluid pressure law equation (5), where the density of the displacing fluid and its height is used in order to calculate the necessary pressure.

$$P = \rho g h \quad (5)$$

It is noteworthy that the motion of the penetrating liquid will not affect this pressure in the inlet, so there is no need to change it during the simulation.

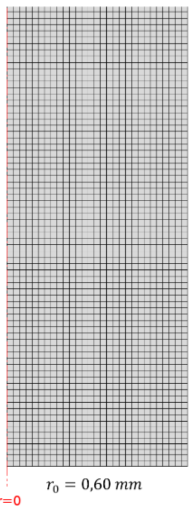

Figure 1. Simple tube mesh.

According to the experiment study [7], Oil (density 965 kg/m^3 and viscousity 97 mPa.s) and Glycerin (density 1225 kg/m^3 and viscousity 97 mPa.s) are set as the displacing and penetrating fluid, respectively.

3.2.Validation

Mesh independency has been investigated through checking the height of the capillary fluid in several different times for different mesh sizes and then, they compared with the results obtained from the experiment [7]. The optimum mesh that has been used in this study for different geometries, has an error less than 5% in comparison with the experiment results for each point. Results of experimental and simulation study are showed in Figure 2.

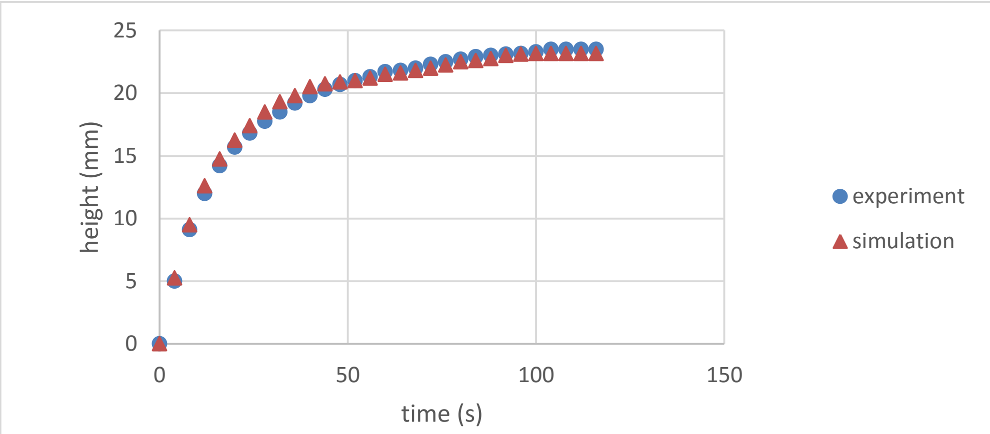

Figure 2. Height of the capillary fluid for different times is plotted for both experiment and simulation.

According to equations (6) and (7)

$$RMSD = \sqrt{\frac{\sum_{t=1}^{n}(x_{1.t}-x_{2.t})^2}{n}} \tag{6}$$

$$NRMSD = \frac{RMSD}{x_{max}-x_{min}} \tag{7}$$

root-mean-square deviation (RMSD) of the results is 0.42 and normalized root-mean-square deviation (NRMSD) is 1.83%, which means that the data obtained from the simulation is very close to the one obtained from the experiment.

3.3.Changing the geometry

To investigate the effect of the tube's shape on the capillary fluid rising, we changed the capillary tube's outer wall to two sinusoidal walls. The shape of walls is described by equation (8) and (9).

$$r = -0.025 * sin(2 * pi * z) + 0.6 \tag{8}$$

$$r = -0.050 * sin(2 * pi * z) + 0.6 \tag{9}$$

Wavelengths of the both sines are the same and the difference is just in their amplitude which affects the diameter of the tube as a function of height. Mean radius of walls and length of the both tubes are 0.6mm and 25mm, respectively. Meshes for both new shapes are depicted in the Figure 3.

