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Evaluation of fiber's misorientation effect on compliance and load carry capacity of shaped composite beams

A N Polilov and N A Tatus'

Mechanical Engineering Research Institute named by A.A. Blagonravov of the Russian Academy of Sciences, 4 Maly Kharitonyevsky Pereulok, Moscow, 101990, Russia

E-mail: polilovan@mail.ru, nikalet@mail.ru

Abstract. The goal of this paper is analysis of design methods for composite beams and plates with curvilinear fiber trajectories. The novelty of this approach is determined by the fact that traditional composite materials are typically formed using prepregs with rectilinear fibers only. The results application area is associated with design process for shaped composite structure element by using of biomechanical principles. One of the related problems is the evaluation of fiber's misorientation effect on stiffness and load carry capacity of shaped composite element with curvilinear fiber trajectories. Equistrong beam with constant cross-section area is considered as example, and it can be produced by unidirectional fiber bunch forming, impregnated with polymer matrix. Effective elastic modulus evaluation methods for structures with curvilinear fiber trajectories are validated. Misorientation angle range (up to 5o) when material with required accuracy can be considered as homogeneous, neglecting fiber misorientation, is determined. It is shown that for the beams with height-to-width ratio small enough it is possible to consider 2D misorientation only.

1. Introduction

The international term "bio-inspired method" refers to a methods inspired by Nature. The main slogan of new scientific area named "strength biomechanics" is "Nature study". By analogy with the structure of knots, rational equally stressed curvilinear fiber trajectories "flowing" around the hole can be constructed for the fiber reinforced polymers (FRP) laminates [1-3]. The structure of a bamboo stalk shows us the need to consider different fracture modes of composite tube elements in order to specify rational equistrong dimensions of tube or bamboo construction [4-6].

One of the major advantages of fiber composites is the ease of complex shape components fabrication when considerable loads and high temperatures are not required as for metals. For example, a modern version of pultrusion method – pulforming allows to create some workpiece from unidirectional fibers, impregnated by polymer matrix, and then it is possible to form it into desired shape, in absence of fibers cutting. Due to these features there is an interest to the design of uniform strength beams as elastic elements for effective substitution of steel multi-leaf springs [7]. It is shown in [8] using special FRP strength criteria [9] that it is possible to design the shaped beam which satisfies the strength and stiffness requirements with three times less weight in comparison with rectangular beam. And because of fiberglass lower elastic modulus and density the weight of this beam can be reduced approximately 15 times, compared with steel beam. The analogy of the bending flexibility of shaped beams with constant cross section area and branching composite structures like a

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treetop with a constant sum of branches section areas (Leonardo's rule) [10, 11] has been studied in [12].

This article is aimed to analyze the effects, which should be considered in case of misorientation of fibers, packed in beam body along curvilinear trajectories, corresponding to complex beam shape and stress state.

Constarea beam (beam with constant cross-section area) is chosen as a most effective form for composite design, because of absence of cut fibers and constant volume fraction of fibers. The following problems are considered:

1. Changing of design relations based on reasonable simplifying hypotheses and taking into account fiber misorientation.
2. Determination of the angle range where fiber misorientation can be neglected with required calculation accuracy.
3. Changing of uniform strength beam shape taking into account fiber misorientation.

Standard triangular, parabolic, constarea uniform stressed beams for homogeneous material are known. It is required to determine shape of uniform stressed beam by iteration process to optimal design using fiber uniform stress criterion. Last problem can be solved only by computer modeling of complex spatial fiber trajectories.

