

Plane waves in magneto-thermoelastic anisotropic medium based on (L-S) theory under the effect of Coriolis and centrifugal forces

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Abstract. The objective of this research is to illustrate the effectiveness of the thermal relaxation time based on the theory of Lord-Shulman (L-S), Coriolis and Centrifugal Forces on the reflection coefficients of plane waves in an anisotropic magneto-thermoelastic medium. Assuming the elastic medium is rotating with stable angular velocity and the imposed magnetic field is parallel to the boundary of the half-space. The basic equations of a transversely isotropic rotating magneto-thermoelastic medium are formulated according to thermoelasticity theory of Lord-Shulman (L-S). Next to that, getting the velocity equation which is illustrated to show existence of three quasi-plane waves propagating in the medium. The amplitude ratios coefficients of these plane waves have been given and then computed numerically and plotted graphically to demonstrate the influences of the rotation on the Zinc material.

1. Introduction

Recently, many results have been shown in the literature to the generalized thermo elasticity theories that depicted the second sound influence that allows the waves propagating with finite speed. Therefore, generalized theories were proved to shown more factual outcomes than the conventional models of thermoelasticity. The generalized thermoelasticity established on relaxation time effects was first presented by Muller [1]. Then, a more explicit theory where the temperature proportions are introduced through the constitutive relations was considered in the references [2] and [3]. Furthermore, Lord and Shulman's theory of generalized thermoelasticity utilizing one relaxation time expects the propagation of waves is finite [4]. These theories were developed by Dhaliwal and Sherief [5] to include the impacts of anisotropy. One may go through Hernarski and Ignaczak [6] for presenting and review of the generalized thermoelasticity.

The earliest works on the topic can exist in [7] and [8]. Abd-alla et al. [9], [10] and [11] displayed the phenomenon of the reflection and transmission or reflection only of waves propagating in a plane in the context of many assumptions under suitable boundary conditions. Othman and Song [12] and [13] have presented some problems in different hypotheses about magneto-thermo-elastic waves when the medium of the reflected wave is homogeneous and isotropic. Ting [14] investigated a surface wave propagation in an anisotropic medium under the effect of rotation.

Wave propagations in a generalized thermoelastic medium with additional variables such as magnetic field, anisotropy, microstructure, temperature and other variables supply essential details about the presence of new or changed waves. These details are beneficial for experimental seismologists in improvements earthquake estimation. Some relevant investigations on the reflection and transmission



wave propagation in a plane based on anisotropic the generalized thermoelastic media are considered by various authors such as Kumar and Singh [15] and Singh and Yadav [16].

Recently, Abd-alla et al. [17-21] studied several problems with regards to the reflections or the reflections and refractions phenomenon in anisotropic smart piezoelectric or in thermo-piezoelectric materials with the presence of the pre-stresses or not.

2. Formulation of the problem

Assuming a homogeneous and transversely isotropic elastic structure is rotating with steady angular velocity $\Omega = \Omega n$, with n is a unit vector showing the direction of the rotation axis. The equation of motion, in this case, has two supplementary terms [12]: the first one is "Centripetal acceleration", which may be written as $\Omega \times (\Omega \times u)$. This term arises due to time-varying motion only. The second term is "Corioli's acceleration" that is given as $2\Omega \times \dot{u}$. Accordingly, the principal equations for magneto-thermoelastic structure in the context of Lord-Shulman model may be written as (without heat sources, body forces and body couples):

(a) The equation of motion for (L-S) model with Lorentz force f and temperature T in rotating media may be written as:

$$\nabla \cdot \sigma + f = \rho[\ddot{u} + \{\Omega \times (\Omega \times u) + (2\Omega \times \dot{u})\}] - \gamma \nabla T \quad (1)$$

Where

$$f = J \times B \quad (2)$$

(b) The simplified linear equations of Maxwell (in S.I. units) for electrodynamics of steadily dynamic solid for a homogeneous and electrically conducting structure are:

$$\text{curl} H = J, \quad \text{curl} E = -\dot{B}, \quad \text{div} B = 0, \quad \nabla \cdot D = \rho_e, \quad (3)$$

Considering the relationships between the vector of magnetic induction B and the vector of total magnetic field H is given as $B = \mu_e H$. So, in our case we will assume that $\rho_e = 0$ in Eqs. (3).