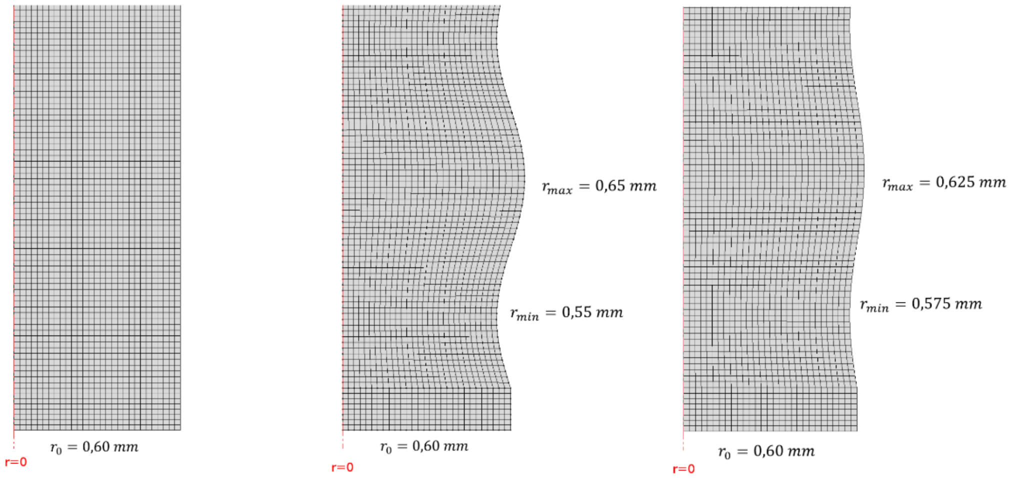

Figure 3. Meshes for different geometries.

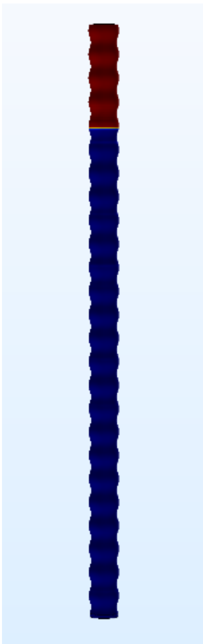

Figure 4. Final result for sinusoidal wall.

Because of the fact that the fluids and the general physics of the problem are the same, there is no need to change the boundary conditions for new shapes.

4. Results and discussion

Computation of the new geometries has been done and an image of the final result for a sinusoidal wall is depicted in Figure 4.
In order to compare the results better, height and time are scaled using equations (10) and (11).

$$Z^* = \frac{z}{h} \tag{10}$$

$$t^* = \frac{t}{\frac{8\mu_c h}{\Delta\rho g r^2}} \tag{11}$$

Where r and h and μ_c are radius of tube, final height and the larger viscosity, respectively [8].

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Scaled height of the imbibing fluid versus scaled time for both of the new wall types is plotted in a log-log diagram (Figure 5).
Changing the shape of the wall has affected the capillary rising in different aspects. For simple capillary tube, the nonlinear pattern in capillary rising is obvious. According to *Walls et al*[7], the height of the capillary fluid for the condition which the viscosities are equal, is an exponential function of time (equation (12)).

$$Z = h(1 - e^{\frac{-\Delta\rho g a^2}{8\mu L}t}) \tag{12}$$

But by changing the geometry, the fluid didn't follow the same pattern and the regime of motion changed. The necessary times for the fluid to reach the equilibrium in these new geometries is decreased in comparison with the simple tube, and are so close to each other.
By fitting linear curves on the sinusoidal wall's data, r-squares for fitted lines were 0.99 (for wall with maximum radius of 0.65mm) and 0.98 (for wall with maximum radius of 0.625mm). The slopes of the lines in the log-log plot was 0.89 and 0.93, so the height of the capillary fluid will have a polynomial relation with time (equations (13) and (14)) for these sinusoidal wall capillary tubes.