2. Shaped constarea beam

Earlier, in [7, 8], equally strong, shaped composite leaf springs (figure 1) are modeled by a cantilever beam, the dimensions of which (width w and height h) are assumed to be variable according to the laws:

$$w(x) = w(0)(1-x^*)^{\alpha},\ h(x) = h(0)(1-x^*)^{\beta},\ x^* = x/L \tag{1}$$

In the rational design of leaf spring it is necessary to find sizes of root section $h(0)$, $w(0)$ and parameters α, β of their variation for the simultaneous implementation of controversial requirements for rigidity and for strength (load carry capacity):

$$C = \frac{P}{v} = \frac{E\,w(0)\,h^3(0)}{4\delta_v L^3}, \tag{2}$$

$$\frac{6P_{\max}L}{w(0)\,h^2(0)} = \sigma^{\max} \le \sigma^*. \tag{3}$$

The contradiction of the requirements (2) and (3) is that, to increase the load carry capacity (to reduce stresses), the thickness of the beam should be increased and, to reduce rigidity (to increase the accumulated elastic energy), the thickness of the beam should be decreased. The best design corresponds to the fulfillment of both (2) and (3) equations.

The additional condition of equal strength is

$$\frac{6P(L-x)}{w(x)\,h^2(x)} = \frac{6PL}{w(0)\,h^2(0)} \tag{4}$$

and leads from equations (1) to the linear ratio

$$\alpha + 2\beta = 1. \tag{5}$$

The deflection v of the shaped beam under load P (figure 1) can be easily defined from the equality of force work $1/2Pv$ and elastic energy accumulated in the beam as follows:

$$v(L)=\int_0^L \frac{P(L-x)^2\,\mathrm{d}\,x}{E(x)I(x)}=\frac{PL^3}{3EI(0)}\cdot\frac{1}{1-\alpha/3-\beta}=v_0\delta_v \tag{6}$$

$$I(x)=\frac{w(x)h^3(x)}{12}=\frac{w(0)h^3(0)}{12}\left(1-x^*\right)^{\alpha+3\beta}=I(0)\left(1-x^*\right)^{\alpha+3\beta}$$

The coefficient of the deflection form of equation (6) is:

$$\delta_v=\frac{v(L)}{v_0}=\frac{1}{1-\alpha/3-\beta}. \tag{7}$$

That is equal to the ratio of the maximum deflection of the shaped beam to the deflection of the rectangular beam v_0 with the same dimensions of the root section.

The constarea beam, which is best for the fibrous structure, is designed to retain the cross-section area and the number of fibers in each section, i.e. there are no cut fibers. The equations (1) shows the constant section area should meet the condition:

$$\alpha+\beta=0. \tag{8}$$

So the values α and β are –1 and 1 respectively, based on the equal strength condition (5). For these values, it is clear that a constarea beam has three times higher flexibility than a rectangular beam (see (7)). The necessary dimensions of the spring root section can be found based on simultaneous fulfilling conditions (2) and (3) as follows:

$$h(0)=\frac{2C\sigma^* L^2}{3EP_{\max}}\delta_v=h_0\delta_v,\; w(0)=\frac{27P_{\max}^3E^2}{2\sigma^{*3}C^2L^3\delta_v^2}=\frac{w_0}{\delta_v^2}, \tag{9}$$

where h_0, w_0 are the constant dimensions of a rectangular beam section that meet conditions (2) and (3). The mass of the shaped beam is easily calculated by the integral:

$$m(L)=\int_0^L \rho w(x)h(x)\,\mathrm{d}\,x=-\rho w(0)h(0)L\int_0^L\left(1-x^*\right)^{\alpha+\beta}\mathrm{d}\left(1-x^*\right)=\frac{\rho w(0)h(0)L}{1+\alpha+\beta}=m(0)\delta_m, \tag{10}$$

$$\delta_m=\frac{1}{1+\alpha+\beta}.$$

A possible mass of the spring is found from equations (9), (10) that fulfill the conditions of rigidity (2) and strength (3):

$$m(L)=\rho w(0)h(0)L\delta_m=\frac{9\rho P_{\max}^2E}{\sigma^{*2}C}\cdot\frac{\delta_m}{\delta_v} \tag{11}$$

where $\delta_m/\delta_v=\delta_\Sigma$ is the coefficient of the mass decrease of the shaped beam compared to a rectangular beam. It can be seen from (11) that the mass of the beam depends on both the material density ρ and the elasticity modulus E. This effect makes GFRP (fiberglass plastic) the most efficient construction material for the elastic elements.