(c) The generalized heat conduction equation based on the (L-S) model takes the form:

$$K \nabla^2 T = T_o \gamma (1 + t_o \frac{\partial}{\partial t}) \nabla \dot{u} + \rho C_e (\dot{T} + t_o \ddot{T}) \quad (4)$$

(d) The constitutive relation between the strain and the displacement are:

$$\varepsilon = \frac{1}{2} (\nabla u + \nabla u^T) \quad (5)$$

(e) Constitutive relation between stress, strain and thermal fields are:

$$\sigma = c \varepsilon - \gamma (T - T_o) \quad (6)$$

(f) The generalized Ohm's law is:

$$J = \sigma_o [E + (\dot{u} \times B)] \quad (7)$$

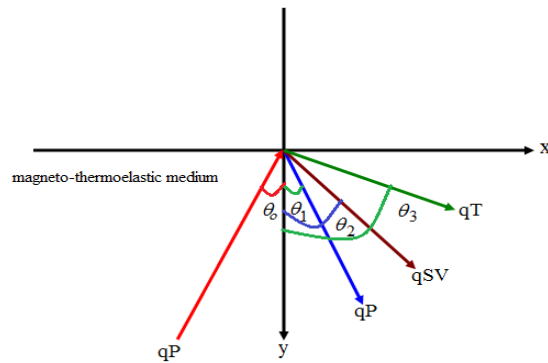


Figure 1. Geometry of the problem.

where the elastic part of the stress tensor σ is in equation (6), for the transversely isotropic materials, it may be seen as in [18]. The superimposed dot gives the derivatives of time and the symbols manifesting in the previous relations are listed in Table 1. The complete imagination of the problem is presented in Figure 1.

We consider the problem of a magneto-thermoelastic half-space ($y \leq 0$) with neglecting the conduction current J because of the small effect that is of temperature gradient.

We assume that $H = h + H_o$ with $H = (0, 0, H_o)$. The perturbed magnetic field h is so small such that the multiplication of h , u and their derivatives may be ignored during linearizing of the equations. Also, the magnetic field for a constant intensity $H = (0, 0, H_o)$ will be considered to act parallel to the bounding plane (take as the direction of the z -axis). Therefore, all the above-mentioned quantities will be functions of the time variable t and of the coordinates x and y [7].

The solution of equations (8), (9) and (10) can be chosen for a harmonic wave propagated for (L-S) model contains only one thermal relaxation time $t_o > 0$ in the following form:

$$\{u, v, T\} = (A, B, C) \exp[ik(x \sin \theta - y \cos \theta - ct)] \quad (8)$$

Where the wave normal locates in the xy -plane, and displays an angle θ with the y -axis.

From Eq. Therefore, we may have a system of homogeneous equations consisting of three Eqs. as:

$$A[D_1 - \zeta \Omega^*] + B[D_2 + 2\frac{\Omega}{\omega} i\zeta] + \tau C^* \sin \theta = 0, \quad (9)$$

$$A[D_2 - 2\frac{\Omega}{\omega} i\zeta] + B[D_3 + \zeta \Omega^*] - \tau_n \bar{\gamma} C^* \cos \theta = 0, \quad (10)$$

$$i\lambda(\varepsilon \zeta \sin \theta)A - i\lambda(\bar{\gamma} \varepsilon \zeta \cos \theta)B + \frac{1}{\omega}(\zeta - D_5)C^* = 0, \quad (11)$$

Where we have used

$$D_1 = (c_{11} + \mu_e H_o^2) \sin^2 \theta + c_{66} \cos^2 \theta, \quad D_2 = -(c_{12} + c_{66} + \mu_e H_o^2) \sin \theta \cos \theta,$$

$$D_3 = (c_{22} + \mu_e H_o^2) \cos^2 \theta + c_{66} \sin^2 \theta, \quad D_4 = K_1 \sin^2 \theta + K_2 \cos^2 \theta,$$