$$Z_{(r_{out}=0.65mm)} \propto t^{0.89} \tag{13}$$

$$Z_{(r_{out}=0.625mm)} \propto t^{0.93} \tag{14}$$

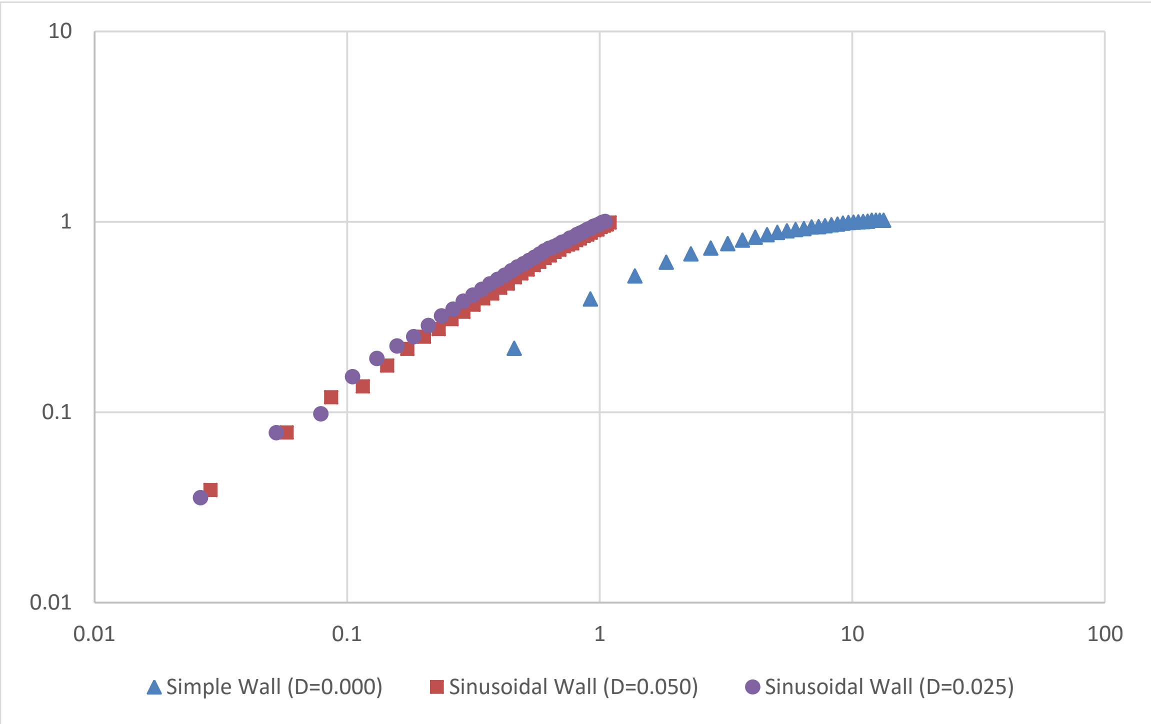

Figure 5. Height of the penetrating fluid versus time for different geometries.

5. Conclusion

Motion of fluid in a capillary tube is a function of many parameters. Viscosity of the fluids, gravity and tube's shape are three important ones. Although many researches have been done about vertical capillary rising, most of them neglected at least one of those three. In this paper, capillary tube's shape effect on fluid rising for a sinusoidal geometry is investigated by considering both gravity and viscosity of the displacing fluid. The results are compared to a simple capillary tube's data. According to them, shape of the capillary tube can affect not only the necessary time for the fluid to reach the steady state, but also can change the fluid motion regime. By changing the geometry, the penetrating fluid does not follow the same pattern and the regime of motion changes. By fitting linear curves on the sinusoidal wall's data, the slopes of the line in the log-log plot are 0.89 and 0.93, so the height of the capillary fluid had a polynomial relation with time which was described by equations 13 and 14.
Nowadays, capillary tubes can be used for variety of applications. Thermal mass flow metering [9], measuring viscosity [10], Microbial detection of enrofloxacin in milk [11] and aerosol generating [12] are some them. Capillary tubes are also used for destruction of volatile organic compounds used in a semiconductor industry [13] and self-assembly of rod like bio-nanoparticles [14]. We understood that new geometries affect the rising regime in a vertical capillary tube, and according to their various applications, this new phenomenon can be used in the future for manufacturing better equipment.
Although many of the parameters that have effect on the fluid capillary motion are studied until now, but it seems that there is a long way to completely understand the behavior of the fluid in this field.

Acknowledgments

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