For steel $E^s\approx 210$ GPa, strength $\sigma^{*s}\approx 800$ MPa and density $\rho^s=7800$ kg/m^3. For a unidirectional GFRP $E^s\approx 45$ GPa, strength $\sigma^{*s}\approx 800$ MPa and $\rho^s=2500$ kg/m^3. Thus, according to (11), in the ideal case, the fiberglass spring can be approximately 15 times lighter compared to the steel one.

An interesting conclusion can be drawn from analyzing the influence of an equally strong spring upon reducing its mass:

$$\delta_\Sigma = \frac{3-\alpha-3\beta}{3(1+\alpha+\beta)}, \tag{12}$$

and resulting from equation (5)

$$\delta_\Sigma = \frac{2-\beta}{3(2-\beta)} = \frac{1}{3}, \tag{13}$$

i.e. by simultaneously fulfilling the conditions of rigidity and strength, any equally strong ideal beam turns out to be three times lighter than a rectangular beam. The same result is obtained for ideal branching. This is the best limit case; due to the form, no larger reduction in mass can be obtained.

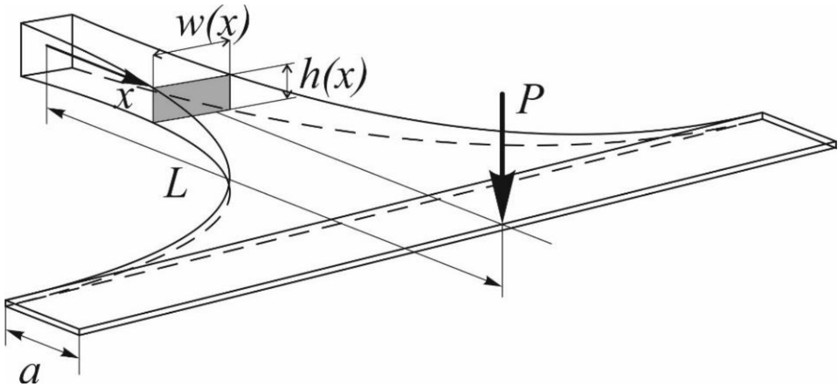

Figure 1. Cantilever constarea beam with constant width-thickness end area.

3. Fibers trajectories in constarea beam

Changing of cross-section dimensions of "ideal" constarea beam is described by power law dependences (1). For taking into account effect of misorientation of fibers on deflection it is necessary to accept some model of "fibers" distribution in the beam.

In this work it is proposed to use "spreading" principle, then in every point direction only is determined as for infinitely thin fiber. Trajectories in this case are conformed with beam shape (1) and are determined by initial coordinates $y(0)$, $z(0)$

$$y(x) = y(0)\left(1-x^*\right)^\alpha, \; z(x) = z(0)\left(1-x^*\right)^\beta, \tag{14}$$

where $\alpha = -1$, $\beta = 1$. Trajectories slope in planes xy and xz are characterized by derivatives of the functions (14)

$$\frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{y(0)}{\left(1-x^*\right)^2}, \; \frac{\mathrm{d}\,z}{\mathrm{d}\,x} = z(0)^*.$$

Model of misorientation angle calculation is shown in figure 2. "Fiber" element along axis 1 has projections onto axes $\mathrm{d}\,x$, $\mathrm{d}\,y$, $\mathrm{d}\,z$. Angle φ between axis 1 and axis x is determined from apparent dependences:

$$\cos\varphi = (1+A)^{-1/2}, \; \mathrm{tg}\,\varphi = \sqrt{A}, \; A = \left(\frac{\mathrm{d}\,y}{\mathrm{d}\,x}\right)^2 + \left(\frac{\mathrm{d}\,z}{\mathrm{d}\,x}\right)^2. \tag{15}$$

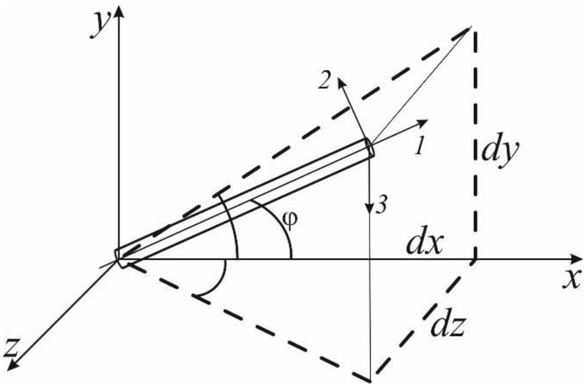

Figure.2. Scheme for calculation of the local misorientation angle of fiber.