By using the following of dimensionless quantities

$$\zeta = \rho c^2, \quad \omega = Kc, \quad \bar{\gamma} = \frac{\gamma_2}{\gamma_1}, \quad \Omega^* = 1 + \frac{\Omega^2}{\omega^2}, \quad C^* = \frac{\omega \gamma_1 C}{k}, \quad \varepsilon = \frac{T_o \gamma_1^2}{\rho C_e}, \quad \lambda_n = nt_o + \frac{i}{\omega},$$

$$\lambda_1 = t_o + \frac{i}{\omega}, \quad D_5 = \frac{D_4}{\tau_1 C_e}, \quad \alpha = \frac{(i/\omega)}{t_o + (i/\omega)} = \frac{(i/\omega)}{\lambda_1}.$$

To get the non-trivial solutions of the system of Eqs. (9), (10) and (11), the determination of the factor of matrix must vanish, therefore,

$$|L_{ij}| = 0 \quad (12)$$

Where

$$L_{11} = D_1 - \zeta \Omega^*, \quad L_{12} = D_2 + 2i\zeta \frac{\Omega}{\omega}, \quad L_{13} = \lambda_n \sin \theta, \quad L_{12} = D_2 - 2i\zeta \frac{\Omega}{\omega}, \quad L_{22} = D_3 - \zeta \Omega^*, \\ L_{23} = \lambda_n \bar{\gamma} \cos \theta, \quad L_{31} = i\omega \alpha \varepsilon \zeta \sin \theta, \quad L_{32} = -i\omega \alpha \bar{\gamma} \varepsilon \zeta \cos \theta, \quad L_{33} = \zeta - D_5.$$

This yields the cubic equation in ζ as:

$$A_0 \zeta^3 + A_1 \zeta^2 + A_2 \zeta + A_3 = 0 \quad (13)$$

Where,

$$A_0 = 4 \left(\frac{\Omega}{\omega} \right)^2 - \Omega^{*2}, \quad A_1 = \Omega^* (D_1 + D_3) + (\Omega^{*2} - 4 \left(\frac{\Omega}{\omega} \right)^2) D_5 - i\lambda_n \varepsilon \omega \alpha (\sin^2 \theta + \bar{\gamma}^2 \cos^2 \theta) \Omega^* \\ A_2 = D_2^2 - D_1 D_3 + \Omega^* (D_1 D_5 + D_3 D_5) + i\lambda_n \varepsilon \omega \alpha (2\bar{\gamma} D_2 \sin \theta \cos \theta + \bar{\gamma}^2 D_1 \cos^2 \theta + D_3 \sin^2 \theta), \\ A_3 = (D_1 D_3 - D_2^2) D_5,$$

Here, the three roots $\zeta_j = \rho c_j^2$, ($j=1,2,3$) of equation (13) corresponding to the phase velocities c_j , ($j=1,2,3$) of QP, QSV and QT waves, respectively, which is has a complex value. If we use the relation $c_j^{-1} = V_j^{-1} - i\omega q_j$, ($j=1,2,3$), then it will be obvious that V_j and q_j are the velocity of propagation and the coefficients of attenuation of the QP, QSV and QT waves [16].

3. Boundary conditions

The required boundary conditions at the free surface $y=0$ are vanishing of the normal stresses, tangential stresses and normal component of the heat flux vector, i.e.,

$$\sigma_{yy} + \sigma_{yy}^* = 0, \quad \sigma_{yx} + \sigma_{yx}^* = 0 \quad \text{and} \quad T_{,y} = 0 \text{ on } y=0, \quad (14)$$

Where

$$\sigma_{yy} = c_{12} u_{,x} + c_{22} v_{,y} - \beta_2 (T + t_1 \dot{T}), \quad \sigma_{yy}^* = -\mu_e H_o^2 (u_{,x} + v_{,y}), \quad \sigma_{yx} = c_{44} (u_{,y} + v_{,x}), \quad \sigma_{yx}^* = 0.$$

4. Reflection from free surface

In this section, we shall derive the relations between the reflection coefficients, when (QP or QT or QSV) wave becomes incident at a thermoelastic solid half-space ($y \geq 0$) with thermally insulated and stress-free surface $y=0$. The positive y -axis is taken into the half-space. For the incident wave at the free surface, there will be three reflected waves, i.e., reflected QP, reflected QT and reflected QSV. Accordingly, if the wave normal to the incident wave (QP or QT or QSV) makes angle θ_o with the positive direction of the y -axis and those of reflected QP, QT and QSV waves make θ_1 , θ_2 and θ_3 with the same direction. The complete geometry showing the incident and reflected waves is depicted in Figure 1. The appropriate displacement components and temperature field which must satisfy the boundary conditions (14) at $y=0$ are as below:

$$u = A_o e^{\eta_o} + A_1 e^{\eta_1} + A_2 e^{\eta_2} + A_3 e^{\eta_3}, \quad (15)$$

$$v = F_o A_o e^{\eta_o} + F_1 A_1 e^{\eta_1} + F_2 A_2 e^{\eta_2} + F_3 A_3 e^{\eta_3}, \quad (16)$$

$$T = G_o A_o e^{\eta_o} + G_1 A_1 e^{\eta_1} + G_2 A_2 e^{\eta_2} + G_3 A_3 e^{\eta_3}, \quad (17)$$

With

$$\eta_o = ik_o (x \sin \theta_o - y \cos \theta_o - c_o t), \quad \eta_l = ik_l (x \sin \theta_l + y \cos \theta_l - c_l t), \quad l=1,2,3$$

Where c_o is the velocity of the incident QP or QT or QSV wave, k_l , ($l=0,1,2,3$) are complex wave numbers, $c_l = \zeta_l / \rho$ ($l=0,1,2,3$).

Substituting from Eqs. (15-17) into Eq.(12) and Eq. (13), we get in the case of incident wave

$$F_o = -N_1 / N_2, \quad G_o = (k_o / \omega \lambda_n)(N_3 / N_2) \quad (18)$$

Where

$$\begin{aligned} N_1 &= \gamma_1 \sin \theta_o [(D_{2o} - 2i(\Omega / \omega) \zeta_o] + \gamma_2 \cos \theta_o (D_{1o} - \zeta_o \Omega^*), \\ N_2 &= \gamma_2 \cos \theta_o [(D_{2o} + 2i(\Omega / \omega) \zeta_o] + \gamma_1 \sin \theta_o (D_{3o} - \zeta_o \Omega^*), \\ N_3 &= [(4(\Omega / \omega)^2 - \Omega^{*2}) \zeta_o^2 + \Omega^* (D_{1o} + D_{3o}) \zeta_o + D_{2o}^2 - D_{1o} D_{3o}], \end{aligned}$$

With

$$\begin{aligned} D_{1o} &= (c_{11} + \mu_e H_o^2) \sin^2 \theta_o + c_{66} \cos^2 \theta_o, \quad D_{2o} = -(c_{12} + c_{66} + \mu_e H_o^2) \sin \theta_o \cos \theta_o, \\ D_{3o} &= (c_{22} + \mu_e H_o^2) \cos^2 \theta_o + c_{66} \sin^2 \theta_o. \end{aligned}$$

In the case of reflected waves, one may get the following displacement components and temperatures field which are suitable to satisfy the boundary conditions (14) at $y=0$.

$$u = A_\ell e^{\eta_\ell}, \quad v = F_\ell A_\ell e^{\eta_\ell} \quad \text{and} \quad T = G_\ell A_\ell e^{\eta_\ell}, \quad \ell = 1, 2, 3 \quad (19)$$

With

$$F_\ell = -N_{1\ell} / N_{2\ell}, \quad G_\ell = -(k_\ell / \omega \lambda_n)(N_{3\ell} / N_{2\ell}), \quad (20)$$

$$\begin{aligned} N_{1\ell} &= \gamma_1 \sin \theta_\ell [(D_{2\ell} - 2i(\Omega / \omega) \zeta_\ell] - \gamma_2 \cos \theta_\ell (D_{1\ell} - \zeta_\ell \Omega^*), \\ N_{2\ell} &= \gamma_2 \cos \theta_\ell [(D_{2\ell} + 2i(\Omega / \omega) \zeta_\ell] - \gamma_1 \sin \theta_\ell (D_{3\ell} - \zeta_\ell \Omega^*), \\ N_{3\ell} &= [(4(\Omega / \omega)^2 - \Omega^{*2}) \zeta_\ell^2 + \Omega^* (D_{1\ell} + D_{3\ell}) \zeta_\ell + D_{2\ell}^2 - D_{1\ell} D_{3\ell}], \end{aligned}$$