If misorientation angle is known, it is possible to calculate local value of longitudinal elastic modulus E_x, which depends on coordinates x, $y(x)$, $z(x)$, connected by equations (14) with initial coordinates of each "fiber" $y(0)$, $z(0)$. The "fiber" tensor modulus E_{11}^0 is much greater than other three moduli E_{22}^0, E_{12}^0, E_{66}^0. Then formula for elastic modulus transformation is extremely simple:

$$E_x(x,y,z,\varphi) = E_1^0 \cos^4 \varphi = E_1^0 / (1+A)^2 . \tag{16}$$

As effective elastic modulus for layer, section or whole beam the value of modulus for homogeneous element with the same dimensions and the same summary stiffness is understood. In the case of beam tension we can suppose that deformations in each section are homogeneous, i.e. deformations are the same for each structural element and the effective modulus can be estimated by simple averaging:

$$E_x = \frac{1}{N}\sum_{i=1}^{N} E_1^0 \cos^4 \varphi_i , \tag{17}$$

where i – "fiber" number in section, N – total number of "fibers".

More complex bending problem can be solved only based on some kinematic hypotheses:

1. Deformations are changing linearly with height.
2. In each layer (across the width – along y axis) deformations are uniform, so the effective modulus of the layer can be determined by (17).
3. If height of section is small in comparison with width, only 2D misorientation can be considered, and effective modulus in each layer can be assumed constant, thereby averaging (17) can be used for whole section.

For bending of beam with dimensions $w(x)$, $h(x)$ it is possible to estimate the effective modulus:

$$E_x^{\text{eff}}(x) w(x) h^3(x) = 48 \int_{-w/2}^{w/2} \mathrm{d}\, y \int_{-h/2}^{h/2} E(x,y,z) z^2 \, \mathrm{d}\, z . \tag{18}$$

Neglecting small "fibers" misorientation across the height for beams with small height-to-width and height-to-length ratios it is possible to estimate the effective modulus by averaging along width only, and considering that this averaged value is constant along section

$$E_x^{\text{eff}}(x) b(x) = \int_{-b/2}^{b/2} E(x,y) \, \mathrm{d}\, y . \tag{19}$$

Form of homogeneous "uniform strength" constarea beam is chosen and the maximum stress on the surface $\sigma_x = E_x^{\text{eff}}\varepsilon$ reaches the tensile strength $\sigma(0)$ simultaneously in every section. For taking into account the misorientation effect it is necessary to consider the different local "fiber" strength $\sigma(\varphi)$ and reduced local stress $\sigma_x(\varphi)$, associated with a decrease of the local modulus of elasticity $E_x(\varphi)$. The deformation ε is proposed to be uniform along the width. So the local stress change for critical state can be expressed as

$$\sigma_x(\varphi) = \varepsilon E_x(\varphi) = \sigma(0) E_x(\varphi) / E_x^{\text{eff}} .$$

For the tensile strength at an angle φ to the fibers two linear criteria are affirmed in [9] for fibers rupture and for splitting along fiber-matrix interface:

$$\sigma_1 + m_1 \cdot \sigma_6 = \sigma(0),\ \sigma_2 + m_2 \cdot \sigma_6 = \sigma(90) .$$

The parameters of the second criterion are usually determined from tests of unidirectional composite specimens, cut at different angles to fibers. For qualitative estimates only small misorientation angles are considered, and then first criterion for fiber rupture is valid:

$$\sigma(\varphi_i) = \frac{\sigma(0)}{\cos^2\varphi_i + m_1 \cos\varphi_i \sin\varphi_i} . \tag{20}$$