Where

$$\begin{aligned} D_{1\ell} &= (c_{11} + \mu_e H_o^2) \sin^2 \theta_\ell + c_{66} \cos^2 \theta_\ell, \quad D_{2\ell} = (c_{12} + c_{66} + \mu_e H_o^2) \sin \theta_\ell \cos \theta_\ell, \\ D_{3\ell} &= (c_{22} + \mu_e H_o^2) \cos^2 \theta_\ell + c_{66} \sin^2 \theta_\ell. \end{aligned}$$

Using Eqs. (15-17) which corresponding to incident and reflected waves in the boundary conditions (14) yield

$$\begin{aligned} &-(c_{22} - \mu_e H_o^2)(F_o A_o (ik_o \cos \theta_o) - F_1 A_1 (ik_1 \cos \theta_1) - F_2 A_2 (ik_2 \cos \theta_2) \\ &- F_3 A_3 (ik_3 \cos \theta_3)) + (c_{12} - \mu_e H_o^2)(A_o (ik_o \sin \theta_o) + A_1 (ik_1 \sin \theta_1) + \\ &A_2 (ik_2 \sin \theta_2) + A_3 (ik_3 \sin \theta_3)) - \gamma_2 (1 - it_1 \omega)(G_o A_o + G_1 A_1 + G_2 A_2 + G_3 A_3) = 0, \end{aligned} \quad (21)$$

$$\begin{aligned} &-ik_o (A_o \cos \theta_o - A_o F_o \sin \theta_o) + ik_1 (A_1 \cos \theta_1 + A_1 F_1 \sin \theta_1) \\ &+ ik_2 (A_2 \cos \theta_2 + A_2 F_2 \sin \theta_2) + ik_3 (A_3 \cos \theta_3 + A_3 F_3 \sin \theta_3) = 0, \end{aligned} \quad (22)$$

$$-ik_o (G_o A_o \cos \theta_o) + ik_1 (G_1 A_1 \cos \theta_1) + ik_2 (G_2 A_2 \cos \theta_2) + ik_3 (G_3 A_3 \cos \theta_3) = 0. \quad (23)$$

Since the phases of the waves should be the same for each values of x , then we obtain the following relations which must be valid

$$\eta_o = \eta_1 = \eta_2 = \eta_3,$$

and so

$$k_o \sin \theta_o = k_1 \sin \theta_1 = k_2 \sin \theta_2 = k_3 \sin \theta_3$$

And

$$k_o c_o = k_1 c_1 = k_2 c_2 = k_3 c_3.$$

Therefore, the Snell's law may be deduced as

$$\frac{\sin \theta_o}{V_o} = \frac{\sin \theta_1}{V_1} = \frac{\sin \theta_2}{V_2} = \frac{\sin \theta_3}{V_3}, \quad (24)$$

Multiplying Eqs. (24-26) by the factor $\frac{i}{k_o A_o}$, using Snell's law (27). putting

$$\frac{A_i}{A_o} = W_i, \quad (25)$$

one may get the following system of three non-homogenous equations,

$$\sum_{j=1}^3 a_{ij} W_j = b_i, \quad i = 1, 2, 3 \quad (26)$$

Where

$$\begin{aligned} a_{1L} &= F_L (c_{22} - \mu_e H_o^2) \left[\left(\frac{V_o}{V_L} \right)^2 - \sin^2 \theta_o \right]^{1/2} + (c_{12} - \mu_e H_o^2) \sin \theta_o + \frac{\omega \lambda_n \gamma_2}{k_o} G_L, \\ a_{2L} &= \left[\left(\frac{V_o}{V_L} \right)^2 - \sin^2 \theta_o \right]^{1/2} + F_L \sin \theta_o, \quad a_{3L} = G_L \left[\left(\frac{V_o}{V_L} \right)^2 - \sin^2 \theta_o \right]^{1/2}, \\ b_1 &= (c_{22} - \mu_e H_o^2) F_o \cos \theta_o - (c_{12} - \mu_e H_o^2) \sin \theta_o - \frac{\omega \lambda_n \gamma_2}{k_o} G_o, \\ b_2 &= \cos \theta_o - F_o \sin \theta_o, \quad b_3 = G_o \cos \theta_o, \quad L = 1, 2, 3. \end{aligned}$$

From Eqs. (26). the analytical expressions of amplitude ratios W_1, W_2 and W_3 for incident QP waves may be obtained. While the analytical expressions of amplitude ratios Y_1, Y_2 and Y_3 for incident QSV are related to amplitude ratios W_j with the relations:

$$Y_1 = F_1^* W_1, \quad Y_2 = F_2^* W_2, \quad Y_3 = F_3^* W_3 \quad (27)$$

Furthermore, the analytical expressions of amplitude ratios Z_1, Z_2 and Z_3 for incident QT are related to amplitude ratios W_j with the relations:

$$Z_1 = G_1^* W_1, \quad Z_2 = G_2^* W_2, \quad Z_3 = G_3^* W_3 \quad (28)$$

Where

$$F_j^* = \frac{F_j}{F_o}, \quad G_j^* = \frac{G_j}{G_o}, \quad j = 1, 2, 3. \quad (29)$$

It is noted that the definitions of the previous amplitude ratios are controversial. The reason is that only the amplitude ratios of the horizontal parts of the mechanical displacement have been taken into account and no contribution comes from the vertical displacement nor from the temperature fields. A general expression of the amplitude ratios, in which all the contributions coming from both the vertical displacement and the temperature fields will be considered in the future works (see for example [20]).

5. Numerical results and discussion

For numerical calculations, the material of Zinc (Zn) is chosen which has the following physical information [22].

$$c_{11} = 1.628 \times 10^{11} \text{ Nm}^{-2}$$

$$c_{12} = 0.362 \times 10^{11} \text{ Nm}^{-2}$$

$$c_{22} = 1.628 \times 10^{11} \text{ Nm}^{-2}$$

$$C_e = 3.9 \times 10^2 \text{ JKg}^{-2} \text{ deg}^{-1}$$

$$\gamma_1 = 5.75 \times 10^6 \text{ Nm}^{-2} \text{ deg}^{-1}$$

$$\gamma_2 = 5.07 \times 10^6 \text{ Nm}^{-2} \text{ deg}^{-1}$$

$$K_1 = 1.24 \times 10^2 \text{ Wm}^{-2} \text{ deg}^{-1}$$

$$K_2 = 1.24 \times 10^2 \text{ Wm}^{-2} \text{ deg}^{-1}$$

$$\rho = 7.14 \times 10^3 \text{ Kg m}^{-3}$$

$$T_o = 296 \text{ K}$$

The absolute values of the reflection coefficients for various quasi-plane waves are numerically considered against the incidence angle of (QP or QSV or QT) waves for various values of the thermal relaxation time and rotation variables from Eqs. (26), (27), (28) and (29). These changes of the reflection coefficients are given graphically in the Figures 2 and 3. Therefore, with the help of the Figures, one may summarize the following observations:

(i) From Figure 2a, it is noted that W_1 starts with the value 1.049 when $\theta_o = 0^\circ$ then decrease to reach the minimum value 0.78 at $\theta_o = 28^\circ$ wherever there is no effect for the variation to t_o . After that, W_1 grows monotonically from 1.049 at $\theta_o = 29^\circ$ up to the maximum value 1.724 at $\theta_o = 66^\circ$ and then reduces down to the minimum value 1.413 at $\theta_o = 90^\circ$. The influence of the t_o appears in the period $\theta_o = (45^\circ - 90^\circ)$, where there is an inverse proportion to the increased of thermal relaxation time t_o .

Figures. 2b and 2c, display the reflection coefficients W_2 and W_3 as a function of θ_o . In these situations, the specific attitudes of these Figures are roughly analogous to the standard normal distribution in probability. It is noticed that the maximum of the modulus of W_1 is 0.268 that occurs around $\theta_o = 54^\circ$, and similarly, the curves corresponding to the change of t_o offer relevant variations only round the maximum. The same effect also happens for W_3 , but the effect of the thermal relaxation time be less and it becomes negligible when $\theta_o = 90^\circ$.