For a unidirectional GFRP the angle φ_0 at change of fracture modes 1-2 is about $5°$. For clarity, in such small range of angles the trigonometric functions can be represented by power series: $\cos^2\varphi \approx 1-\varphi^2$, $\cos^4\varphi \approx 1-2\varphi^2$, $\cos\varphi_0 \cdot \sin\varphi_0 \approx \varphi - 2\varphi^3/3$. The effective modulus of elasticity is estimated by the average value of $\cos^4\varphi$:

$$\frac{E_x^{\text{eff}}}{E_1^0} = \left(\cos^4\varphi\right)^* = \frac{\int_0^{\varphi_0} \cos^4\varphi \, d\varphi}{\varphi_0} = 1 - \frac{2}{3}\varphi_0^2 .$$

Strength $\sigma(\varphi)$ in the range $\varphi_i \le \varphi_0$ can be estimated in following form:

$$\frac{\sigma(\varphi)}{\sigma_x(\varphi)} = \frac{\sigma(0)E_1^0(\cos^4\varphi_0)^*}{\sigma(0)E_1^0(\cos^4\varphi_0)(\cos^2\varphi_0 + m_1\cdot\cos\varphi_0\cdot\sin\varphi_0)} \approx 1 - m_1\varphi_0 + 7\frac{\varphi_0^3}{3} - 2m_1\varphi_0^3 + o\left(\varphi_0^3\right) \approx 1.013 ,$$

for accepted values $m_1 \approx \varphi_0 \approx 0.1$. It means that for these composite material properties the "equal strength" condition is satisfied even with a small reserve. This ratio may be slightly less than one, but it is important to note that the strength reduction for small fiber misorientation is insignificant.

In order to "design" beam shape with the "equal strength" condition an iterative computational simulation of trajectories and rebuilding of beam shape by using at least two of the above criteria is necessary, and for more correct results another criterion for interlaminate shear fracture [9] $\sigma_x + m\cdot\tau_{xy} = c$ has to be added.

Decrease of the effective elastic modulus of the material is recommended for elastic elements such as leaf springs, if it does not decrease the strength. Simplified example shows that misalignment can have a positive effect in the elastic elements. It is important to note that a small misalignment is useful to increase of unidirectional composites splitting resistance, which is especially important during cyclic loading.

More rigorous estimates are obtained with use of the finite element method upon misoriented inhomogeneous structure and this method does not require the hypotheses of width and/or height uniform deformation.

4. Bending flexibility increase due to branching

The analogy of the bending flexibility of shaped beams with constant cross section area and branching composite structures like a treetop with a constant sum of branches section areas (Leonardo's rule) is studied. In ideal case shaping or branching provide threefold bend flexibility growth with strength retaining, i.e., threefold increase of accumulated elastic energy for the fixed applied load and the same mass of the elastic element. It is shown that the use of unidirectional GFRP (glass fiber reinforced plastic) in shaped elastic beams makes it possible to reduce mass approximately 15 times in comparison with the steel analog.

An interesting method for increasing the bending flexibility of the composite elastic elements is to observe the structure of the top of an apple tree. In notes of Leonardo da Vinci [11, 12] the following statement is already expressed: "The sum of the squares of the diameters of branches is the same before and after branching".

Let us imagine the simplest branching model, in which the console cylinder beam is loaded at the end with a concentrated force P (figure 3) branched into N similar cylinder rods that preserve the total section area as follows:

$$d^2 = N d_{N1}^2 \tag{21}$$

where d_{N1} is the diameter of each of the similar branches; the second suffix number indicates the number of the branch point, in this case it is only one. The distance from the seal in which it is reasonable to make the branch is selected based on the condition of equal maximum bending stresses in the root section and at the branch point as follows:

$$\frac{PL}{\pi d^3} = \frac{PL_{N1}}{N\pi d_{N1}^3} \Rightarrow L_{N1} = LN^{-1/2} \Rightarrow \lambda_{N1} = N^{-1/2} \tag{22}$$