(ii) For the second group, the results are given in Figures. (3a, 3b, 3c), where changes of the dimensionless rotation come as $\Omega' = 4.0, 4.4, 4.8$, with the fixed values of both $H_o = 10^6 \text{ A/m}$ and $t_o = 3 \text{ PS}$. In this case, the Figures 3a displays the effectiveness of variation in the rotation Ω' (respectively in, 3b and 3c), on the reflection coefficient W_1 as function of θ_o . It has been observed that W_1 has the same behavior as in Figure 2a. In details, the value of W_1 decreases in the two intervals $\theta_o = (0^\circ, 29^\circ)$ and $\theta_o = (61^\circ, 90^\circ)$. However, the opposite happens in the interval $\theta_o = (30^\circ, 60^\circ)$. An absolute maximum for W_1 becomes visible about $\theta_o = 62^\circ$. Figures. 3b and 3c have a similar attitude as Figures. 2b and 2c, respectively. However, reflection coefficients are directly proportional to an increased rotation.

6. Conclusion

This mathematical model illustrates the characteristics of reflection of quasi-plane wave propagation in a transversely isotropic magneto-thermoelastic structure. The solution is given for suitable boundary conditions in the context of the generalized theory of thermo elasticity based on Lord-Shulman Model. In this work existence of three quasi-plane waves QP, QSV and QT waves are found. Furthermore, the reflection coefficients ratios depend upon the rotation, angle of reflection, angle of incidence and anisotropy in the structure. This research is significant to solve problems with parameters like anisotropic elastic structure and rotation coexist, for example, the seismic wave propagation inside the earth, the characteristics of earthquakes, nuclear arms and nuclear devices and underground nuclear explosions. Furthermore, the methods which are used in this paper may be applied in some recent interesting practical works and future research directions (see for example [23] -[25]).