So the bending stiffness changes after branching and specify the deflection of the initial cylinder beam is specified as v. After branching into N parts

$$v_{N1} = \frac{P}{E}\left[\frac{1}{I}\int_0^{L-L_{N1}} (L-x)^2\,\mathrm{d}x + \frac{1}{I_{N1}}\int_{L-L_{N1}}^{L} (L-x)^2\,\mathrm{d}x\right] = v\left[1+(N-1)\lambda_{N1}^3\right],$$

$$v = \frac{PL^3}{3EI},\ I = \frac{\pi d^4}{64},\ I_{N1} = N\frac{\pi d_{N1}^4}{64} = \frac{I}{N},\ \lambda_{N1} = \frac{L_{N1}}{L} \tag{23}$$

$$\delta_v = v_{N1}/v = 1+(N-1)N^{-3/2} \xrightarrow[N\to 1;\ N\to\infty]{} 1$$

As a result of branching, the tree becomes more flexible, which means it better resists the wind loads. It is interesting to note that branching does not provide any advantages of flexibility for either the trivial case ($N=1$ in (23)) or a large number of branches ($N\to\infty$). This means that there is some optimum number of branches for the maximum increase in the bending flexibility:

$$\mathrm{d}\delta_v/\mathrm{d}N = 0 \Rightarrow 2N^{3/2} - 3N^{1/2}(N-1) = 0 \Rightarrow N_{\text{opt}} = 3 \tag{24}$$

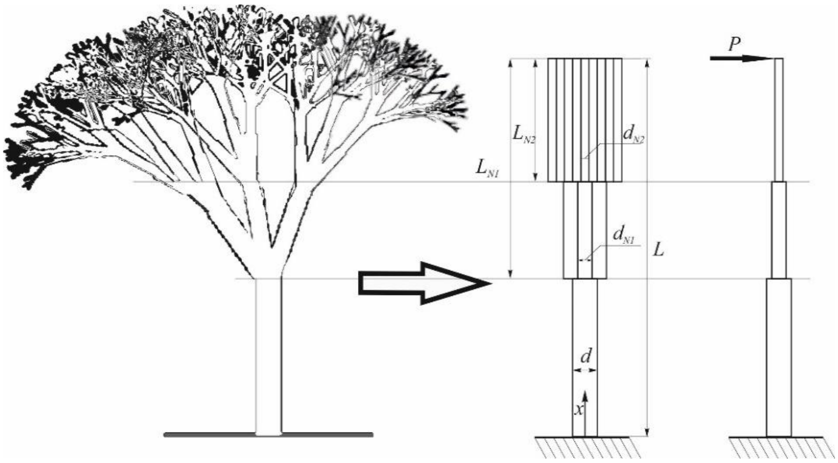

Figure. 3. Calculation model and pattern of variations in the number of branches for increasing flexibility under conditions of equal stress.

For $N=3$: $\lambda_{31}=3^{-1/2} \Rightarrow v_{31}/v=1+2/3^{3/2} \approx 1.38$. This is the maximum possible value of the deflection coefficient for retaining equal strength for one branch.

The flexibility increases with growth of branches number. A general formula can be written for n points of branching into N branches at each point while retaining the condition of equal strength (22) for each subsequent branch as follows:

$$\delta_v=\frac{v_{Nn}}{v}=1+\sum_{i=1}^{n}\frac{N^i-N^{i-1}}{N^{3i/2}}=1+\sum_{i=1}^{n}\frac{1-N^{-1}}{N^{i/2}}. \tag{25}$$

These power series certainly converge, and let us settle on the optimal case of $N=3$. Then, the following expression for additional deflection represents a geometric progression sum with the initial member $a_1=2/3\sqrt{3}$ and $q=1/\sqrt{3}$. The sum of this progression is: $a_1\left(1-q^n\right)/\left(1-q\right)$.

With unlimited number growth ($n\to\infty$) of equally strong branches into three branches ($N=3$), the deflection coefficient

$$\delta_v=\frac{v_{3n}}{v}\xrightarrow[n\to\infty]{}1+\frac{a}{\left(1-1/\sqrt{3}\right)}=\frac{2}{3\left(\sqrt{3}-1\right)}\approx 1.91.$$

This is the maximum coefficient of flexibility growth while retaining the strength that can be obtained during branching into a number (three) of similar branches. The situation is analogous to the case when the constarea beam profile (figure 1) could only be changed step by step, not smoothly.