7. References

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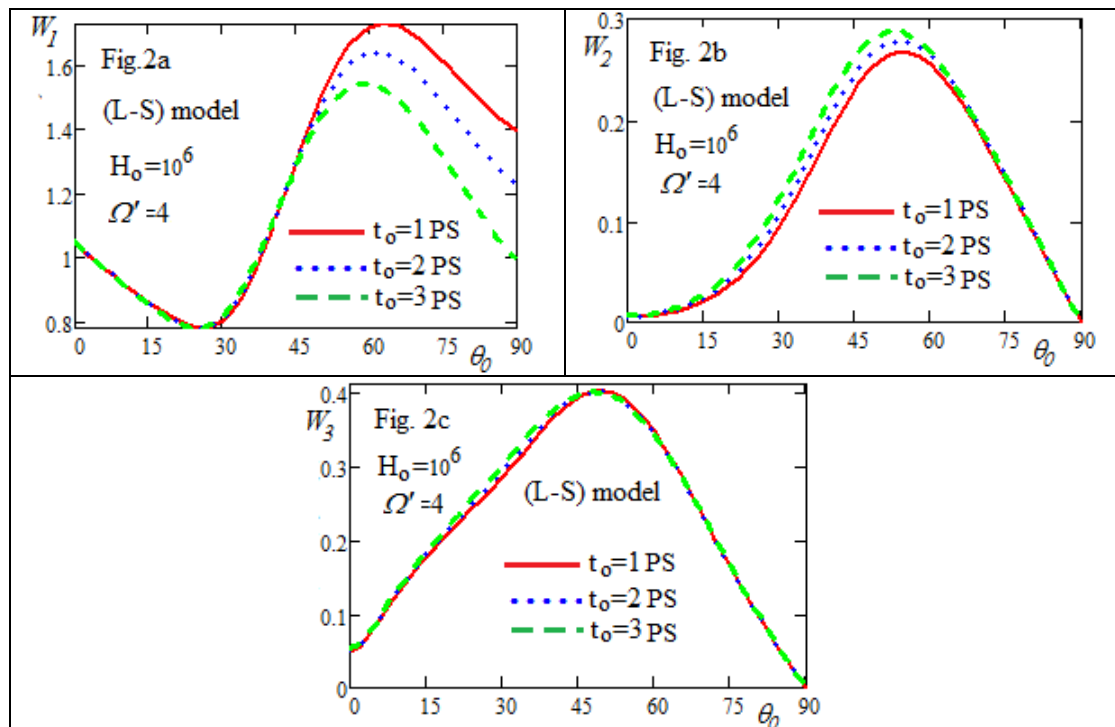
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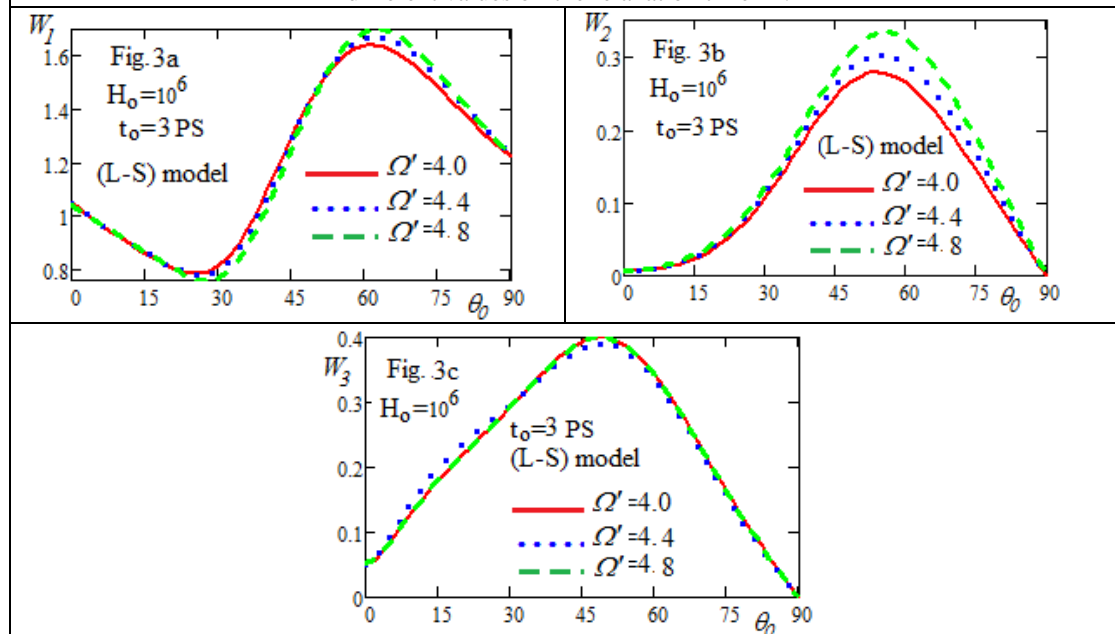
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Table 1. Nomenclature

ρ	mass density
ρ_e	charge density
\mathbf{u}	mechanical displacement
$\boldsymbol{\varepsilon}$	strain tensor
$\boldsymbol{\sigma}$	stress tensor
\mathbf{c}	fourth order tensor tiffnesss
\mathbf{B}	magnetic induction vector
\mathbf{H}	total magnetic field vector
\mathbf{E}	electric field vector
\mathbf{f}	electromagnetic body force (Lorentz force)
\mathbf{J}	electric current density vector
\mathbf{H}_o	perturbed magnetic field vector \mathbf{h}
T	temperature
T_0	reference uniform temperature of the body
\mathbf{K}	heat conduction second order tensor
$\boldsymbol{\Omega}$	rotation vector
$\boldsymbol{\gamma}$	thermal elastic coupling second order tensor
C_e	specific heat at constant strain
t_o	thermal relaxation time
\mathbf{n}	vector of the symmetric axis
\mathbf{N}	second order tensor $\mathbf{n} \otimes \mathbf{n}$
\mathbf{I}	identity tensor of second order
$\lambda, \mu, \mu_0, \alpha, \beta$	elastic constitutive parameters
μ_e	magnetic permeability of the medium
σ_o	electric conductivity of the medium
ω	circular frequency
c	complex phase velocity
θ	angle of propagation measured from the normal to the half-space



Figures (2a,2b,2c). Represent the reflection coefficients $W_1(2)$, $W_2(3)$, and $W_3(4)$ against θ_0 for different values of the relaxation time t_0 .



Figures (3a,3b,3c). Represent the reflection coefficients $W_1(5)$, $W_2(6)$, and $W_3(7)$ against θ_0 for different values of the rotation Ω' .