In order to estimate the limit value of the deflection coefficient for continuous (fractal) branching, it is necessary to present a number N of branches as permanently changing value while retaining the total area $d^2=N(x)d^2(x)$ and equal stress $LN(x)d^3(x)=(L-x)d^3$. So $N(x)=\left(L/(L-x)\right)^2 \Rightarrow I(x)=I/N(x)$ and the deflection

$$v^*_{N(x)}=\frac{P}{EI}\int_0^L (L-x)^2 N(x)\,\mathrm{d}\,x=\frac{PL^3}{EI}=3v \tag{26}$$

Thus, in the ideal case, the branching cylinder rod with the same strength has three times greater flexibility compared to a homogenous rod (see (13)).

A branching composite beam can be used as an effective elastic element, e.g., in space-based structures (with no dimension limits), while for the fixed weight three times greater elastic energy is accumulated. If the weight efficiency of these elements is compared to the steel springs, then the low-modulus and high-strength GFRP can provide a gain by more than 15 times.

5. Conclusions

Traditional composite technology is based on the application of rectilinear (layout fabrics, prepregs) or spiral (winding bundles, belts) fibers trajectories. Therefore, methods of stiffness and strength engineering estimation of profiled composite components with complex fiber layup trajectories are not well developed. This article is intended to fill this gap. On the basis of the proposed approaches to the estimation of effective elastic modulus and strength, composite elements can be calculated taking into account the local variable misorientation of the fibers. The range of misorientation angles about 5^{o} leads to negligible refinements. Computer simulation of the trajectories of fibers and rather cumbersome calculation of stresses is necessary for strength evaluation in the general case. This will clarify the form of a «uniform strength» composite elements in comparison with the known homogeneous analogs.

Acknowledgments

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References

[1] Cho H R and Rowlands R E 2009 Optimizing fiber direction in perforated orthotropic media to reduce stress concentration *Journal of Composite Materials* **43** 1177–98
[2] Malakhov A V and Polilov A N 2016 Design of composite structures reinforced curvilinear fibers *Composites: Part A* **87** 23–8
[3] Malakhov A V and Polilov A N 2017 Design algorithm of rational fiber trajectories in arbitrarily loaded composite plate *J. Machinery Manufacture and Reliability* **46** 479–87
[4] Wegst U G K and Ashby M F 2007 The structural efficiency of orthotropic stalks, stems and tubes *J. Mat. Sci.* **42** 9005–14
[5] Polilov A N and Plitov I S 2015 Rational dimensions of segments of bamboo stems and composite tubes subjected to compression, flexure and torsion *J. Machinery Manufacture and Reliability* **44** 239–48
[6] Polilov A N, Malakhov A V, Tatus' N A and Shabalin V V 2011 Biomechanical principles of designing a quasi-unidirectional composite structures and their fasten places *Bul. Nizhny Novgorod Lobachevsky Univ.* **4** 2437–40
[7] Polilov A N, Tatus' N A and Shabalin V V 2011 Peculiarities of constructing elastic elements in the form of shaped composite beams *J. Machinery Manufacture and Reliability* **6** 532–7
[8] Polilov A N, Tatus' N A and Plitov I S 2013 Estimating the effect of misorientation of fibers on stiffness and strength of profiled composite elements *J. Machinery Manufacture and Reliability* **42** 390–7
[9] Polilov A N and Tatus' N A 2008 FRP strength criteria for some experimental effects substantiation *Eng. & Automation Probl.* **3** 103–10
[10] Minamino R and Tateno M 2014 Tree branching: Leonardo da Vinci's rule versus biomechanical models *PlosOne* **9** e9535
[11] Eloy C 2011 Leonardo's rule, self-similarity and wind-induced stresses in trees *Phys. Rev. Lett.* **107** 258101
[12] Polilov A N and Tatus' N A 2017 Designing branching or shaped composite elements by analogy with the structure of treetops *J. Machinery Manufacture and Reliability* **46** 385–93